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MODIFIED HYPERBOLIC MODEL FOR CAPTURING UNDRAINED SHEAR BEHAVIOR

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ABSTRACT: The hyperbolic and modified hyperbolic models are proposed to predict the stress~strain response of the uncemented, naturally cemented and induced cemented clays under undrained shear so as to suggest the simple analysis and assessment. The hyperbolic responses of stress ratio and shear strain (η, ε_s) as well as of effective mean principal stress and shear strain (p', ε_s) are introduced to assess the undrained shear behavior of the uncemented and naturally cemented clay. The modified hyperbolic responses are employed for the induced cemented clays. These models consist of the parameters, which control the constitutive behavior of the uncemented, naturally cemented, and induced cemented clays in undrained situation. These parameters are easily determinable from standard triaxial tests. The predicted and laboratory responses are in good agreement.

Key Words: Cemented clay, cement admixed clay, modified hyperbolic model, undrained shear behavior

INTRODUCTION

The inherent nature and diversity of the geotechnical process involved in soil formation are responsible for the wide variability in the in-situ state of soils. The stress to which they have been subjected to, the environment in which they are formed, and the time that has elapsed in the geotechnical time scale at different stages of their formation have been recognized as potential factors to impart their effects to the soft clay deposits. Due to the time effect, the soil is left to age and creep; hence the bonds develop at particle contacts and it possibly turns into soft rock. The bond is called "Natural Cementation Bond". For instance, the natural soft clay at normally consolidated state is stable at high void ratio with low compressibility and possesses the apparent pre-consolidation stress higher than the present in-situ vertical pressure.

For the improvement of the soft ground by the chemical admixture such as in-situ deep mixing technique, the natural clay is disturbed by mixing wings and mixed with cement or lime. The natural cementation is destroyed and taken over by the admixture cementation. The clay-cement mixture is called "Cement Admixed Clay" or "Induced Cemented Clay".

The explicit nature of stress-strain response of the cemented soils both naturally and induced cemented soils mostly depends on fabric and nature of bonding (Horpibulsuk et al., 2003) in addition to the usual factors such as current state, stress path and drainage conditions. Despite the availability of a good number of constitutive relations concerning the behavior of clays, there are still a large number of problems which have not been satisfactorily tackled. Most often cemented soils are treated as overconsolidated soils because they also exhibit similar features like strain softening, higher initial stiffness, etc. But a recent study revealed that the undrained shear behavior is very different from that of the overconsolidated uncemented soils (Miura et al., 2001 and Horpibulsuk et al., 2004). Strain softening is observed in relation to cemented samples under effective confining pressures even far higher than the mean effective yield stress. Most important difference is that the softening is associated with positive pore pressure in undrained shear and with positive volumetric strain in drained shear, whereas the same would not happen in case of the uncemented clay samples. Many researchers have explained this behavior by using the theory of plasticity (Nayak and Zienkiewicz, 1972; Prevost and Hoeg, 1975; Bannerjee and Stipho, 1979; Dragon and Mroz, 1979; Kawahara et al., 1981; Matsumoto and Ko, 1982). All the above models are basically enveloped for overconsolidated uncemented soils and do not address the special problem of softening in cemented soils. However, more recently Oka et al. (1989) proposed a constitutive model for natural soft clay with strain softening by extending Oka and Adachi's (1985) approach. Vatsala (1989)

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hypothesized that the load carrying capacity of cemented soils can be split into two components: an uncemented or remoulded resistance and cementation bond resistance. However, the deformation of soil is essentially due to changes in stress increments on equivalent unbonded soil skeleton. Gens and Nova (1993), Adachi and Oka (1995) pointed out that for modeling the behavior of cemented soil, it is necessary to consider the behavior of an equivalent uncemented state. Most of the constitutive variables involved in these plasticity based models are intricate and require elaborate experimental program. Thus, there is a need for development of a realistic and simple model comprising of easily determinable constitutive parameters which are capable of capturing the most important aspects such as strain softening behavior of soft clays.

The hyperbolic non-linear elastic model was first applied for the cohesive soils by Kondner (1963) as well as Duncan and Chang (1970). It is widely used in finite element applications even to date because of its simplicity. However, Kondner's approach does not address the strain softening behavior, which is the typical of naturally and induced cemented clays. The aim of this paper is to present the simple practical procedure for representing the softening behavior of cemented clays during undrained shear with constants being determined by simple laboratory test and subsequently used for the stress condition. Depending on the specific field situation, it may be possible to analyze the problem with much simpler model.

SOIL SAMPLE AND EXPERIMENTAL STUDY

Ariake clay in Saga, Japan collected from a depth of 2 m was used for this experimental investigation. This soft soil is gray silty clay generally composed of 55 percent clay, 44 percent silt and only 1 percent sand. The clay is highly plastic with natural water content of 140 percent. The bulk density and specific gravity of the soils are 14.0 kN/m^3 and 2.60, respectively. The liquid and plastic limits are in the order of 120 and 57 percent, respectively. The groundwater is located at about 1 meter from the ground surface, leading to the effective in-situ overburden pressure of about 18 kPa. The apparent pre-consolidation pressure is 40 kPa. The undrained shear strength (S_u) is 20 kPa and the effective strength parameters in compression are $c' = 0$ and $\phi' = 38°$. The pH of the pore water is about 8.8 and the salinity is about 1.0 g/l.

To obtain the induced cemented clay samples, the clay paste was passed through 2-mm sieve for removal of shell pieces and other bigger size particles. The water content of the clay was adjusted to 180%. This intentional increase in water content is to simulate water content increase taking place in wet method of dispensing cement admixture in deep mixing and the significant increase in water contents taking place in jet grouting. The clay along with Type I Portland cement at cement content, C, varying from 6% to 18% is thoroughly mixed to obtain a uniform dispersion in the slurry. Thus the clay water/cement ratio, w_c/C, which is defined as the ratio of clay water content to cement content, varies from 30 to 10. It is shown that this parameter is the prime parameter governing the engineering bahvior of cement admixed clay (Miura et al., 2001 and Horpibulsuk et al., 2003). The lower the w_c/C, the greater the strength and yield surface. The mixing time was arbitrarily fixed at 10 min as has done by Miura et al., 2001; and Horpibulsuk, 2001. The uniform paste was then transferred to cylindrical containers of 50 mm diameter and 100 mm height, taking care to prevent any air entrapment. After 24 hours the cylindrical samples were dismantled. All the cylindrical samples were wrapped in vinyl bags and stored in a humidity chamber of constant temperature (20±2°C) and cured for 28 days.

Isotropically consolidated undrained triaxial compression (CIUC) tests were run on all samples. A backpressure of 190 kPa was maintained to ensure high levels of degree of saturation at all levels of testing. The rate of compression was fixed at 0.0075 mm/min. All the tests were conducted according to the procedure recommended by Head (1998).

ANALYSIS OF TEST DATA

The test results are analysed using the effective mean normal and deviator stress parameters $p'$and q'as given by:

$$p' = \frac{\sigma_1' + 2\sigma_3'}{3} \quad (1)$$

$$q = \sigma_1' - \sigma_3' \quad (2)$$

where σ'_1 is axial effective stresses on a cylindrical sample, and σ'_3 is radial effective stresses on a cylindrical sample.

The shear strain, ε_s (%) and volumetric strain, ε_v (%) are expressed by:

$$\varepsilon_s = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \tag{3a}$$

$$\varepsilon_v = (\varepsilon_1 + 2\varepsilon_3) \tag{3b}$$

where ε_1 is an axial strain, ε_3 is a radial strain. For undrained test $\varepsilon_v = 0$ and hence $\varepsilon_s = \varepsilon_1$.

EFFECT OF STRESS LEVEL ON STRESS-STRAIN CHARACTERISTICS

The relationships between void ratio and mean effective stress (e, log p') of the induced clay at different cement contents (C) are presented in Fig. 1. The mean effective yield stresses, p'_y values are 60, 220, 380 and 1800 kPa for samples at cement contents of 6, 9, 12 and 18 percent, respectively.

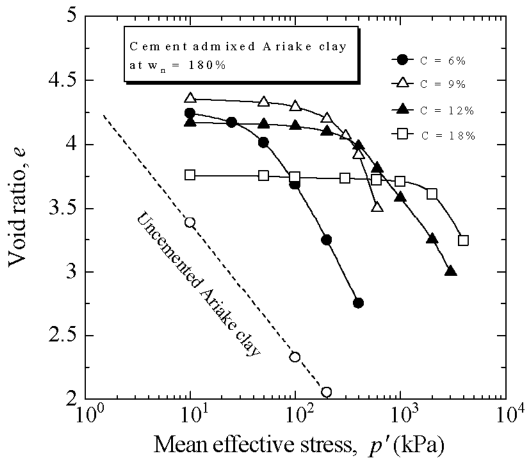

Fig. 1 Plots of (e, log p') of cement admixed clay at initial water content of 180%.

The stress level is an essential parameter controlling the stress-strain characteristics of uncemented and cemented clays (Nagaraj and Miura, 2001 and Horpibulsuk et al., 2004). Horpibulsuk et al. (2003 and 2004) revealed that undrained shear behavior of normally consolidated uncemented clay is mainly dependent upon the clay fabric. The dismembering of the clay clusters in the fabric brings about the interlocking when the clay is in overconsolidated state. For the cemented clay, both in naturally or induced cemented states, the stress-strain-strength characteristics are governed by the fabric and cementation. At the effective confining pressures lower than the yield stress, the (q, ε_s) relation is irrespective of effective confining pressure same whereas the (Δu, ε_s) changes as the effective confining pressure varies. This is because the change in fabric in consolidation process is less. At effective confining pressures higher than the yield stress, the (q, ε_s) and (Δu, ε_s) relations are dependent upon the effective confining pressure owing to the large change in fabric during consolidation. However, at both stress levels the stress softening would be seen due to the break-up of the clay sample after peak stress.

UNCEMENTED AND NATURALLY CEMENTED ARIAKE CLAYS

It has been known that for the naturally cemented clay, the hyperbolic relations of (q, ε_s) and (Δu, ε_s) are not evident since the softening behavior is recognized even at post-yield state (Nagaraj and Miura, 2001). However, it is herein found that the (η, ε_s) and (p', ε_s) at post-yield state show the hyperbolic relation similar to that of uncemented clay (*vide* Figs. 2 – 5) in which η is the stress ratio.

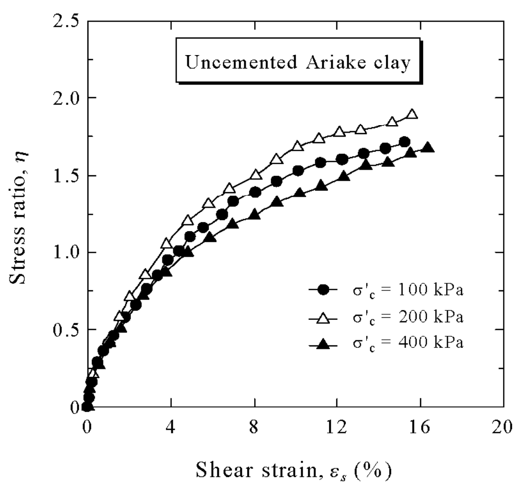

Fig. 2 Stress ratio~shear strain relationships of uncemented Ariake clay.

The hyperbolic manner merits analyzing the stress~strain and pore pressure response under the undrained shear. The variations of the stress ratio, η and the mean effective stress, p' with the shear strain in terms of the hyperbolic relation take the form as

$$\eta = \frac{\varepsilon_s}{a_1 + b_1 \varepsilon_s} \tag{4}$$

$$(p'_0 - p') = \frac{\varepsilon_s}{a_2 + b_2 \varepsilon_s} \qquad (5)$$

where p'_0 is the initial mean effective stress (kPa), and a_1, b_1, a_2, and b_2 are constants.

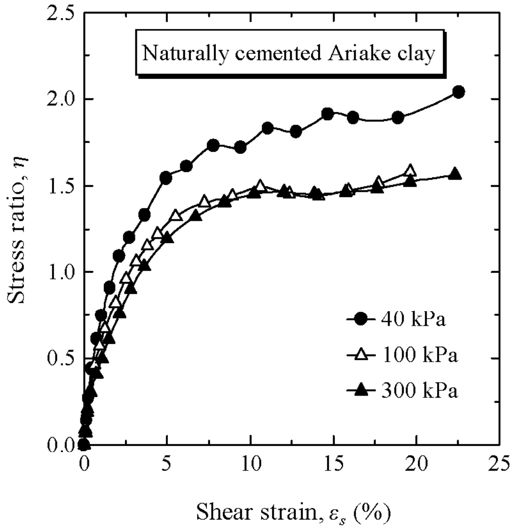

Fig. 3 Stress ratio~shear strain relationships of naturally cemented Ariake clay.

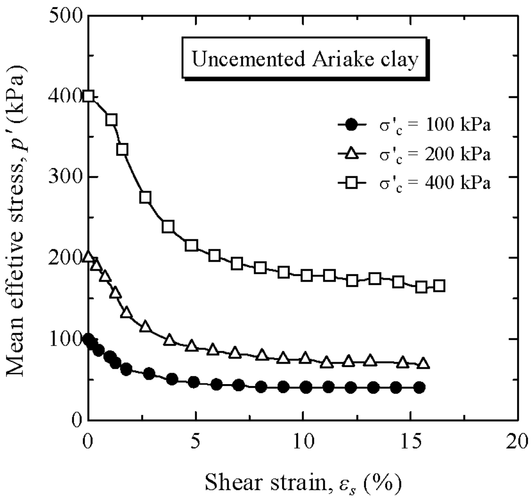

Fig. 4 Mean effective stress~shear strain relationships of uncemented Ariake clay.

The higher the a_1 and b_1, the lower the slope of the (η, ε_s) relation and the value of the stress ratio. Similarly, the higher the a_2 and b_2, the lower the slope of the (p', ε_s) relation and the change in mean effective stress. The parameters can be obtained by plotting ($\frac{\varepsilon_s}{\eta}$, ε_s) and ($\frac{\varepsilon_s}{(p'_0 - p')}$, ε_s) relations. The plots are shown in Figs. 6 through 9.

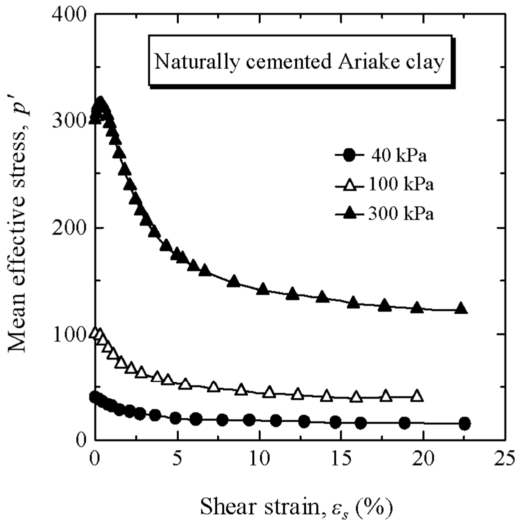

Fig. 5 Mean effective stress~shear strain relationships of naturally cemented Ariake clay.

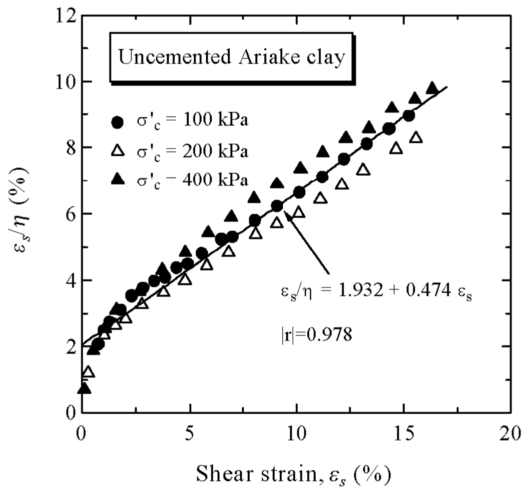

Fig. 6 Transformed stress ratio and shear strain of uncemented Ariake clay.

When the parameters in Eqs. (4) and (5) are determined, the (q, ε_s) and the (Δu, ε_s) relations can be assessed. In the conventional triaxial test, the excess pore water pressure (Δu) at any shear strain is calculated from $\Delta u = p'_d - p'$ where p'_d is the mean effective stress along the drained path and equal to $q - p'_0/3$.

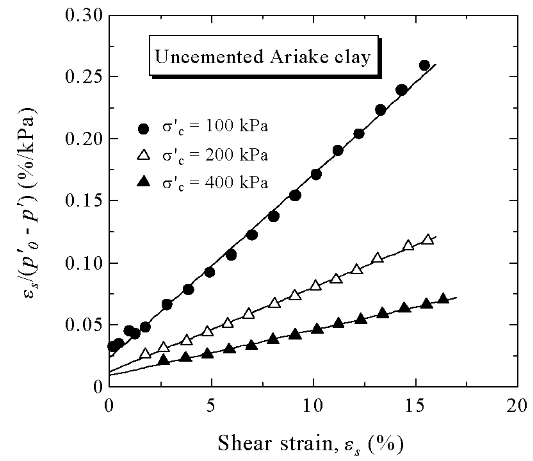

Fig. 7 Transformed mean effective stress and shear strain of uncemented Ariake clay.

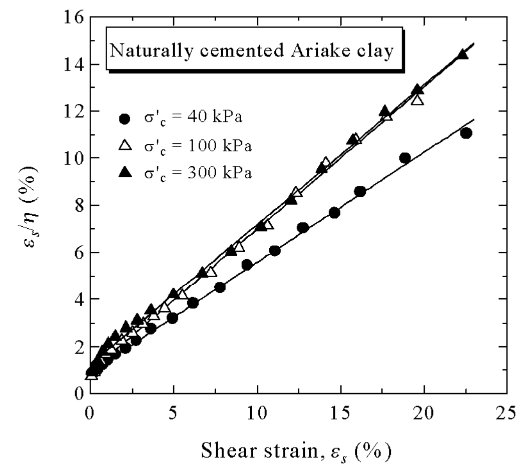

Fig. 8 Transformed stress ratio and shear strain of naturally cemented Ariake clay.

For meaningful application of the relations, it is preferable to determine the exact nature of the constants in terms of effective mean stress, p'. It is found that the hyperbolic model can be well applied for the uncemented and naturally cemented samples. This application is very useful for the normally consolidated uncemented (remolded) clay because the stress ratio~shear strain relationships of all samples exhibit the same feature (Atkinson and Bransby, 1978). The slight difference in the locations of the curves is probably due to the testing error which is acceptable. This leads to the same set of parameters (a_1 = 1.932 and b_1 = 0.474) being applied. The parameters a_2 and b_2 are presented in terms of initial effective confining pressures in a form of a power function as follows (*vide* Fig. 10).

$$a_2 = 0.572\left(p'_0\right)^{-0.706} \tag{6a}$$

$$b_2 = 1.438\left(p'_0\right)^{-1.000} \tag{6b}$$

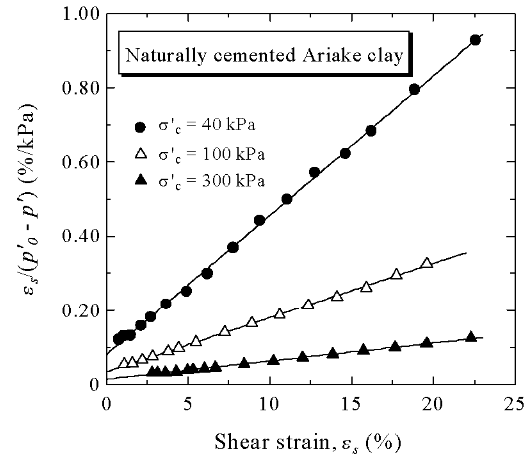

Fig. 9 Transformed mean effective stress and shear strain of naturally cemented Ariake clay.

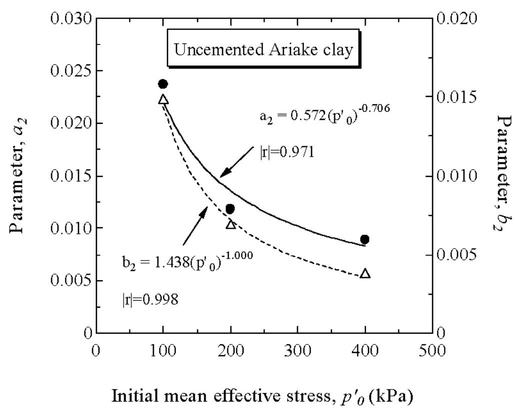

Fig. 10 Variation of parameters a_2 and b_2 with initial mean effective stress for uncemented Ariake clay.

For the naturally cemented clay, the (η, ε_s) relationship is not unique due to the cementation effect. The relations of these constants in terms of p' are shown in Equations 7 and 8, as well as Figs. 11 and 12.

$$a_1 = 0.281 + 0.166\ln\left(p'_0\right) \tag{7a}$$

$$b_1 = -0.018 + 0.197\ln\left(p'_0\right) \tag{7b}$$

$$a_2 = 1.051\left(p'_0\right)^{-0.718} \tag{8a}$$

$$b_2 = 1.613\left(p'_0\right)^{-1.020} \tag{8b}$$

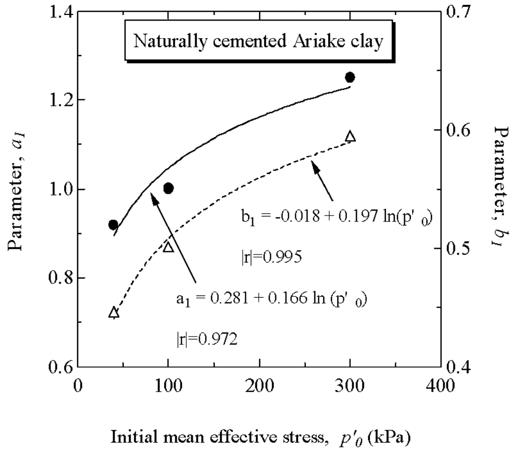

Fig. 11 Variation of parameters a_1 and b_1 with initial mean effective stress for naturally cemented Ariake clay.

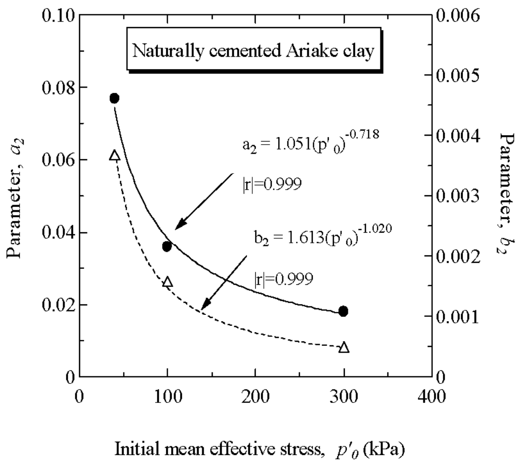

Fig. 12 Variation of parameters a_2 and b_2 with initial mean effective stress for naturally cemented Ariake clay.

It must be kept in mind that the relationships between parameters and the effective stress are valid only in the range of effective confining pressure considered. Figs. 13 and 14 show the predicted (q, ε_s) and (Δu, ε_s) relations of the uncemented and naturally cemented samples. It is shown that the predicted curves are in agreement with the experimental curves moreover the strain softening behavior can be captured.

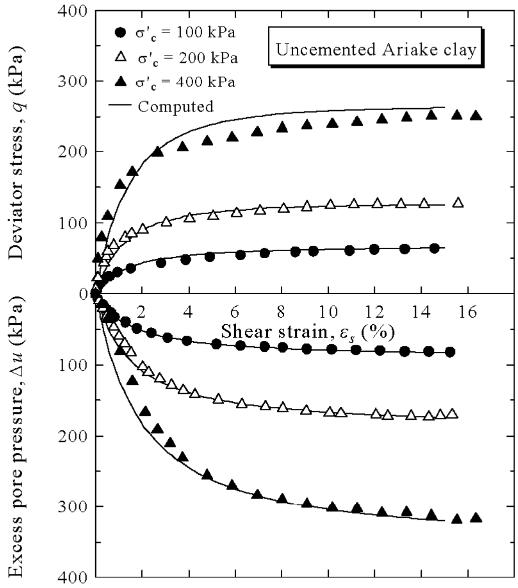

Fig. 13 Calculated and experimental (q–Δu–ε_s) curves for uncemented Ariake clay.

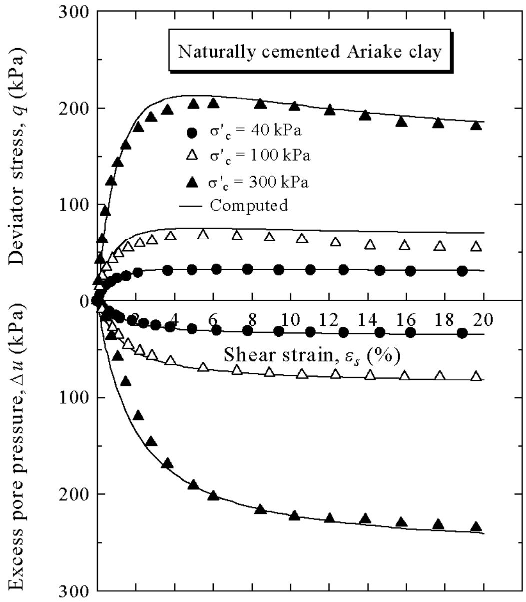

Fig. 14 Calculated and experimental (q–Δu-ε_s) curves for naturally cemented Ariake clay.

CEMENT ADMIXED ARIAKE CLAY

The prediction of undrained behavior of the cement admixed Ariake clay samples made up at cement contents of 6% and 18% is being presented.

For 6% cement samples, since the w_c/C is so high, the amount of cement is not enough to harden the clay. The role of the cement is only to weld the clay fabric, resulting in the increase in yield stress and strength (Nagaraj et al., 1998; and Nagaraj and Miura, 2001). As such, they can be considered as the highly naturally cemented clay and the analysis would herein be done at post-yield state. The other can be regarded as the elements of the soil-cement columns. Due to the aim of this presentation is to aid geotechnical engineers to predict the undrained behavior of the soil-cement columns, the prediction would be done only for the induced cemented samples subjected to the effective confining pressures in the engineering practice (σ'_c < 400 kPa).

The cement admixed samples at low cement content of 6% follow hyperbolic relation in mean effective stress~shear strain (p', ε_s) variation, whereas it is initially hyperbola and softens after the peak state for high cement content samples at 18% cement (*vide* Figs. 15 and 16).

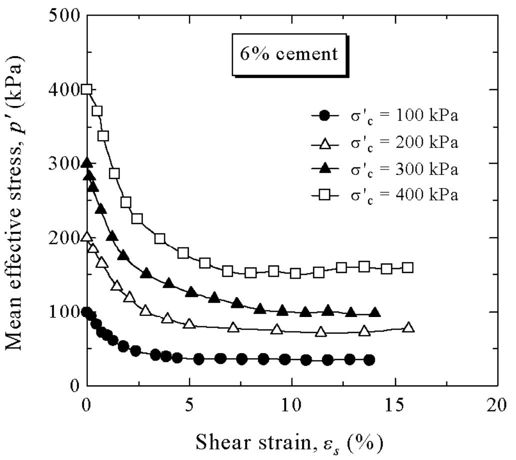

Fig. 15 (p', ε_s) relationships of cement admixed Ariake clay at 6% cement.

For the stress ratio~shear strain (η, ε_s) relation, it found that the hyperbolic relationship with softening behavior is evident for the cemented samples (*vide* Figs. 17 and 18). As a result, the modified hyperbolic model is required in this analysis. The analysis based on the (p', ε_s) and (η, ε_s) relations is more advantage than that based on the (q, ε_s) and (Δu, ε_s) relations because all values of mean effective stress and stress ratio are positive even after the peak values, whereas the negative pore pressure is recognized for the samples subjected to very low effective confining pressures. The modified hyperbolic relationship cannot be applied to assess the negative value. The variation of stress ratio and the mean effective stress with the shear strain in terms of modified hyperbolic relation takes the form as:

$$\eta = \frac{\varepsilon_s}{a_1 + b_1 \varepsilon_s^{n_1}} \tag{9}$$

$$(p'_0 - p') = \frac{\varepsilon_s}{a_2 + b_2 e_s^{n_2}} \tag{10}$$

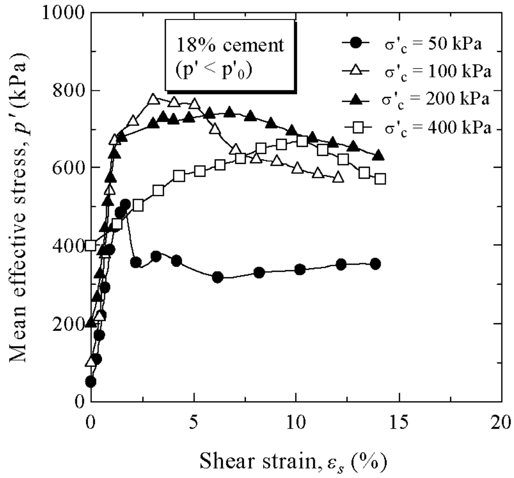

Fig. 16 (p', ε_s) relationships of cement admixed Ariake clay at 18% cement (p' < p'_0).

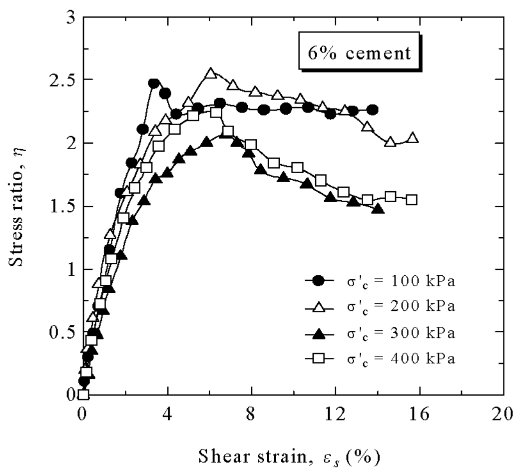

Fig. 17 (η, ε_s) relationships of cement admixed Ariake clay at 6% cement.

For 6% cement samples, as the hyperbolic model can be applied to the (p', ε_s) relationship, n_2 of Equation (10) is taken as unity. The modified hyperbolic relation is applied to the (η, ε_s) relationship with n_1 of 2.0. The variation of parameters with the mean effective stress is as shown below and Figs. 19 and 20.

$$a_1 = 0.284 p_0'^{0.269} \quad (11a)$$

$$b_2 = 0.035 \quad (11b)$$

$$a_2 = 0.002 \left(p_0'\right)^{-0.665} \quad (12a)$$

$$b_2 = 1.582 \left(p_0'\right)^{-1.108} \quad (12b)$$

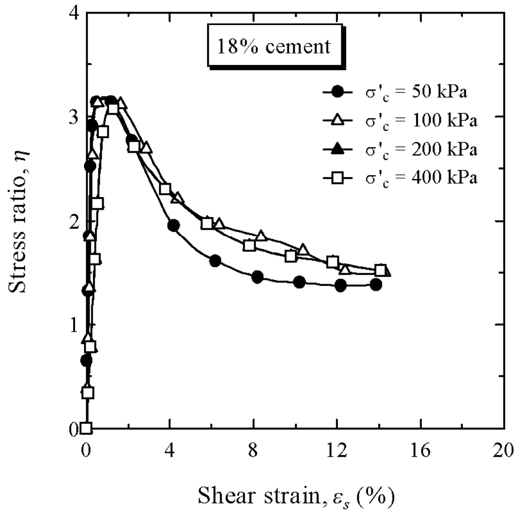

Fig. 18 (η, ε_s) relationships of cement admixed Ariake clay at 18% cement.

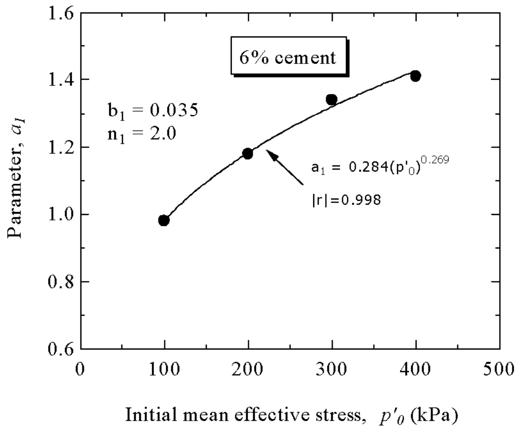

Fig. 19 Variation of parameter a_1 with initial mean effective stress for 6% cement samples.

It is now to examine the variation in the parameters for very high degree of cementation. The investigation is done by employing the results of the 18% cement samples at effective confining pressures ranging from 50 to 400 kPa.

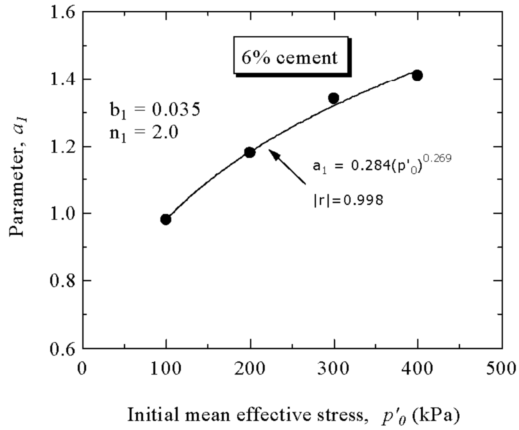

Fig. 19: Variation of parameter a_1 with initial mean effective stress for 6% cement samples.

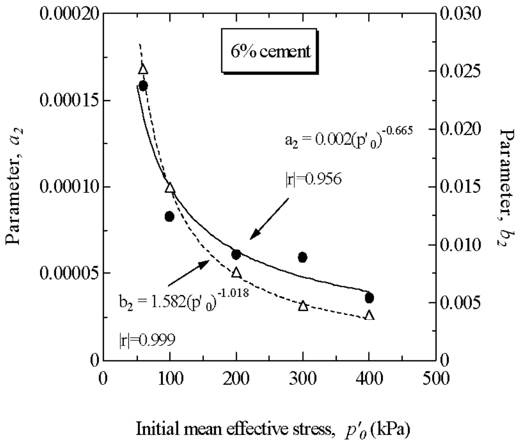

Fig. 20 Variation of parameters a_2 and b_2 with initial mean effective stress for 6% cement samples.

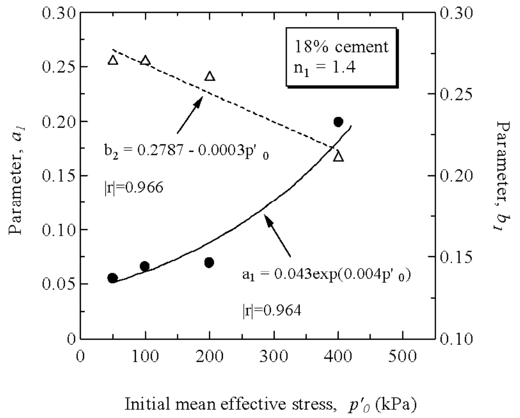

Fig. 21 Variation of parameters a_1 and b_1 with initial mean effective stress for 18% cement samples.

From the analysis, the n_1 and n_2 can be taken as 1.4 for all samples. The other parameters are introduced as below, as well as Figs. 21 and 22.

$$a_1 = 0.043\exp(0.004p'_0) \quad (13a)$$

$$b_1 = 0.2787 - 0.003(p'_0) \quad (13b)$$

$$a_2 = 6\times10^{-4}(p'_0)^{-0.0069} \quad (13a)$$

$$b_2 = 1.0\times10^{-5} + 2.5\times10^{-4}\ln(p'_0) \quad (13b)$$

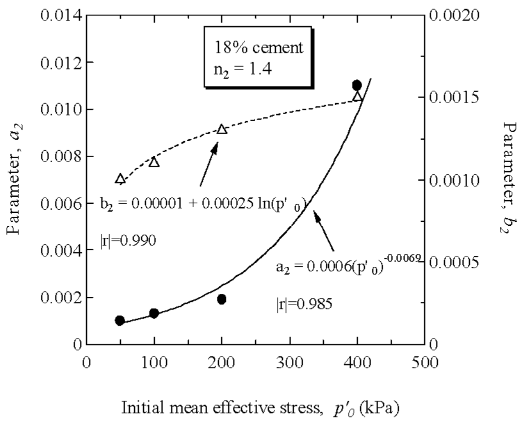

Fig. 22 Variation of parameters a_2 and b_2 with initial mean effective stress for 18% cement samples.

The predicted deviator stress and excess pore pressure versus shear strain responses of all samples are presented in Figs. 23 and 24. They show that the predicted and laboratory curves are in good agreement.

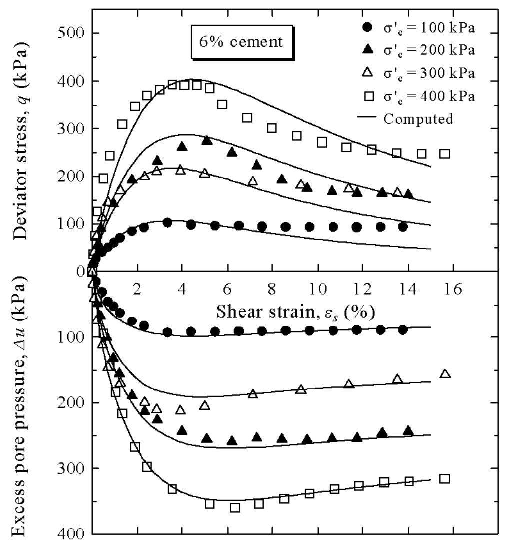

Fig. 23 Calculated and experimental (q–Δu–ε_s) curves for 6% cement samples.

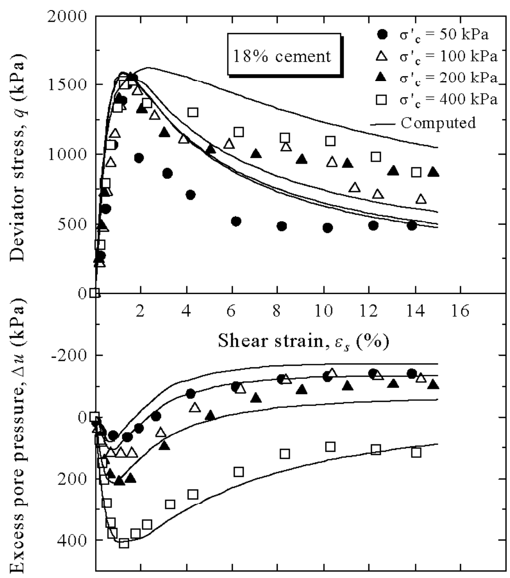

Fig. 24 Calculated and experimental (q–Δu–ε_s) curves for 18% cement samples.

CONCLUSIONS

This paper deals with the simple assessment of the stress-strain relationship of uncemented, naturally cemented and induced cemented clay. The conclusions can be drawn as follows.

1. Since the engineering behavior of normally consolidated uncemented clay is governed by the fabric, the (η, ε_s) relation is irrespective of effective confining pressure. The same set of parameters a_1 and b_1 can be used for different confining pressure.
2. For the naturally cemented clay, the (q, ε_s) and (Δu, ε_s) relations cannot be predicted by the hyperbolic model proposed by Duncan and Chang (1970) since their model does not capture softening behavior. However, the (p', ε_s) relation is hyperbolic and the modified hyperbolic relation of (η, ε_s) is realized for highly naturally cemented clay. By Combination of (η, ε_s) and (p', ε_s) relations, the softening behavior of (q, ε_s) and (Δu, ε_s) relations can be assessed.
3. For the very highly induced cemented clay which is used for ground improvement, both (η, ε_s) and (p', ε_s) relations are not hyperbolic. However, the analysis based on the (η, ε_s) and (p', ε_s) relations is more advantage than that based the (q, ε_s) and (Δu, ε_s) relations since all values of mean effective stress and stress ratio are positive even after the

peak values, whereas the negative pore pressure is recognized for the samples subjected to very low effective confining pressures. The modified hyperbolic relationship cannot be applied to assess the negative value.

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