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A LINEAR PROGRAMMIMG MODEL FOR TIDAL RIVER WATER QUALITY MANAGEMENT

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ABSTRACT: This paper presents the formulation of a mathematical model for tidal river water quality management, considering tidal effect on pollutant transport in a tidal river. The linear programming optimization and finite element method are used in model formulation. The objective function of the model is to maximize total BOD load which can be discharged into the river. The decision variables are the ratio of remaining BOD after treatment to the generated BOD of all controllable sources of BOD load discharging into the tidal river. The BOD and DO constraint inequalities are formulated such that at any time step the BOD values at identified nodal points are not more the specified limits and the DO values are not less than the specifies limits. Since the objective function and all the constraints are linear functions, this optimization problem is in the form of linear programming and the well known *Simplex method* can be used to solve the problem. To demonstrate the application of the model, it is applied to determine the optimal management plan for allocating the degree of treatment among the central wastewater treatment plants of large municipalities located along the Thachin river in the central region of Thailand. The construction plan includes seven wastewater treatment plants located at different sites along the river. This case study can demonstrate effectiveness of the model in determining the optimal water quality management plan for a tidal river and providing the optimal solution for wastewater management.

Keywords: Water quality management; Wastewater treatment allocation; Linear programming; Finite element method; Tidal river water quality

INTRODUCTION

Human activities usually produce wastewaters which are normally disposed into nearby water bodies. In order to maintain water quality in those water bodies within the acceptable levels, it is necessary that the generated wastewaters be treated to some degrees prior to disposal. Even though, discharges of effluent from several wastewater treatment plants (WTP) to the same water body may still cause the risk of exceeding its assimilative capacity. Some pollutant concentrations violate water quality criteria, especially during low flow period. For this reason, proper control of pollutant load discharging plays an important role in water quality management.

Nowadays, a lot of mathematical models have been developed to help decision makers establish their management strategies effectively. Water quality models are usually used to describe relationships between pollutant loadings and water quality response in the water body (Qin et al. 2009). Generally, the models are based on the principle of conservation of mass, considering physical and biochemical reactions, diffusion and advection processes of specific pollutants in a water body. In the past few decades, many studies presented the integration of optimization techniques such as linear programming, nonlinear programming, dynamic programming, etc. with water quality models to determine the most effective water quality management solution. Most of them addressed problems of economical pollutant discharge control. Jenq et al. (1983) integrated the linear programming with mass balance equations of total phosphorus in lake and stream to determine the minimum cost of eutrophication control in the lake. Fujiwara et al. (1987) integrated the linear programming with the Streeter-Phelps model to solve the problem of wastewater treatment cost saving when the main stream, tributaries and storm water were considered as random variables. Hanley et al. (1998) integrated the integer and linear programming with a water quality software, MIKE 11, to solve the problem of the economical water pollution control in estuary. Kuo et al.

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(2008) integrated the dynamic programming with the mass balance equations of kinetic cycle of chlorophyll-a, phosphorus and nitrogen for a complete-mixed lake to determine the optimal nutrient removal rates for lake eutrophication management. Cho et al. (2004) integrated the genetic algorithm with the QUAL2E simulation model to solve the problem of wastewater treatment cost optimization. Li and Huang (2009) integrated an inexact two-stage stochastic quadratic programming with the Streeter-Phelps model to solve the problem of water quality management under uncertainty.

In most of the above mentioned works, the issue of tidal effect on pollutant transport that could affect water quality along the river has not been given much attention. In this study, to reduce errors in predicting water quality in a tidal river, the relation of tidal action and pollutant transport in the river is considered in water quality simulation that is involved in solving an optimization problem.

For river water quality management, modeling method should be able to quantify waste load allocation of various point sources along the river such that the river water quality standards are maintained. In this work, dispersion models based on the two-dimensional vertically averaged mass balance equations of biochemical oxygen demand (BOD) and dissolved oxygen (DO) are formulated using the finite element method with Galerkin's weighted residual technique; then the obtained finite element equations are employed in the formulation of BOD and DO constraint inequalities. In order to achieve the optimal management objective, the management objective is established such that the total BOD loading discharged into the tidal river is maximized, while the BOD and DO at various locations in the river are maintained within the acceptable limits at any time. The pollutant sources are separated into 2 groups, controllable and uncontrollable sources. It is required that all controllable pollutant sources receive some degrees of treatment prior to discharging into the river. The degrees of treatment at those WTP's are considered as decision variables of the optimization model. Since the objective function and constraints are linear functions of the selected decision variables, this optimization problem can be arranged in the form of linear programming optimization which can be solved by the well known Simplex method.

The developed model is applied to the lower and middle sections of the Thachin River in central Thailand, where tidal effect is dominant. Time varying depths and velocities of flow at various points along the river which are input data of the model are obtained from the two-dimensional hydrodynamic model. In this case study, seven municipalities located along the Thachin River are supposed to construct wastewater treatment plants to reduce their BOD loads. The model is applied to determine proper allocation of waste loads considering degrees of treatment at various WTPs. This demonstration can show the effectiveness of the model in providing precisely optimal solution of the problem under tidal action.

WATER QUALITY MODEL FORMULATION

Governing Equations

This study focuses on the control of BOD loadings from WTPs, referred to as controllable BOD load, and the prediction of BOD and DO concentrations in receiving water. Thus, the basic governing equations of water quality models in this study are two-dimensional vertically averaged mass balance equations of BOD and DO as shown in Equations (1) and (2), respectively.

$$\frac{\partial B}{\partial t}+\frac{\partial (uB)}{\partial x}+\frac{\partial (vB)}{\partial y}-\frac{1}{h}\left[\frac{\partial}{\partial x}\left(hK_x\frac{\partial B}{\partial x}\right)+\frac{\partial}{\partial y}\left(hK_y\frac{\partial B}{\partial y}\right)\right] + k_1 B + k_s B - R_{bc} - R_{bu} = 0 \quad (1)$$

$$\frac{\partial D}{\partial t}+\frac{\partial (uD)}{\partial x}+\frac{\partial (vD)}{\partial y}-\frac{1}{h}\left[\frac{\partial}{\partial x}\left(hK_x\frac{\partial D}{\partial x}\right)+\frac{\partial}{\partial y}\left(hK_y\frac{\partial D}{\partial y}\right)\right] + k_1 B - k_2(D_s - D) - R_d = 0 \quad (2)$$

where B is the BOD concentration (g/m^3); D is the DO concentration (g/m^3); t is the time (s); x and y are the coordinates (m); u and v are the corresponding vertically averaged velocity components (m/s); h is the water depth (m); K_x and K_y are the dispersion coefficients in the x- and y-directions, respectively (m^2/s); k_1 is the BOD decaying rate (s^{-1}); k_2 is the atmospheric reaeration coefficient (s^{-1}); k_s is the BOD removal rate by sedimentation (s^{-1}); D_s is the saturated DO concentration (g/m^3); R_{bc} is the controllable BOD load (g/m^3.s); R_{bu} is the uncontrollable BOD load (g/m^3.s); R_d is other DO source or sink (g/m^3.s), e.g., DO produced by photosynthesis of phytoplankton and other aquatic plants, DO consumed by benthic demand, etc.

Boundary Conditions

Boundary conditions of the two-dimensional mass balance equation can be separated into 2 types, i.e., 1) the shoreline boundary, where the normal substance discharge flux is specified, and 2) the open boundary, where the substance concentration is specified.

Weighted Residual Equations

In the weighted residual method, the unknown variable in the governing equation is replaced by an approximated function which is expressed in terms of the values of that variable at nodal points identified in the study domain. This approximation will introduce some error or residual. The notion in the weighted residual method is to force this residual to be zero on the average sense, by introducing a weighting function and setting the integral of the product between the weighting function and residual over the entire study domain equal to zero (Huebner 1995). This results in the so-called weighted residual equation. For BOD and DO dispersion equations in Equations (1) and (2), the following weighted residual equations are obtained.

$$\iint_\Omega W_b[\frac{\partial \tilde{B}}{\partial t}+\frac{\partial(u\tilde{B})}{\partial x}+\frac{\partial(v\tilde{B})}{\partial y}-\frac{1}{h}\left[\frac{\partial}{\partial x}\left(hK_x\frac{\partial \tilde{B}}{\partial x}\right)+\frac{\partial}{\partial y}\left(hK_y\frac{\partial \tilde{B}}{\partial y}\right)\right] + k_1\tilde{B}+k_s\tilde{B}-R_{bc}-R_{bu}]dA = 0 \quad (3)$$

and

$$\iint_\Omega W_d[\frac{\partial \tilde{D}}{\partial t}+\frac{\partial(u\tilde{D})}{\partial x}+\frac{\partial(v\tilde{D})}{\partial y}-\frac{1}{h}\left[\frac{\partial}{\partial x}\left(hK_x\frac{\partial \tilde{D}}{\partial x}\right)+\frac{\partial}{\partial y}\left(hK_y\frac{\partial \tilde{D}}{\partial y}\right)\right] + k_1\tilde{B}-k_2(D_s-\tilde{D})-R_d]dA = 0 \quad (4)$$

in which $\tilde{B}$ and $\tilde{D}$ are approximated BOD and DO concentrations; W_b and W_d are weighting functions introduced for BOD and DO equations, respectively. Equations (3) and (4) can be expanded to

$$\begin{aligned}&\iint_\Omega W_b[\frac{\partial \tilde{B}}{\partial t}+\frac{\partial(u\tilde{B})}{\partial x}+\frac{\partial(v\tilde{B})}{\partial y}+k_1\tilde{B}+k_s\tilde{B}-R_{bc}-R_{bu}]dA\\&-\iint_\Omega W_b\left[\frac{K_x}{h}\frac{\partial h}{\partial x}\frac{\partial \tilde{B}}{\partial x}+\frac{K_y}{h}\frac{\partial h}{\partial y}\frac{\partial \tilde{B}}{\partial y}\right]dA\\&-\iint_\Omega[\frac{\partial}{\partial x}(W_bK_x\frac{\partial \tilde{B}}{\partial x})+\frac{\partial}{\partial y}(W_bK_y\frac{\partial \tilde{B}}{\partial y})]dA\\&+\iint_\Omega[K_x\frac{\partial W_b}{\partial x}\frac{\partial \tilde{B}}{\partial x}+K_y\frac{\partial W_b}{\partial y}\frac{\partial \tilde{B}}{\partial y}]dA = 0 \quad (5)\end{aligned}$$

and

$$\begin{aligned}&\iint_\Omega W_d[\frac{\partial \tilde{D}}{\partial t}+\frac{\partial(u\tilde{D})}{\partial x}+\frac{\partial(v\tilde{D})}{\partial y}+k_1\tilde{B}-k_2(D_s-\tilde{D})-R_d]dA\\&-\iint_\Omega W_d\left[\frac{K_x}{h}\frac{\partial h}{\partial x}\frac{\partial \tilde{D}}{\partial x}+\frac{K_y}{h}\frac{\partial h}{\partial y}\frac{\partial \tilde{D}}{\partial y}\right]dA\\&-\iint_\Omega[\frac{\partial}{\partial x}(W_dK_x\frac{\partial \tilde{D}}{\partial x})+\frac{\partial}{\partial y}(W_dK_y\frac{\partial \tilde{D}}{\partial y})]dA\\&+\iint_\Omega[K_x\frac{\partial W_d}{\partial x}\frac{\partial \tilde{D}}{\partial x}+K_y\frac{\partial W_d}{\partial y}\frac{\partial \tilde{D}}{\partial y}]dA = 0 \quad (6)\end{aligned}$$

Apply Green's theorem and rearrange Equations (5) and (6), we obtain

$$\begin{aligned}&\iint_\Omega W_b[\frac{\partial \tilde{B}}{\partial t}+\frac{\partial(u\tilde{B})}{\partial x}+\frac{\partial(v\tilde{B})}{\partial y}+k_1\tilde{B}+k_s\tilde{B}-R_{bc}-R_{bu}]dA\\&-\iint_\Omega W_b\left[\frac{K_x}{h}\frac{\partial h}{\partial x}\frac{\partial \tilde{B}}{\partial x}+\frac{K_y}{h}\frac{\partial h}{\partial y}\frac{\partial \tilde{B}}{\partial y}\right]dA\\&+\iint_\Omega[K_x\frac{\partial W_b}{\partial x}\frac{\partial \tilde{B}}{\partial x}+K_y\frac{\partial W_b}{\partial y}\frac{\partial \tilde{B}}{\partial y}]dA\\&-\int_S W_b[K_x(\frac{\partial \tilde{B}}{\partial x})dy-K_y\frac{\partial \tilde{B}}{\partial y}dx] = 0 \quad (7)\end{aligned}$$

and

$$\begin{aligned}&\iint_\Omega W_d[\frac{\partial \tilde{D}}{\partial t}+\frac{\partial(u\tilde{D})}{\partial x}+\frac{\partial(v\tilde{D})}{\partial y}+k_1\tilde{B}-k_2(D_s-\tilde{D})-R_d]dA\\&-\iint_\Omega W_d\left[\frac{K_x}{h}\frac{\partial h}{\partial x}\frac{\partial \tilde{D}}{\partial x}+\frac{K_y}{h}\frac{\partial h}{\partial y}\frac{\partial \tilde{D}}{\partial y}\right]dA\\&+\iint_\Omega[K_x\frac{\partial W_d}{\partial x}\frac{\partial \tilde{D}}{\partial x}+K_y\frac{\partial W_d}{\partial y}\frac{\partial \tilde{D}}{\partial y}]dA\\&-\int_S W_d[K_x(\frac{\partial \tilde{D}}{\partial x})dy-K_y\frac{\partial \tilde{D}}{\partial y}dx] = 0 \quad (8)\end{aligned}$$

The term $K_x(\partial\tilde{B}/\partial x)dy-K_y(\partial\tilde{B}/\partial y)dx$ and the term $K_x(\partial\tilde{D}/\partial x)dy-K_y(\partial\tilde{D}/\partial y)dx$ represent the BOD and DO discharge fluxes through the boundary which can be written as Q_bdL and Q_ddL, respectively, in which Q_b and Q_d are the BOD and DO inflow rates per unit length of the boundary. Then, Equations (7) and (8) can be written as

$$\begin{aligned}&\iint_\Omega W_b[\frac{\partial \tilde{B}}{\partial t}+\frac{\partial(u\tilde{B})}{\partial x}+\frac{\partial(v\tilde{B})}{\partial y}+k_1\tilde{B}+k_s\tilde{B}-R_{bc}-R_{bu}]dA\\&-\iint_\Omega W_b\left[\frac{K_x}{h}\frac{\partial h}{\partial x}\frac{\partial \tilde{B}}{\partial x}+\frac{K_y}{h}\frac{\partial h}{\partial y}\frac{\partial \tilde{B}}{\partial y}\right]dA\\&+\iint_\Omega[K_x\frac{\partial W_b}{\partial x}\frac{\partial \tilde{B}}{\partial x}+K_y\frac{\partial W_b}{\partial y}\frac{\partial \tilde{B}}{\partial y}]dA-\int_S W_bQ_bdL = 0 \quad (9)\end{aligned}$$

and

$$\begin{aligned}&\iint_\Omega W_d[\frac{\partial \tilde{D}}{\partial t}+\frac{\partial(u\tilde{D})}{\partial x}+\frac{\partial(v\tilde{D})}{\partial y}+k_1\tilde{B}-k_2(D_s-\tilde{D})-R_d]dA\\&-\iint_\Omega W_d\left[\frac{K_x}{h}\frac{\partial h}{\partial x}\frac{\partial \tilde{D}}{\partial x}+\frac{K_y}{h}\frac{\partial h}{\partial y}\frac{\partial \tilde{D}}{\partial y}\right]dA\\&+\iint_\Omega[K_x\frac{\partial W_d}{\partial x}\frac{\partial \tilde{D}}{\partial x}+K_y\frac{\partial W_d}{\partial y}\frac{\partial \tilde{D}}{\partial y}]dA-\int_S W_dQ_ddL = 0 \quad (10)\end{aligned}$$

The approximated solutions $\tilde{B}$ and $\tilde{D}$ can be expressed in terms of the BOD and DO concentrations at nodal points as follow.

$$\widetilde{B} = \Sigma N_i B_i = \boldsymbol{N^T B} \quad (11)$$

$$\widetilde{D} = \Sigma N_i D_i = \boldsymbol{N^T D} \quad (12)$$

in which B_i and D_i are the BOD and DO at nodal points in the study domain, N_i is a function of independent variables known as interpolation function, and $\boldsymbol{N^T}$ is transpose of matrix $\boldsymbol{N}$.

Velocities u and v, water depth h, and DO saturation concentration D_s can also be expressed in the same manner, i.e.,

$$u = \Sigma N_i U_i = \boldsymbol{N^T U} \quad (13)$$

$$v = \Sigma N_i V_i = \boldsymbol{N^T V} \quad (14)$$

$$h = \Sigma N_i H_i = \boldsymbol{N^T H} \quad (15)$$

$$D_s = \Sigma N_i D_{si} = \boldsymbol{N^T D_s} \quad (16)$$

In the Galerkin's method, the interpolation function N_i (i = 1, 2,…, n) is used as the weighting function, i.e.,

$$W_b = N_i \quad (17)$$

$$W_d = N_i \quad (18)$$

Substituting the above expressions into Equations (9) and (10) and rearrange, we will obtain the sets of BOD and DO weighted residual equations written in the compact form as follow:

$$\boldsymbol{M}\frac{d\boldsymbol{B}}{dt} + \left(\boldsymbol{M_{ux}} + \boldsymbol{M_{vy}} - \boldsymbol{M_{hx}} - \boldsymbol{M_{hy}} + \boldsymbol{M_{kx}} + \boldsymbol{M_{ky}} + \boldsymbol{M_{k1}} + \boldsymbol{M_{ks}}\right)\boldsymbol{B} - \boldsymbol{M_{rbc}} - \boldsymbol{M_{rbu}} - \boldsymbol{M_{qb}} = \boldsymbol{0} \quad (19)$$

and

$$\boldsymbol{M}\frac{d\boldsymbol{D}}{dt} + \left(\boldsymbol{M_{ux}} + \boldsymbol{M_{vy}} - \boldsymbol{M_{hx}} - \boldsymbol{M_{hy}} + \boldsymbol{M_{kx}} + \boldsymbol{M_{ky}} + \boldsymbol{M_{k2}}\right)\boldsymbol{D} - \boldsymbol{M_{k2}}\boldsymbol{D_s} + \boldsymbol{M_{k1}}\boldsymbol{B} - \boldsymbol{M_{rdc}} - \boldsymbol{M_{rdu}} - \boldsymbol{M_{qd}} = \boldsymbol{0} \quad (20)$$

in which

$$\boldsymbol{M} = \iint_\Omega \boldsymbol{NN^T} dA \quad (21)$$

$$\boldsymbol{M_{ux}} = \iint_\Omega \boldsymbol{NN^T U}\frac{\partial \boldsymbol{N^T}}{\partial x} dA \quad (22)$$

$$\boldsymbol{M_{vy}} = \iint_\Omega \boldsymbol{NN^T V}\frac{\partial \boldsymbol{N^T}}{\partial y} dA \quad (23)$$

$$\boldsymbol{M_{hx}} = \iint_\Omega \frac{K_x}{\boldsymbol{N^T H}}\boldsymbol{N}\frac{\partial \boldsymbol{N^T}}{\partial x}\boldsymbol{H}\frac{\partial \boldsymbol{N^T}}{\partial x} dA \quad (24)$$

$$\boldsymbol{M_{hy}} = \iint_\Omega \frac{K_y}{\boldsymbol{N^T H}}\boldsymbol{N}\frac{\partial \boldsymbol{N^T}}{\partial y}\boldsymbol{H}\frac{\partial \boldsymbol{N^T}}{\partial y} dA \quad (25)$$

$$\boldsymbol{M_{kx}} = \iint_\Omega K_x \frac{\partial \boldsymbol{N}}{\partial x}\frac{\partial \boldsymbol{N^T}}{\partial x} dA \quad (26)$$

$$\boldsymbol{M_{ky}} = \iint_\Omega K_y \frac{\partial \boldsymbol{N}}{\partial y}\frac{\partial \boldsymbol{N^T}}{\partial y} dA \quad (27)$$

$$\boldsymbol{M_{k1}} = \iint_\Omega k_1 \boldsymbol{NN^T} dA \quad (28)$$

$$\boldsymbol{M_{ks}} = \iint_\Omega k_s \boldsymbol{NN^T} dA \quad (29)$$

$$\boldsymbol{M_{k2}} = \iint_\Omega k_2 \boldsymbol{NN^T} dA \quad (30)$$

$$\boldsymbol{M_{rbc}} = \iint_\Omega R_{bc} \boldsymbol{N} dA \quad (31)$$

$$\boldsymbol{M_{rbu}} = \iint_\Omega R_{bu} \boldsymbol{N} dA \quad (32)$$

$$\boldsymbol{M_{rdc}} = \iint_\Omega R_{dc} \boldsymbol{N} dA \quad (33)$$

$$\boldsymbol{M_{rdu}} = \iint_\Omega R_{du} \boldsymbol{N} dA \quad (34)$$

$$\boldsymbol{M_{qb}} = \int_S Q_b \boldsymbol{N} dS \quad (35)$$

$$\boldsymbol{M_{qd}} = \int_S Q_d \boldsymbol{N} dS \quad (36)$$

In more compact form, Equations (19) and (20) can be written as

$$\boldsymbol{M}\frac{d\boldsymbol{B}}{dt} + \boldsymbol{FB} - \boldsymbol{M_{rbc}} - \boldsymbol{M_{bu}} = \boldsymbol{0} \quad (37)$$

$$\boldsymbol{M}\frac{d\boldsymbol{D}}{dt} + \boldsymbol{GD} + \boldsymbol{M_{k1}}\boldsymbol{B} - \boldsymbol{M_{du}} = \boldsymbol{0} \quad (38)$$

in which

$$\boldsymbol{F} = \boldsymbol{M_{ux}} + \boldsymbol{M_{vy}} - \boldsymbol{M_{hx}} - \boldsymbol{M_{hy}} + \boldsymbol{M_{kx}} + \boldsymbol{M_{ky}} + \boldsymbol{M_{k1}} + \boldsymbol{M_{ks}} \quad (39)$$

$$\boldsymbol{G} = \boldsymbol{M_{ux}} + \boldsymbol{M_{vy}} - \boldsymbol{M_{hx}} - \boldsymbol{M_{hy}} + \boldsymbol{M_{kx}} + \boldsymbol{M_{ky}} + \boldsymbol{M_{k2}} \quad (40)$$

$$\boldsymbol{M_{bu}} = \boldsymbol{M_{rbu}} + \boldsymbol{M_{qb}} \quad (41)$$

$$\boldsymbol{M_{du}} = \boldsymbol{M_{rdc}} + \boldsymbol{M_{rdu}} + \boldsymbol{M_{qd}} - \boldsymbol{M_{k2}}\boldsymbol{D_s} \quad (42)$$

In the finite element method, the study domain is divided into a number of elements. In each element, the unknown variables and model parameters are expressed in terms of the values at nodal points in that element. The integral over the whole study domain can be obtained from assembling the element integrals. Several element configurations can be selected, e.g., linear triangular element, isoparametric element, etc. For each configuration, a local coordinate system is set and the interpolation function is defined. Then, element integrals in the matrix form can be determined which are called element matrices. These element matrices are assembled to form system matrices in Equations (21) - (36).

OPTIMIZATION MODEL FORMULATION

Objective Function

In this study, the objective function of the model is to maximize the total BOD load which can be discharged into the receiving water. Here, BOD loadings are classified as controllable BOD load and uncontrollable BOD loads. The first type is defined for the remaining BOD load in the effluent from the planned wastewater treatment plants whereas the second type is defined for BOD loads from other discharges, including return flow from agricultural areas, discharges from aquaculture ponds, effluent from the existing wastewater treatment plants, as well as various non-point sources. The latter type is considered as uncontrollable BOD load because their treatment levels are not optimized in this objective function. That is the total BOD load to be optimized in this objective function is referred to the controllable BOD load which can be discharged into the water body in addition to the existing BOD load which are considered uncontrollable.

Let Z represent total controllable BOD load, then the objective function can be expressed as

$$\text{Maximize } Z = \sum_{e=1}^{m}\left[R_{bc}^{e}\right] = \sum_{e=1}^{m}\left[L_c^e P_c^e\right] \tag{43}$$

in which R_{bc}^{e} is the amount of controllable BOD load discharged into the e^{th} element, L_c^e is the total amount of controllable BOD load generated at the e^{th} element, and P_c^e is ratio of the remaining BOD load after treatment to the total amount of controllable BOD load generated in the e^{th} element, i.e., $P_c^e = R_{bc}^e / L_c^e$. Thus, the value of P_c^e is equal to $1 - E^e$, where E^e is the efficiency of wastewater treatment in the e^{th} element. A set of P_c^e values which maximize the value of Z is then the optimal $\boldsymbol{P_c}$ to be determined in this optimization model.

Constraints

Constraints of this optimization model include:

$$\boldsymbol{B_{t_i}} \leq \boldsymbol{B_s} \tag{44}$$

$$\boldsymbol{D_{t_i}} \geq \boldsymbol{D_s} \tag{45}$$

$$\boldsymbol{P_{c,min}} \leq \boldsymbol{P_c} \leq \boldsymbol{P_{c,max}} \tag{46}$$

where $\boldsymbol{B_{t_i}}$ and $\boldsymbol{D_{t_i}}$ are matrices of BOD and DO concentrations at time t_i ; $\boldsymbol{B_s}$ and $\boldsymbol{D_s}$ are matrices of the specified BOD and DO limited concentrations, respectively; $\boldsymbol{P_c}$ is matrix of P_c^e ; $\boldsymbol{P_{c,min}}$ and $\boldsymbol{P_{c,max}}$ are matrices of the minimum and maximum values of P_c^e , respectively, based on the practical ranges of BOD load removal rates of WTPs.

Formulation of BOD Constraint Inequality

From Equation (37), replace $\boldsymbol{M_{rbc}}$ by the product of the ($n\times m$) matrix $\boldsymbol{M_{rm}}$ and the ($m\times 1$) matrix $\boldsymbol{R_{bc}}$ which is the column matrix of controllable BOD loadings R_{bc}^e in all elements in the study domain. Then, replace $\boldsymbol{R_{bc}}$ by the product of the ($m\times m$) diagonal matrix $\boldsymbol{L_{ct}}$ and the ($m\times 1$) matrix $\boldsymbol{P_c}$. We will obtain $\boldsymbol{M_{rbc}} = \boldsymbol{M_{rm} L_{ct} P_c}$.

Substitute $\boldsymbol{M_{rbc}}$ in Equation (37) by $\boldsymbol{M_{rm} L_{ct} P_c}$, we obtain

$$\boldsymbol{M}\frac{d\boldsymbol{B}}{dt} + \boldsymbol{FB} - \boldsymbol{M}_{rm}\boldsymbol{L}_{ct}\boldsymbol{P}_c - \boldsymbol{M}_{qb} = \boldsymbol{0} \tag{47}$$

Applying Euler's method yields

$$\boldsymbol{M}_t(\boldsymbol{B}_{t_2} - \boldsymbol{B}_{t_1}) + \boldsymbol{F}_{t_1}\boldsymbol{B}_{t_1} - \boldsymbol{M}_{rm}\boldsymbol{L}_{ct_1}\boldsymbol{P}_c - \boldsymbol{M}_{qb,t_1} = \boldsymbol{0} \tag{48}$$

where $\boldsymbol{M}_t = \boldsymbol{M}/(t_2 - t_1) = \boldsymbol{M}/\Delta t$.

From Equation (48), $\boldsymbol{B}_{t_2}$ can be determined from

$$\boldsymbol{B}_{t_2} = \boldsymbol{M}_t^{-1}(\boldsymbol{M}_t - \boldsymbol{F}_{t_1})\boldsymbol{B}_{t_1} + \boldsymbol{M}_t^{-1}\boldsymbol{M}_{rm}\boldsymbol{L}_{ct_1}\boldsymbol{P}_c + \boldsymbol{M}_t^{-1}\boldsymbol{M}_{qb,t_1} \tag{49}$$

Let $\boldsymbol{M}_t - \boldsymbol{F}_{t_1} = \boldsymbol{M}_{f,t_1}$. Then, we obtain

$$\boldsymbol{B}_{t_2} = \boldsymbol{M}_t^{-1}\boldsymbol{M}_{f,t_1}\boldsymbol{B}_{t_1} + \boldsymbol{M}_t^{-1}\boldsymbol{M}_{rm}\boldsymbol{L}_{ct_1}\boldsymbol{P}_c + \boldsymbol{M}_t^{-1}\boldsymbol{M}_{qb,t_1} \tag{50}$$

To determine BOD values in the next time step, we replace $\boldsymbol{B}_{t_1}$, $\boldsymbol{M}_{f,t_1}$, $\boldsymbol{L}_{ct_1}$ and $\boldsymbol{M}_{qb,t_1}$ in Equation (50) with $\boldsymbol{B}_{t_2}$, $\boldsymbol{M}_{f,t_2}$, $\boldsymbol{L}_{c,t_2}$ and $\boldsymbol{M}_{qb,t_2}$, respectively. Then we obtain

$$B_{t_3} = M_t^{-1} M_{f,t_2} B_{t_2} + M_t^{-1} M_{rm} L_{ct_2} P_c + M_t^{-1} M_{qb,t_2} \quad (51)$$

Substitute B_{t_2} from Equation (50) into Equation (51), we obtain

$$\begin{aligned} B_{t_3} &= M_t^{-1} M_{f,t_2} M_t^{-1} M_{f,t_1} B_{t_1} \\ &+ [M_t^{-1} M_{f,t_2} M_t^{-1} M_{rm} L_{ct_1} + M_t^{-1} M_{rm} L_{ct_2}] P_c \\ &+ M_t^{-1} M_{f,t_2} M_t^{-1} M_{qb,t_1} + M_t^{-1} M_{qb,t_2} \end{aligned} \quad (52)$$

Proceed in a similar manner. Thus, the values of BOD at time t_i, B_{t_i} , can be expressed in term of B_{t_1} as follow.

$$\begin{aligned} B_{t_i} &= M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_2} M_t^{-1} M_{f,t_1} B_{t_1} \\ &+ [M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_2} M_t^{-1} M_{rm} L_{ct_1} \\ &+ M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_3} M_t^{-1} M_{rm} L_{ct_2} \\ &+ \ldots + M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{rm} L_{ct_{i-2}} + M_t^{-1} M_{rm} L_{ct_{i-1}}] P_c \\ &+ M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_2} M_t^{-1} M_{qb,t_1} \\ &+ M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_3} M_t^{-1} M_{qb,t_2} \\ &+ \ldots + M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{qb,t_{i-2}} \\ &+ M_t^{-1} M_{qb,t_{i-1}} \end{aligned} \quad (53)$$

which can be written in more compact form as

$$B_{t_i} = P_{t_i} B_{t_1} + Q_{t_i} + R_{t_i} P_c \quad (54)$$

where

$$P_{t_i} = M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_2} M_t^{-1} M_{f,t_1} \quad (55)$$

$$\begin{aligned} Q_{t_i} &= M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_2} M_t^{-1} M_{qb,t_1} \\ &+ M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_3} M_t^{-1} M_{qb,t_2} \\ &+ \ldots + M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{qb,t_{i-2}} \\ &+ M_t^{-1} M_{qb,t_{i-1}} \end{aligned} \quad (56)$$

$$\begin{aligned} R_{t_i} &= M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_2} M_t^{-1} M_{rm} L_{ct_1} \\ &+ M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{f,t_{i-2}} \ldots M_t^{-1} M_{f,t_3} M_t^{-1} M_{rm} L_{ct_2} \\ &+ \ldots + M_t^{-1} M_{f,t_{i-1}} M_t^{-1} M_{rm} L_{ct_{i-2}} \\ &+ M_t^{-1} M_{rm} L_{ct_{i-1}} \end{aligned} \quad (57)$$

Recall Equation (44), we obtain BOD constraint inequality as follow.

$$P_{t_i} B_{t_1} + Q_{t_i} + R_{t_i} P_c \le B_{st} \quad (58)$$

Formulation of DO Constraint Inequality

From Equation (38), applying Euler's method, we obtain

$$M_t (D_{t_2} - D_{t_1}) + G_{t_1} D_{t_1} + M_k B_{t_1} - M_{qd,t_1} = 0 \quad (59)$$

D_{t_2} can be determined from

$$D_{t_2} = M_t^{-1} (M_t - G_{t_1}) D_{t_1} - M_t^{-1} M_k B_{t_1} + M_t^{-1} M_{qd,t_1} \quad (60)$$

Let $M_t - G_{t_1} = M_{g,t_1}$. We obtain

$$D_{t_2} = M_t^{-1} M_{g,t_1} D_{t_1} - M_t^{-1} M_k B_{t_1} + M_t^{-1} M_{qd,t_1} \quad (61)$$

To determine DO values in the next time step, we replace D_{t_1} , M_{g,t_1} , B_{t_1} and M_{qd,t_1} in Equation (61) with D_{t_2} , M_{g,t_2} , B_{t_2} and M_{qd,t_2} , respectively. Then we obtain

$$D_{t_3} = M_t^{-1} M_{g,t_2} D_{t_2} - M_t^{-1} M_k B_{t_2} + M_t^{-1} M_{qd,t_2} \quad (62)$$

Substitute D_{t_2} from Equation (48) into Equation (49), we obtain

$$\begin{aligned} D_{t_3} &= M_t^{-1} M_{g,t_2} M_t^{-1} M_{g,t_1} D_{t_1} \\ &- M_t^{-1} M_{g,t_2} M_t^{-1} M_k B_{t_1} - M_t^{-1} M_k B_{t_2} \\ &+ M_t^{-1} M_{g,t_2} M_t^{-1} M_{qd,t_1} + M_t^{-1} M_{qd,t_2} \end{aligned} \quad (63)$$

Proceed in a similar manner. Thus, the values of DO at time t_i, D_{t_i} , can be expressed in terms of D_{t_1} and B_{t_1} as follow.

$$\begin{aligned} D_{t_i} &= M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_2} M_t^{-1} M_{g,t_1} D_{t_1} \\ &- [M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_2} M_t^{-1} M_k \\ &+ M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_3} M_t^{-1} M_k P_{t_2} \\ &+ \ldots + M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_k P_{t_{i-2}} + M_t^{-1} M_k P_{t_{i-1}}] B_{t_1} \\ &- M_t^{-1} M_k M_t^{-1} M_{f,t_{i-2}} M_t^{-1} M_{f,t_{i-3}} \ldots M_t^{-1} M_{f,t_1} B_{t_1} \\ &+ M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_2} M_t^{-1} M_{qd,t_1} \\ &+ M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_3} M_t^{-1} M_{qd,t_2} \\ &+ \ldots + M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{qd,t_{i-2}} + M_t^{-1} M_{qd,t_{i-1}} \\ &- M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_3} M_t^{-1} M_k Q_{t_2} \\ &- M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_4} M_t^{-1} M_k Q_{t_3} \end{aligned}$$

$$- \ldots\ldots - M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_k Q_{t_{i-2}} - M_t^{-1} M_k Q_{t_{i-1}}$$
$$- [M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_3} M_t^{-1} M_k R_{t_2}$$
$$+ M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_4} M_t^{-1} M_k R_{t_3}$$
$$+ \ldots\ldots + M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_k R_{t_{i-2}}$$
$$+ M_t^{-1} M_k R_{t_{i-1}}] P_c \qquad (64)$$

which can be written in more compact form as

$$D_{t_i} = S_{t_i} D_{t_1} + X_{t_i} - Y_{t_i} B_{t_1} - Z_{t_i} P_c \qquad (65)$$

where

$$S_{t_i} = M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_2} M_t^{-1} M_{g,t_1} \qquad (66)$$

$$X_{t_i} = M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_2} M_t^{-1} M_{qd,t_1}$$
$$+ M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_3} M_t^{-1} M_{qd,t_2}$$
$$+ \ldots\ldots + M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{qd,t_{i-2}} + M_t^{-1} M_{qd,t_{i-1}}$$
$$- M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_3} M_t^{-1} M_k Q_{t_2}$$
$$- M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_4} M_t^{-1} M_k Q_{t_3}$$
$$- \ldots\ldots - M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_k Q_{t_{i-2}}$$
$$- M_t^{-1} M_k Q_{t_{i-1}} \qquad (67)$$

$$Y_{t_i} = M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_2} M_t^{-1} M_k$$
$$+ M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_3} M_t^{-1} M_k P_{t_2}$$
$$+ \ldots\ldots + M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_k P_{t_{i-2}} + M_t^{-1} M_k P_{t_{i-1}}$$
$$+ M_t^{-1} M_k M_t^{-1} M_{f,t_{i-2}} M_t^{-1} M_{f,t_{i-3}} \ldots\ldots$$
$$\ldots\ldots M_t^{-1} M_{f,t_2} M_t^{-1} M_{f,t_1} \qquad (68)$$

$$Z_{t_i} = M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_3} M_t^{-1} M_k R_{t_2}$$
$$+ M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_{g,t_{i-2}} \ldots M_t^{-1} M_{g,t_4} M_t^{-1} M_k R_{t_3}$$
$$+ \ldots\ldots + M_t^{-1} M_{g,t_{i-1}} M_t^{-1} M_k R_{t_{i-2}}$$
$$+ M_t^{-1} M_k R_{t_{i-1}} \qquad (69)$$

Recall Equation (45), we obtain DO constraint inequality as follow.

$$S_{t_i} D_{t_1} + X_{t_i} - Y_{t_i} B_{t_1} - Z_{t_i} P_c \quad \geq \quad D_{st} \qquad (70)$$

According to the above procedure, a simulation based linear programming model for water quality management under unsteady river flow is obtained. The model is expressed in the standard form of the linear programming as follow.

$$\text{Maximize} \qquad Z = \sum_{e=1}^{m} [L_c^e P_c^e] \qquad (71)$$

Subject to

$$R_{t_i} P_c \quad \leq \quad B_{st} - P_{t_i} B_{t_1} - Q_{t_i} \qquad (72)$$

$$Z_{t_i} P_c \quad \leq \quad S_{t_i} D_{t_1} + X_{t_i} - Y_{t_i} B_{t_1} - D_{st} \qquad (73)$$

$$P_{c,min} \quad \leq \quad P_c \quad \leq \quad P_{c,max} \qquad (74)$$

MODEL TESTING

A test of the formulated model is conducted by applying to a channel with uniform cross section and assumed sinusoidal flow fluctuation. This channel is divided into 50 elements with 102 nodal points as shown in Fig. 1. It is assumed that domestic wastewaters generated along this channel are collected and transported to four wastewater treatment plants (WTPs) to reduce BOD load before discharging into the channel. The location of each WTP and assumed BOD load to each WTP are also shown in Fig. 1. The water quality requirements are specified that along the channel the BOD concentration shall not be more than 1.5 g/m^3 and DO concentration shall not be less than 6.0 g/m^3. In this example, the value of P_c (the ratio of the remaining BOD load after treatment to the generated BOD load) at each WTP is set within the range of 0.10 - 0.40.

It is assumed that tidal fluctuation at the downstream end of the channel follows the expression $\eta_0 = a_0 \sin \omega t$. Then, flow velocity and water level fluctuation along the channel can be determined using equations developed by Ippen (1966). It is also assumed that $K_x = K_y = 50$ m^2/s, $k_1 = 0.1$ d^{-1}, $k_2 = 0.2$ d^{-1}, $k_s = 0.01$ d^{-1}, and BOD and DO concentrations at the upstream end are 1.5 g/m^3 and 6.0 g/m^3, respectively.

The simplex method is used to solve the formulated linear programming model under the given conditions, the obtained results, as presented in Table 1, show that the P_c value of each WTP being within the specified range (i.e., 0.10 - 0.40).

According to the obtained P_c values, BOD and DO concentrations at various points in the channel also meet the water quality requirements as shown in Fig.2 and Fig.3. These results show that the model can provide a set of P_c values that satisfy all constraints of the problem. The next step is to check whether these P_c values can provide maximum total BOD loading into the channel or not.

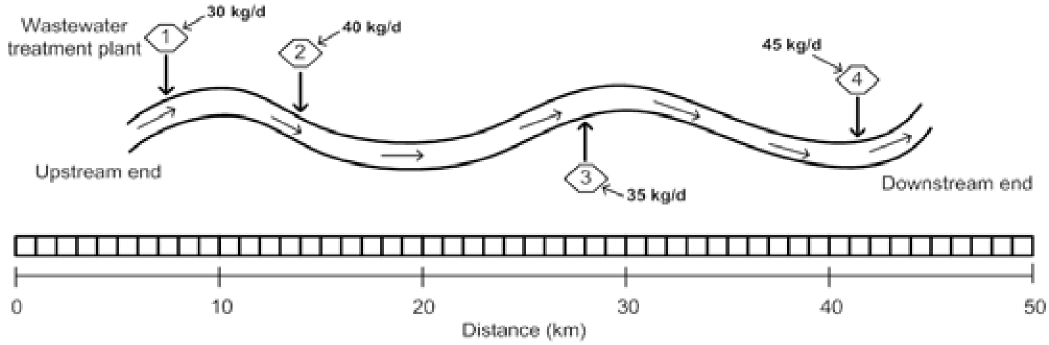

Fig. 1 A tidal river with uniform cross section and the finite element grid

Table 1 Discharge points and influent BOD loads

WTPs	1	2	3	4
P_c values	0.1949	0.1112	0.1849	0.1099
BOD loadings (kg/d)	5.847	4.448	6.472	4.945
Overall BOD loading = 21.712 kg/d				

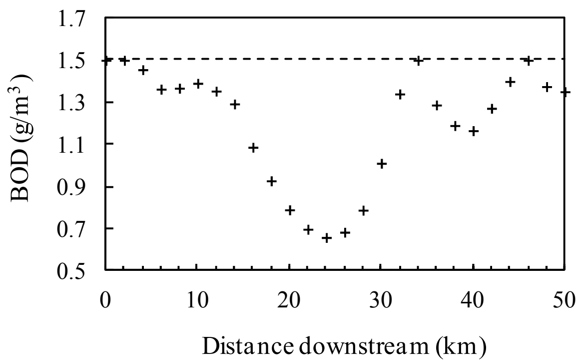

Fig. 2 Predicted BOD (+) and BOD standard (---)

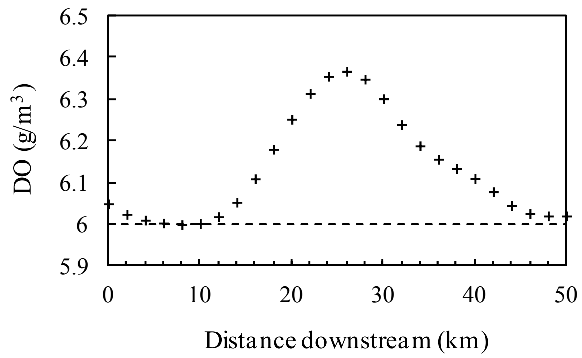

Fig. 3 Predicted DO (+) and DO standard (---)

Table 2 List of P_c changes of each test

Test	P_c values assigned to wastewater treatment plant				Overall BOD Loading (kg/d)
	1	2	3	4	
1	0.1720	0.1112	0.1849	0.1250	21.704
2	0.1949	0.1112	0.1650	0.1250	21.695
3	0.1	0.1112	0.2660	0.1099	21.704
4	0.1949	0.1112	0.1	0.1750	21.670
5	0.2930	0.1112	0.1	0.1099	21.684
6	0.1949	0.1850	0.1	0.1099	21.692
7	0.1	0.1112	0.1849	0.1730	21.704

From Fig.2 and Fig.3, it is found that there are some points in the channel where BOD and DO values exactly meet the standards – we call these points as critical points. The existence of the critical points implies that the set of P_c values and the overall BOD loading listed in Table 1 represent the optimal solution and the maximum value, respectively.

To prove this implication, we try changing the P_c values in the P_c set while keeping the overall BOD loading equal 21.712 kg/d, such as that shown in Table 2, and then determine the distributions of BOD and DO along the channel. The results of these tests reveal that the obtained BOD and DO values violate the water quality requirements particularly at the points nearby the WTPs where P_c values are increased. These results imply that other P_c sets are invalid to this problem. Thus, it can be said that the set of P_c values in Table 1 is the optimal solution and the overall BOD loading to the river corresponding to the optimal P_c set is the maximum amount. As examples, we show the distributions of BOD and DO values of some tests (as listed in Table 2) in Fig. 4 and Fig. 5, respectively.

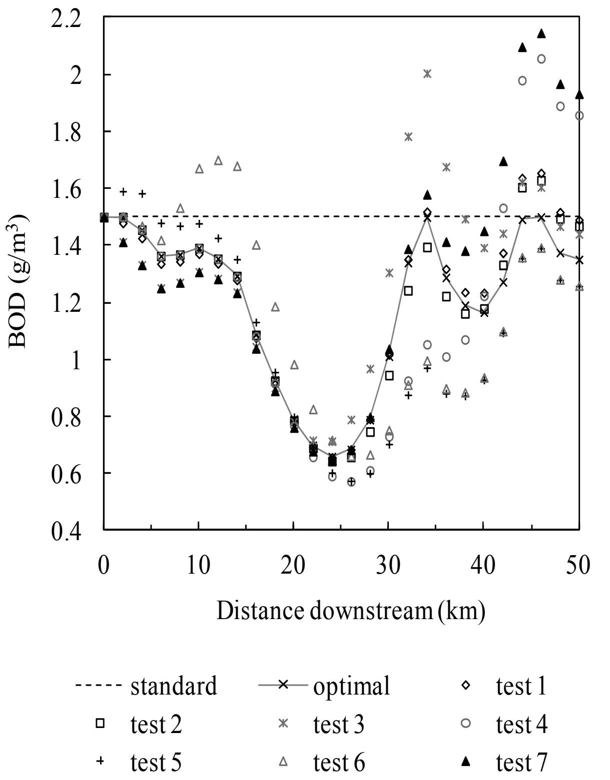

Fig. 4 Computed BOD for various P_c sets compared with BOD standard

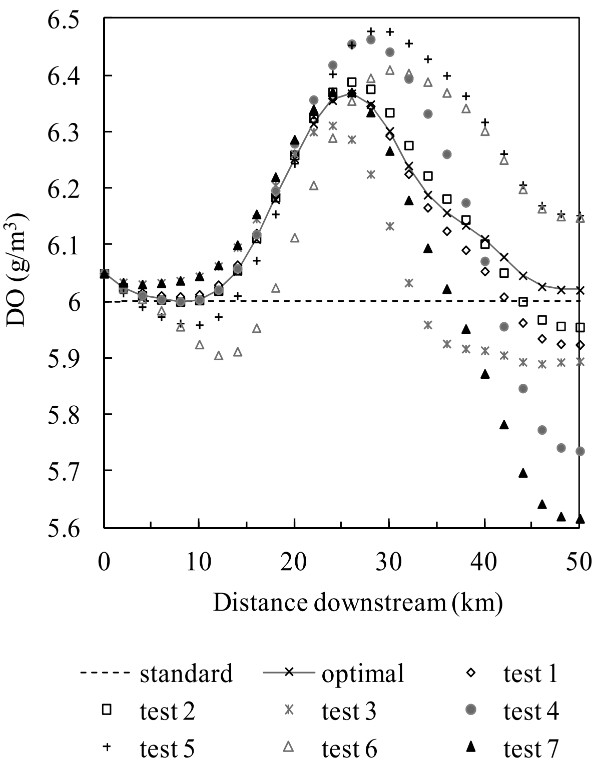

Fig. 5 Computed DO for various P_c sets compared with DO standard

CASE STUDY

Study Area

The Thachin River is one of important waterways in central Thailand (see Fig. 6). It supports a variety of uses, including agriculture, aquaculture, water supply in urban areas and industries. The return flow from all these users which is directly discharged into the river usually contains a large amount of BOD load, causing BOD and DO levels in the water body, especially in the middle and lower parts of the river, often violating Thai water quality standards for surface water. Thus, in this study, a case of wastewater management for improving water quality in the middle and lower parts of the Thachin River is focused.

The study area covers the river length of 202 km with the upstream end at Phophraya Regulator in Suphanburi Province and the downstream end at the river mount in Sumut Sakhon Province. Most of inflowing water in the study area is discharged from the Phophraya Regulator, flowing downstream to the sea. Tidal movement at the river mount also affects flow patterns in the study area. The influence of tides during high flow and low flow periods are about 120 and 180 km from the river mount, respectively (Simachaya and Healthcote 1999). Tidal motion causes water to move in and out of the river mount in a periodic fashion (Chapra 1997). As a result, velocities and depths of flow at various points of the river fluctuate all the time with the movement of tides.

Wastewater Treatment System in the Study Area

At present time, there are two existing WTPs for purifying domestic wastewater from some areas of the study domain. One plant is located in Muang Suphanburi District and another plant is located in Muang Nakhon Pathom District, each being stabilization pond. To demonstrate the application of the formulated model to determine the proper allocation of wastewater treatment in WTPs, we propose a WTPs construction plan for domestic wastewater control in year 2025. This plan proposes the construction of five WTPs located in different sites along the study domain. Thus, we will have seven WTPs to treat domestic wastewater which is going to be discharged to the study area. We assign that all WTPs can be primary or secondary treatment with BOD load removal rate of 30-80 percent. In other words, Pc values are specified within the range of 0.2 - 0.7. Here, we assign the amount of BOD load entering each WTP based on the amount of generated wastewater in the zone where that WTP is located. Sites of WTPs are listed in Table 3.

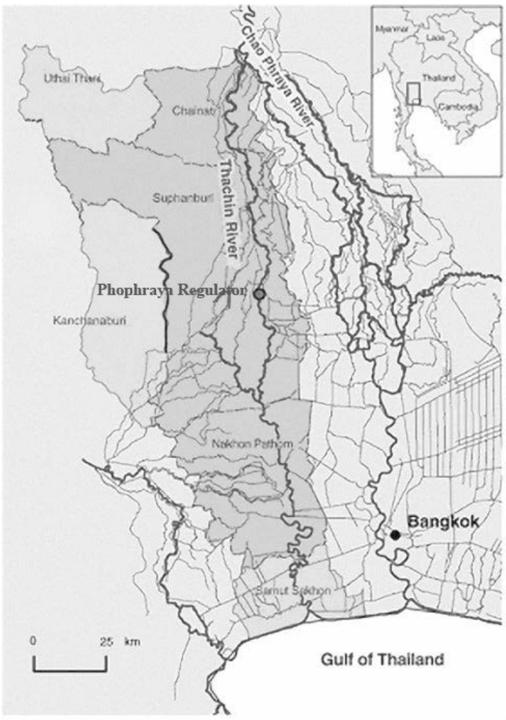

Fig. 6 Thachin River in central Thailand (Schaffner, 2009)

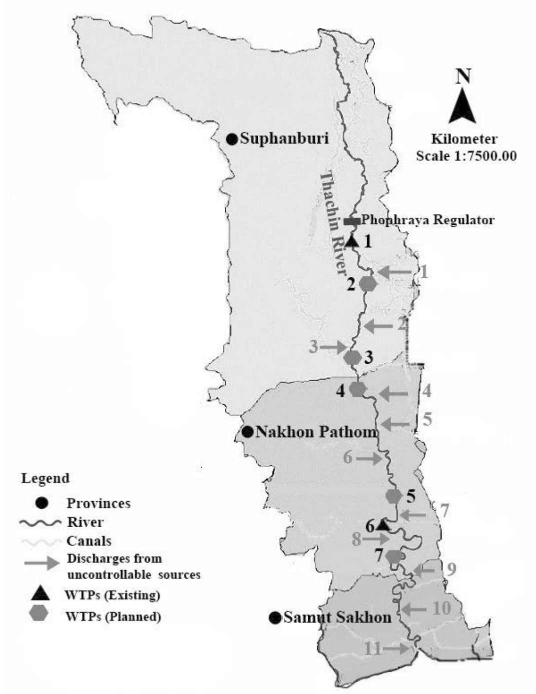

Fig. 7 Discharge points of effluents from WTPs and uncontrollable BOD sources in the study area

Table 3 Estimated BOD load in influents to WTPs

Discharge point number	Sites of WTPs (Districts)	BOD load (kg/d)
1	Muang Suphanburi	2,267.13
2	Bangplama	2,204.17
3	Songphinong	3,602.30
4	Banglen	4,625.24
5	Nakhon Chaisi	7,732.66
6	Nakhon Pathom	10,012.21
7	Muang Samut Sakhon	12,457.24

Model Application to the Case Study

To apply the model to the case study, the river from Phophraya Regulator to the river mount is divided at every 2 km along the length to obtain 101 quadrilateral elements with 204 corner nodal points. Discharge points of WTPs in the wastewater treatment system and discharges from other pollutant sources, i.e., pig farms, aquacultures and industries, (considered as sources of uncontrollable BOD load) are assigned corresponding to the domain discretization. Discharge points of effluents from WTPs and the various uncontrollable BOD sources are depicted in Fig. 7. Amount of BOD load in influents to WTPs and in discharges from uncontrollable sources corresponding to discharge numbers in Fig. 7 are listed in Tables 3 and 4, respectively. For the sake of simplicity, the same values of model parameters, as presented in Table 5, are assigned to every nodal point of the finite element grids. In case of hydrodynamic data, we use an available two-dimensional hydrodynamic model based on the finite element method to determine velocities and depths of flow at different nodal points at various times and then use these values as input data of the formulated model. However, details of the hydrodynamic model and its application are not described in this paper.

All necessary data are input to the model and then the computation for the optimal solution is conducted using the simplex algorithm. Specifying that BOD and DO values at every hour meet the standards, we obtain optimal P_c values for various WTPs as listed in Tables 6.

Based on the optimal P_c, BOD and DO along the river are predicted using the available dispersion models to find the point and the time that the river water quality is critical. The predicted values confirm that both BOD and DO at all points of the study domain meet standard levels. A critical water quality occurs at the distance of 196 km downstream of the domain where BOD exactly meets the standard. All data of the predict values are abundant and cannot be totally presented in this paper.

Table 4 Estimated BOD load in uncontrollable sources discharged to the study area

Discharge point number	BOD load (kg/d) from various sources discharged to each discharge point			
	Pig farms and aquacultures	industries	Untreated domestic wastewater	Total
1	1,224.95	0.39	174.30	1,399.64
2	728.86	-	-	728.86
3	521.46	-	-	521.46
4	259.73	-	-	259.73
5	1,036.70	-	-	1,036.70
6	1,018.52	38.36	-	1,056.88
7	1,154.61	1.93	394.95	1,551.49
8	3,595.92	35.83	1,298.71	4,930.46
9	6,524.86	871.73	5,690.30	13,086.89
10	1,498.33	596.10	4,857.25	6,951.68
11	449.50	335.36	-	784.86

Table 5 Model variables

Variables	Values	References
k_1 (d^{-1})	0.1	Chapra (1997)
k_2 (d^{-1})	$5.01v^{0.969}H^{-1.673}$ for $H \leq 3.48$ $3.93v^{0.5}H^{-1.5}$ for $H \geq 3.48$	Lung (2001)
K_x (m^2/s)	$0.05937Q / SB$	Chapra (1997)
K_y (m^2/s)	$0.011U^2B^2 / H^{1.5}g^{0.5}S^{0.5}$	Chapra (1997)
k_s (d^{-1})	0.01	Chapra (1997)
D_s (g/m^3)	468 / (31.6+T)	Lung (2001)

Note: v = velocity (m/s), H = mean depth (m), B = width (m), Q = mean flow (m^3/s), g = acceleration due to gravity (m/s^2), S = channel slope (dimensionless), T = water temperature (°c)

Thus, as examples, we just show the predicted BOD along the river length at a critical time and the predicted BOD at various times at the distance of 196 km downstream (a critical point) as depicted in Fig. 8 and Fig. 9, respectively.

Table 6 Results from model application to the case study

Sites of WTPs (Districts)	Optimal P_c	Remaining BOD load in effluent (kg/d)
Muang Suphanburi	0.20	453.43
Bangplama	0.20	440.83
Songphinong	0.20	720.46
Banglen	0.65	3,006.41
Nakhon Chaisi	0.54	4,175.64
Nakhon Pathom	0.20	2,002.44
Muang Samut Sakhon	0.42	5,232.04

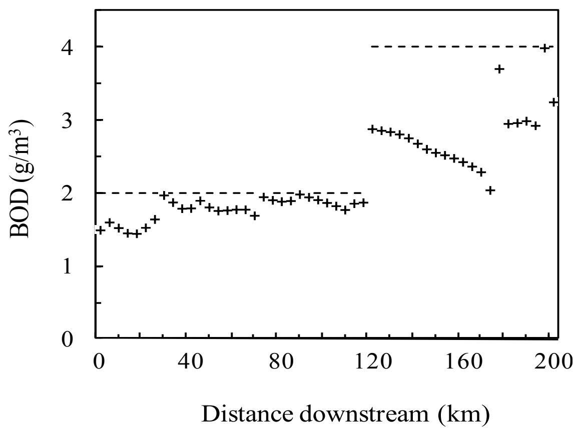

Fig. 8 Predicted BOD (+) along the length of the river at a critical time and BOD standard (---)

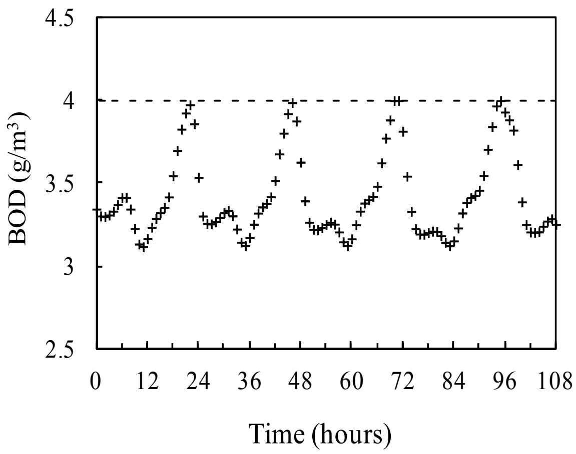

Fig. 9 Predicted BOD (+) at various times at the distance of 196 km downstream (a critical point) and BOD standard (---)

CONCLUSIONS

The formulated linear programming model is used to determine the optimal wastewater treatment policy, in terms of the ratio of the remaining BOD load after treatment to the total amount of BOD load generated at each element, so as to maximize the total amount of BOD load discharged into the water body while maintaining the BOD and DO concentrations within the acceptable limits. The model formulation is made by expressing BOD and DO concentrations at various times in terms of initial BOD and DO concentrations and input BOD loads with varying values of flow velocities and water depths at each time step. Thus, this model can be used to determine an optimal solution of the problem under fluctuating flow condition. In applying the developed model to the Thachin River it is found that the precise values of treatment levels at all WTPs in the proposed WTP construction plan are obtained. This clearly shows the effectiveness of the formulated model in providing useful information for decision makers to allocate proper wastewater treatment policy for the tidal river with fluctuating flow patterns.

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