

Research Paper

Investigation of the safety factor and reliability of the embankment in soil cement column improved ground on Saga lowland

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ABSTRACT

Saga plain constitutes thick deposited soft marine clay which is susceptible for the construction of any civil engineering structures with reference to large and differential settlement problems in the area. This paper presents the case study of the Ariake sea coastal road project failure on Ashikari region in Saga lowland. In this region, the group of soil cement columns as ground improvement technique is adopted for the construction of the highway embankment. The estimation of probability of failure of the embankment on the stabilized soils is approached through the probabilistic analysis for this research. The assessments of total safety factor (Fs) and reliability rely on random variables of geotechnical parameters. These random variables considered for this study are the unit weights of the soil materials (γ) (fill and clay), the undrained shear strength (c_u) in the embankment fill material and undrained shear strength of the stabilized soil and soft soil. Furthermore, the paper also delineates judgment of the geotechnical risk assessment based on real site condition.

1. Introduction

Geotechnical engineering always deals in a substantial way with the procedure of making decision under uncertainties and risks. The uncertainties encountered in geotechnical engineering are regularly related to unexpected values of physico-chemical and mechanical properties which is due to the unevenness of the properties in the soil deposit lying under the soil surface (Hino et al., 2012; 2014). Generally, high variability of design parameters enhances the risk of failures which indeed need to be compromised in order to

accomplish the project. Due to these reasons, geotechnical engineers need to pay attention for minimizing uncertainties which will strengthen the stability of the overall structure or project. The conventional geotechnical practices adopt the selection of safety factors based on experiences.

In Japan, the method of deep mixing method is used to increase the stability and the strength by reducing the settlement of highways or railway embankment. The geotechnical properties, (such as shear strength, unit weight, etc.) of the natural soil have a wide variation in

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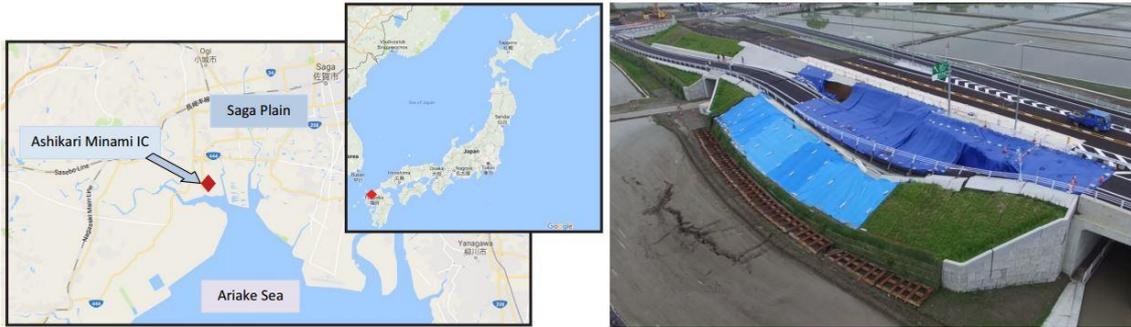


Fig. 1. The location of the failed embankment along the Ariake Sea Coastal Road (left) and the photograph representing the failed embankment along the Ariake Sea Coastal Road on June 23, 2016 (ASCRDO).

geology, physico-chemical properties and mechanical properties. Besides, the application of deep mixing method may also not achieve the uniformity due to the limitation of the deep mixing equipment. The variability of strength properties in the improved soil is considered to be the highest due to variations in sub-surface soil profiles and the complicated mixing process.

Reliability theory applied to geotechnical engineering and the calculation of the factor of safety (Fs) can provide a means of evaluating the combined effects of uncertainties and a method of distinguishing between conditions where uncertainties are particularly high or low (Duncan, 2000). The reliability method allows us to assess the probability of failure which is the assessment of the risk that the failure will occur. In terms of safety factor (Fs), the probability of failure is defined as the probability that the Fs could be less than 1.0 considering unfavorable values of the variables involved in its calculation.

This paper examines the reliability in the design of embankments founded on improved soil by soil cement columns together with assessing the uncertainties on the Fs. The reliability assessment assists to analyze the impact of the various sources of uncertainty associated with estimating the influence of parameters such as unit weight of the embankment fill material and undrained shear strength of the stabilized soil and the stability of the embankment. For this reference, the embankment constructed in Saga Lowland was considered for the research. The reliability analyses were performed using the Taylor series method, Duncan (2000).

The safety factor was computed using Bishop method (Bishop, 1955) for the reliability study. Moreover, this study used practices and concepts for the determination

of the properties and behavior of the stabilized soil and procedures for the geotechnical design of the soil cement columns (Kitazume and Terashi, 2013).

2. Failed Embankment along the Ariake Sea Coastal Road

The Ariake Sea is predominated with a very sensitive clay called Ariake Clay in northern Kyushu, Japan, where the top soil of the Saga plain is recognized with the same deposit. Generally, the thickness of the Ariake clay runs from 10 m to 20 m with maximum value of 30 m thick in some places. The Ariake Sea Coastal Road Project is a construction of a regional expressway with a length of 55 km between Ohmuta City in Fukuoka Prefecture and Kashima City in Saga Prefecture (**Fig. 1**). The thick layers of clay that underlies in this area are among the softest and highly sensitive clay layers in Japan. The deep mixing method mixes in situ soil with a hardening agent (cement, lime, slag, or other binders at depths by augers or blades) as the ground improvement technique to construct the embankment. The use of this ground improvement technique has been widely adopted in Japan in the last two decades and possesses large number of advantages (Chai et al., 2012; 2015; Hino et al., 2012; 2014).

However, it can have some limitations such as high variability in column strengths, relatively high mobilizing cost and lack of standardized quality control methods [9].

The site selection was chosen in Saga Prefecture along the Ariake Sea Coastal Road project near Ashikari Minami interchange in Saga lowland where the embankment was failed on June 23, 2016 (**Fig. 1**).

3. Methodology

3.1 Deterministic design for stability of the embankment stabilized by soil cement column

In Saga plain, the group of soil cement columns as ground improvement technique is adopted for the construction of the highway embankment. In general, embankment slopes are designed using shear strength parameters obtained from tests on samples of the selected material compacted to the design density. Because, the fills are generally built up in layers, the stability analyses of the embankment requires the analysis for all steps in the life of the project including: (a) all phases of construction (b) the end of construction (c) the long-term condition and (d) natural disturbances such as flooding and earthquakes (Abramson and Thomas, 2001).

The purpose of stability analysis is to determine the F_s of a potential failure surface. In this study, we focus on the assessment of the long-term stability of the embankment. In the design procedure for the group of column type improved ground, the slope stability failures are investigated in order to determine the strength of the stabilized soil column. The improvement area ratio using the allowable magnitude of safety factor of 1.3 is adopted for the static condition (Kitazume and Terashi, 2013).

The deterministic evaluation of the (F_s) for the stability of the embankment can be analyzed using limit-equilibrium methods, such as Fellenius method and simplified Bishop method together with the use of numerical analysis.

This paper inspects the slope stability for long-term stability of the embankment in soil cement column improved ground using Bishop method. In this method, failure is assumed to occur along a circular slip surface through the columns and the surrounding soil. The method is used for the determination of the safety factor for both the deterministic and the probabilistic analyses (reliability analysis) (Kitazume and Maruyama, 2006). **Fig. 2** represents the schematic diagram of the slip circle method of the highway embankment with the slice of 0.2 m.

The F_s can be calculated according to Bishop (1955) as follows:

$$F_s = \frac{\sum (c_i \cos \theta_i L_i + w_i \tan \phi_i) \cdot m(\alpha)_i}{\sum (w_i \sin \phi_i)} \quad [1]$$

where the term ($m(\alpha)_i$) in the Eq. (1) is

$$m(\alpha)_i = \cos \alpha_i + \frac{((\sin \alpha_i) \cdot \tan \phi_i)}{F_s}$$

In this study the slope stability analysis has been calculated with the help of Bishop method (Bishop, 1955) using the slice width of 0.2, the critical failure surface was considered corresponding to the surface initiated from the crack on the embankment surface as reported by the ASCRDO, where the failure occurred for the first time. The safety factor calculated using Bishop method is determined to be 1.41.

3.2 Evaluation of the Deterministic and Random Variables Parameters

3.2.1 Embankment (information of the construction of embankment)

The embankment was performed by clay fill material that was established as fill with cohesion of 5 kN/m² material with an internal angle of friction of $\phi_{emb} = 30^\circ$. The average value of total unit weight of the embankment fill was about 19 kN/m². The height of the embankment was 7.5 m. The dimensions of the width of short and long base of the embankment are 25.25 m × 48.7 m, (Fig. 2) and the slope gradient of the right side was initially 1:1.8 (V:H), the west direction, .Then the slope gradient along the left side was changed into 1:0.3 (V:H), the east direction. The traffic load was treated as a deterministic value and set at $Q = 10$ kN/m².

3.2.2 Soft Soil

The cohesion parameters of the wide distribution of the soft soil are varied from 12 kN/m² to 45 kN/m². The values of shear strength parameter were obtained from the unconfined compression tests. The required geotechnical parameters are incorporated in **Table 1**.

3.2.3 The stabilized soil column

The ground was improved by soil cement columns and each of the columns had a diameter of 1.2 m and length of 13 m. The columns were placed in a square pattern with a center-to-center distance of 1.9 m and their design unconfined compression strength (q_u) was determined to be 600 kN/m². The shear strength (C_u) of the soft ground is considered to be 300 kN/m², half of the value of the q_u .

In the soil cement improved ground, the soil cement columns are stronger and stiffer than the soft soil. This leads stress-strain incompatibility between the soft soil and the soil cement columns. Thus, it is required to consider the improved area as a composite ground with the average strength of stabilized soil columns and soft soil.

In the study, the composite ground consisting of the stabilized soil columns and the soft soil is considered to have an average strength (Kitazume and Terashi, 2013) defined by:

$$\tau_{comp} = a_s C_{us} (1 - a_s) \cdot k / C_{uu} \quad [2]$$

The term k introduced in the Eq (2) can be determined by using Eq (3) in which k is defined as:

$$k = \frac{C_{u0}}{C_{uu}} \quad [3]$$

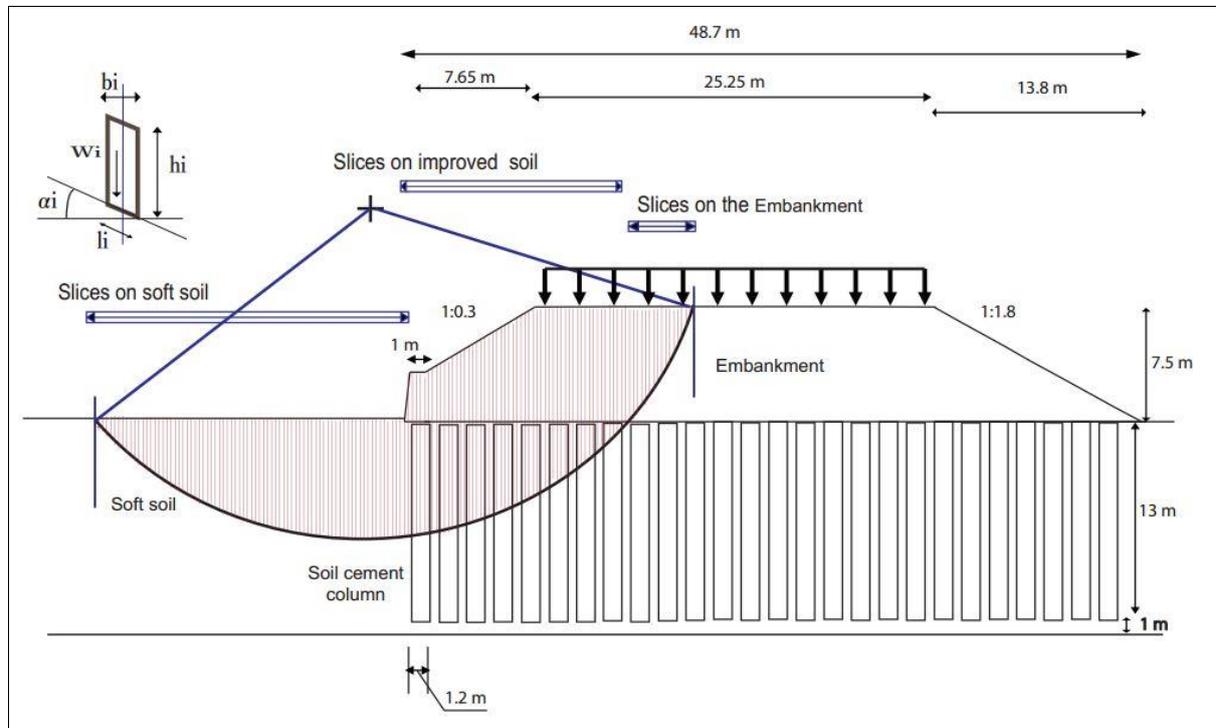


Fig. 2. Schematic representation of the slip circle of the highway embankment with the slice width of 0.2 m

Table 1 . The geotechnical parameters of the embankment

Geotechnical parameters	
Unit weight embankment, $\gamma_{t\text{fill}}$ (kN/m ³)	19
Shear strength of soft ground, $c_{u\text{soil}}$ (kN/m ²)	12
Shear strength of embankment, $c_{u\text{emb}}$ (kN/m ²)	5
Average shear strength of soil cement columns, c_{column} (kN/m ²)	300
Unit weight of SC columns, $\gamma_{t\text{soil}}$ (kN/m ³)	16
Improvement area ratio, a_s	0.3
The height of embankment, H (m)	7.5
The depth of improved area, D (m)	13
Unit weight of soft soil, γ_t (kN/m ³)	13
Internal friction angle of embankment (ϕ)	30

Using the laboratory test conducted in both the soft soil and the soil cement columns, the minimum value was then calculated. The value of the mobilization factor k is determined to be 0.584.

3.3 Estimation of the variability of the Random Variables

3.3.1 The uncertainties of the soil geotechnical parameter

In geotechnical engineering, it is almost impossible to predict the exact value of the mechanical and physico-chemical properties of the soil at any position, so, these parameters are described as random variables. Thus, geotechnical properties of the soft soil such as the unit weights of the soil materials (γ), the undrained shear strength (c_u) in the embankment fill material and undrained shear strength of the stabilized soil and soft soil were considered as random variables.

In most cases, the reliability of an engineering design confronts a great deal of the uncertainties and they are due to the high variability of the soil. Thus, the engineers often take decisions to deal with the irregularities based on the mechanical and the physico-chemical properties.

Therefore, it is necessary to evaluate the uncertainties in these geotechnical properties and make assessment of the geotechnical risk for the project.

In fact, Geotechnical engineering is characterized by the natural/intrinsic variability, measurement errors, and transformation uncertainties. There are three main sources of geotechnical uncertainties: (a) inherent variability, (b) measurement error and (c) transformation uncertainty (Phoon and Kulhawy, 1999a; 1999b), in which :

$$COV(total)^2 = COV(nat)^2 + COV(mesur)^2 + COV(trans)^2 \quad [4]$$

The natural variability is due to the diverse nature of soil which has been the result of association of multiple geological processes and environmental and chemical transformations. The measurement error is generally the effect of the operator, equipment, procedures and arbitrary testing outcomes. The transformation uncertainty is used when field or laboratory measurements are converted using empirical or correlation models into value of the soil parameters.

In the study, the published values of the variability for the soil geotechnical parameter were used. Thus, the minimum coefficient of variability according to the ranges of mean values and coefficient of variability of inherent, measurement error and soil variability were considered in this study (Phoon and Kulhawy, 1999a; 1999b).

3.3.2 *The uncertainties of the stabilized soil variability*

In this study, the variability of strength of soil cement column was assumed to be 15%. According to the Japanese accumulated data, the coefficient of variation in the field strength varies from 15 to 50% for the on-land wet method (Coastal Development Institute of Technology, 2008), (Kitazume and Terashi, 2013).

3.4 *Probabilistic Analysis for Stability of the Embankment*

In the stability analysis, it is common to use the deterministic method in the engineering design procedure. In the deterministic method, each input parameter has a fixed value. If the parameter varies in time or location and, the designer will select a more conservative value and use it for stability analysis. To ensure the safety of the design, a safety factor should be greater than 1, often superior to 1.5 is required (Duncan, 2000).

However, by using the probabilistic method, we acquire a more realistic approach in which each input parameter has a mean and a coefficient of variation obtained from field or laboratory tests or based on field measurements or past experience (Duncan, 2000).

Hence, the probabilistic analysis using Taylor series method was adopted for this research.

The Taylor Series method is a "first-order second moment" (FOSM) analysis. Only the first two "moments" (the mean and the standard deviation) are considered in the analysis. The application of the Taylor Series method in geotechnical engineering has been described by Wolff (1994), U.S. Army Corps of Engineering (1997) and Duncan (2000). The method requires an assumption on the distribution of the safety factor and the determination of coefficient of variability of the safety factor using values calculated through varying each design parameter by adding ± 1 standard deviation. Afterwards, the probability of failure and the reliability index was determined.

The Taylor Series method offers a relatively easier way to assess probabilities of failure, which is the strongest asset of this method. Furthermore, it is necessary for the Taylor Series method to assume the distribution of the safety factor and to determine the best form of this distribution. Silva et al. (2008) gave some empirical evidence that approve the lognormal distribution's assumption for the factor of safety (Phoon et al., 2016).

In the study, a lognormal distribution for the safety factor in the reliability analysis was incorporated. The reliability index can be calculated using the Eq (5) where F_s and COV represent the safety factor determined by the deterministic method and coefficient of variability of the safety factor (Phoon and Ching, 2015). The probability of failure can be calculated using the reliability index by utilizing the equation using Excel program:

$$\text{Reliability index} = \beta(\text{lognormal}) = \frac{\ln \left(\frac{F_s}{\sqrt{1 + (COV)^2}} \right)}{\sqrt{\ln(1 + (COV)^2)}} \quad [5]$$

$$\begin{aligned} \text{The probability of failure} &= P_f(\text{lognormal}) \\ &= 1 - \text{NORMSDIST}(\beta \text{ lognormal}) \end{aligned} \quad [6]$$

4. Results and discussions

4.1 Variability in the random variables

4.1.1 Embankment

The coefficient of variation (COV_{γ} and $COV_{c_{u,emb}}$) was adopted from Table 2 according to Phoon and Kulhawy (1999a; 1999b) as guidelines.

4.1.2 Soft Soil

Since the number of tests performed was less, the coefficients of variation ($COV_{c_{u,soil}}$) were used according to Phoon and Kulhawy (1999a; 1999b) guideline. In these guidelines (Phoon and Kulhawy, 1999a; 1999b) have developed the range of coefficient of variation. (Table 2)

4.1.3 The stabilized soil variability

The variability of the soil cement columns was estimated to be equal to $COV_{c_{u,colm}} = 15\%$, which can be considered as reasonable. The mean of the undrained shear strength of the soil cement columns were evaluated as the half of strength $C_u = q_u/2$.

4.2 Reliability Analyses

Reliability analyses were performed as shown in Fig 2, by considering slices for the three different parts: soft soil, improved soil and embankment. Thus, each section has a different shear resistance contributed to the total strength.

In computing, the uncertainty in safety factor against slip circle failure, the variability of the parameters, unit weight of embankment, average shear strength of soil cement columns, shear strength of soft ground and shear strength of embankment are considered. All these parameters involve some degree of uncertainty. Therefore, the computed value of F_s also involves some uncertainty. It is useful to assess the reliability of F_s , as well as the best estimate of its value. This can be done using the Taylor series method, which is illustrated in the Table 3.

Using the Taylor series method, the coefficient of variation could be estimated the safety factor as 15.6 %. Hence, the reliability index β can be computed as 2.137 and the probability of failure as 1.63% which means it has 98.37% reliability referring equations (5) and (6).

Table 2 The coefficient of variability of geotechnical parameters used in the analysis

Geotechnical parameters	COV(total)
Unit weight embankment γ_{fill} (kN/m ³)	10.2%
Average shear strength of soil cement columns $c_{u,colm}$ (kN/m ²)	15%
Shear strength of soft ground $c_{u,soil}$ (kN/m ²)	12.81%
Shear strength of embankment $c_{u,emb}$ (kN/m ²)	12.81%

Table 3: The Taylor series method and the reliability analysis for the embankment .

Variable	Values			ΔF
Unit weight embankment				
Value plus σ	20.938	F+	1.298	-0.27393717
Value minus σ	17.06	F-	1.572	
Shear strength improved soil				
Value plus σ	108.4	F+	1.58	0.34137885
Value minus σ	81.4	F-	1.239	
Undrained shear strength of soft soil				
Value plus σ	13.537	F+	1.434	0.04734534
Value minus σ	10.463	F-	1.386	
Embankment shear strength				
Value plus σ	5.640	F+	1.4124	0.00466101
Value minus σ	4.3597	F-	1.4078	
Standard deviation of factor of safety		Coefficient of variation of factor of safety		
σ_4	COV Fs			
0.220138819	0.15612111	15.6 %		

Table 4: The reliability index, probability of failure and the embankment performance level [17]

Expected Performance Level	Reliability indices	Probability of Unsatisfactory Performance
High	5	0.0000003
Good	4	0.00003
Above average	3	0.001
Below average	2.5	0.006
Poor	2	0.023
Unsatisfactory	1.5	0.07
Hazardous	1	0.16

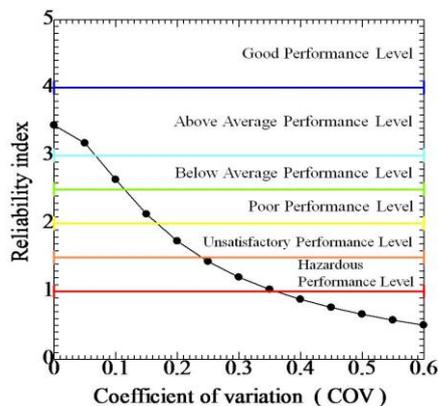


Fig. 3. Influence of the variability of the shear strength of the soil cement column on the reliability index and the embankment performance level

4.3 The risk of failure and performance level,

In order to make the assessment of the stability of the embankment and their performance level, we used U.S. Army Corps of Engineering (1997) in which they have made specific recommendations about target reliability indices and targeted probabilities of failure with their related expected performance levels in geotechnical, water resources and infrastructure projects. Table 4 shows the reliability indices ranging from 1 to 4, the probability of failure that varies from 0.00003% to 16 % and the expected performance levels from hazardous to high performance level (Phoon and Retief, 2015).

According to Table 4 the embankment has a poor performance level which means that the embankment is expected to perform poorly and present major reliability issues (U.S. Army Corps of Engineering, 1997). In terms of slope instability, the probability of unsatisfactory performance is around 0.015, it means that 15 of every 1000 instabilities will result in damage which causes a safety hazard.

The Fig. 3 shows the influence of the variability of the shear strength of the soil cement column on the reliability index and the performance of the embankment. According to the graph, the variability of the shear strength of the soil cement column have a significant influence on the reliability and the performance level of the embankment.

5. Conclusions

This paper presents a reliability analysis for slope stability and an estimation of the risk of failure of the embankment using the Taylor series method and the Bishop method in conjunction with general procedure of limit equilibrium of slope stability analysis. The paper also outlines the effects of uncertainties and their estimation on the assessment of performance and the risk of failure of the embankment.

Although several methods can be applied in reliability analysis the Taylor series method was adopted for the analyses, because of its simplicity and its ease of its applicability. The Taylor series method offers way to calculate the reliability index and the probability of failure of the embankment in term of slope stability. Thus, the results provide a reasonable way to take into account the uncertainties in geotechnical design properties and especially the relatively high variability of the soil cement columns.

The results show that the failed saga embankment present low level of performance and have reliability issues in long terms of slope stability which may also be susceptible to fail in case of natural disaster. Since the stability of the embankment is significantly influenced by the strength properties of the improved part, further research should focus on determining the variability of soil cement columns and their influence of the risk of failure.

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Engineering, Forensic Geotechnical Engineering, Developments in Geotechnical Engineering, Springer India, 467 – 491.		Uncertainty
Phoon K.K. and Ching J., 2015. Risk and Reliability in Geotechnical Engineering, Taylor & Francis Group, 167 – 172.	$COV_{(nat)}$	The coefficient of variation of inherent variability of the measured property
Phoon K.K. and Retief J.V., 2015. Reliability of Geotechnical Structures in ISO2394, Taylor & Francis Group, 210 – 211.	$COV_{(trans)}$	The coefficient of variation of transformation uncertainty of the transformation model;
Phoon, K.K. and Kulhawy, F.H., 1999a. Characterization of geotechnical variability. Canadian Geotechnical Journal, 36 (4): 612–624 .	h	Height of a slice
Phoon, K.K. and Kulhawy, F.H., 1999b. Evaluation of geotechnical property variability. Canadian Geotechnical Journal, 36 (4): 625– 639.	i	Number of slices
Silva .F., Lambe, T.W, and Marr, W.A., 2008. Probability and risk of slope failure. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 134 (12): 1691– 1699.	j	Number of slices
U.S. Army Corps of Engineers., 1997. Introduction to probability and reliability methods for using in geotechnical engineering, ETL 1110-2-547.	li	Length of slice i
Wolff, T.F., 1994. Evaluating the reliability of existing levees, Prepared for the US Army Engineers ,Waterways Experiment Station Geotechnical Lab, Vicksburg, MS.	θ_i	Inclination angle
	wi	Weight of the slice i
	γ	Unit weight
	ϕ	Friction angle
	Q	Traffic load
	c_u	Undrained shear strength
	σ	Standard deviation
	μ	Mean value
	τ	The shear strength the composite ground consisting of stabilized soil columns and unstabilized soil;
	K	The value of the mobilization factor
	C_{uu}	Undrained shear strength of soft soil
	C_{u0}	Undrained shear strength of soft soil mobilized at the peak shear strength of stabilized soil
	C_{us}	Undrained shear strength of stabilized soil

Symbols and abbreviations

COV	Coefficient of variation
Fs	Factor of safety
B	Reliability index
as	Improvement area ratio;
$COV_{(total)}$	The total coefficient of variation of the design property
$COV_{(mesur)}$	The coefficient of variation of measurement

Subscripts

emb	Embankment
nat	Natural
mesur	Measurement
trans	Transformation
colm	Soil cement columns
comp	Composite materials (soil and columns)