

## MECHANICAL BEHAVIOR OF CRUSHED ROCK BASE (CRB) UNDER REPEATED LOADS

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**ABSTRACT:** This paper aims to report the mechanical behavior of a crushed rock base (CRB) subjected to repeated loads from Repeated Loads Triaxial (RLT) tests with various stress paths in order to improve more understanding of roads based materials on mechanistic-empirical pavement design and analysis. As is well known, pavement surface rutting, longitudinal and alligator cracks are normally the main cause of damage in flexible pavements. Factors contributing to such damage are the excessive irreversible and reversible deformation of a base layer including the behavior of a mechanical response of CRB under traffic load is not well understood. In this study, the shakedown concept was utilized to describe and determine limited use of CRB subjected to different stress conditions. In this paper, compacted CRB samples were subjected to the various stress condition defined by the stress ratio (the ratio of a vertical major stress,  $\sigma_1$  and a horizontal minor stress,  $\sigma_3$ ) in order to simulate the real condition of pavement. The study reports that CRB was defined the working stress ratio of 11 in pavement structure and will show rutting deterioration at the large number of load cycles after a stable state. Moreover, the mechanical responses were investigated and the limit ranges of using CRB in pavements were determined.

**Keywords:** Crushed rock base (CRB), unbound granular materials (UGMs)

### INTRODUCTION

Unbound granular materials (UGMs) layer with thin bituminous surfacing is widely used in the Australian road network. Normally, crushed rock base (CRB) is used as a base course material in Western Australia, can be determined as UGMs. The important function of the base course in pavements is to distribute and reduce amount of vertical stresses and strains because of vehicle wheel loads into the sub-base and the sub-grade without unacceptable strain. Consequently, an obvious understanding of shear strength, resilient and permanent strain, and shake down limit characteristics of materials relevant to pavement mechanistic design is very important to obtain the effective uses of such materials. However, Western Australia pavement design still relies on a traditional design procedure which is unreliable enough to explain a relationship between design parameter inputs and pavement performances. Roads need to be investigated to improve pavement analysis and design more precisely than in the past with respect of real behavior and the amount of traffic during the service life. Consequently, a most economical of layer thickness and an appropriate material type for the pavement will be determined.

This paper focuses on applying the mechanical behavior for CRB as a base course material and developing the typical models of CRB for pavement analysis in Western Australia. The empirical design method is unacceptable because the test protocols to require the design parameter inputs from monotonic loading tests rather than cyclic loading tests which are more representative of real traffic loading conditions. A mechanistic design attempts to explain pavement characteristics under real pavement conditions such as load types, material properties of the structure and environments based on design parameters from sophisticated tests which can simulate real pavement conditions into the test protocol (Collins, Wang et al. 1993). The main success of this analytical method is the experimental measurement and appropriate characterization of the mechanical responses from the RLT test which is the basic protocol of this study.

### BACKGROUND

The empirical nature of traditional pavement design procedure is based on experience and the results of simple tests. Such testing results are all static parameters

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and simple index parameters rather than any consideration of multidimensional geometry, realistic material performance and displacement distribution during cyclic loading, stresses and strain distribution in multilayered pavement design. Consequently, the use of empirical approaches becomes sub-standard. Traditional design procedure has been criticized by Wolff, who argued that it is too simplistic and does not take into account the non-linear behavior of UGMs (Wolff and Visser 1994).

Basically, the conventional pavement construction is designed to provide adequate thickness cover the sub layer in such a way that no shear failures and unacceptable permanent deformation takes place in each layer. For pavement design purposes, the stress level which is related with a reversible strain response must be determined and consequently not exceeded, once unacceptable permanent strains are prevented. This has improved the possibility of a critical boundary stress between stable and unstable conditions in a pavement.

The shakedown concept has been used to explain the behavior of conventional engineering structures under repeated cyclic loading. For the theoretical approach of the UGMs' permanent deformation used to describe the behavior of tested materials under RLT tests under macro-mechanical observations of the material response and in the distribution of the plastic strain in the tested material were investigated.

Firstly, the possible employment of the shakedown concept in pavement design was introduced by Sharp and Booker (1984) and Sharp (1983). They explained the application of the shakedown concept based on the tested results of the AASHTO road tests (AASHTO 1986) where in some cases, deterioration was reported due to stiffening or post-compaction after a number of load cycles (Kent 1962). Numerous investigations have been conducted regarding the behavior of UGMs used in flexible pavements. The original shakedown concept maintains that there are three ranges of permanent strain response under repeated loading.

- Plastic shakedown range (Range A). The low loading levels apply and the material response indicates plastic in a few initial cycles, although the ultimate response is elastic after Post-compaction. The strain is completely reversible and does not lead any permanent strains when it reaches a state of stability.
- Plastic creep range (Range B). The applied loading level is low enough to avoid a quick collapse. The material achieves a long-term stable state response with any accumulation of plastic strain (Post-compaction). However the

material will show failure with a large number of load cycles after a stable state.

- Incremental collapse range (Range C). The repeated loading is relatively large so that plastic strain accumulates rapidly with failure occurring in a small number of load cycles after stiffening.

A pavement is likely to show progressive accumulation of permanent strains (rutting) under repeated traffic loading if the magnitude of the applied loads exceeds the limiting value (Range C). If the applied traffic loads are lower than this limit, after any post-compaction stabilization, the permanent strains will level off and the pavement will achieve a stable state of "shakedown" (Ranges A and B) presenting only reversible strain under additional traffic loading (Sharp 1985). This implies an adaptation by the pavement subjected to the working load. This could be due to a change in material response (compaction degree), due to a change in stress state or due to a combination of both effects. With this understanding of material behavior, the shakedown concept typically then determine the load carrying capacity of the structure if it is not to reach excessive permanent strain. For performance prediction, it is of great importance to know whether a given pavement will experience progressive accumulation of permanent strain leading to state of incremental collapse or if the increase in permanent strain will cease, resulting in a stable response (shakedown state).

To implement the RLT measured permanent strain development in the computation of permanent strain development in a pavement structure, the permanent strain in the material under consideration has to be known as a function of both the number of load cycles and the stresses in the materials. Furthermore the shakedown approach should be considered. Lekarp and Dawson (Lekarp and Dawson 1998) suggested that the shakedown approach might also be employed in explaining the permanent strain behavior of UGM. In conclusion, they pointed out that more research is required to determine this shakedown limit.

## MATERIALS

### Crushed Rock

The crushed rock samples used in this study were taken from a local stockpile of Gosnells Quarry and kept in sealed containers. RLT tests were performed on samples as part of the collaboration with Civil Engineering, Curtin University of Technology. The

Table 1 Characterization tests (Main Roads Western Australia 2007)

| Tests*                         | Results               | Tests*                                         | Results   |
|--------------------------------|-----------------------|------------------------------------------------|-----------|
| Liquid Limit (LL)              | 22.4%                 | Coefficient of uniformity (Cu)                 | 22.4      |
| Plastic Limit (PL)             | 17.6%                 | Coefficient of curvature (Cc)                  | 1.4       |
| Plasticity Index (PI)          | 4.8%                  | % fines (<75micron)                            | 5 %       |
| Linear Shrinkage (LS)          | 1.5%                  | Cohesion of CRB (C**)                          | 32 kPa    |
| Flakiness Index (FI)           | 22.5%                 | Internal friction angle of CRB ( $\phi^{**}$ ) | 59°       |
| Maximum dry density (MDD)      | 2.27 t/m <sup>3</sup> | Max. Dry Compressive Strength (MDCS)           | 3,528 kPa |
| Optimum moisture content (OMC) | 5.5%                  | California Bearing Ratio (CBR) (%)             | 180       |

\* Accordance with MRWA (Main Roads Western Australia 2006)

\*\* Drained triaxial compression tests at the 100%OMC condition.

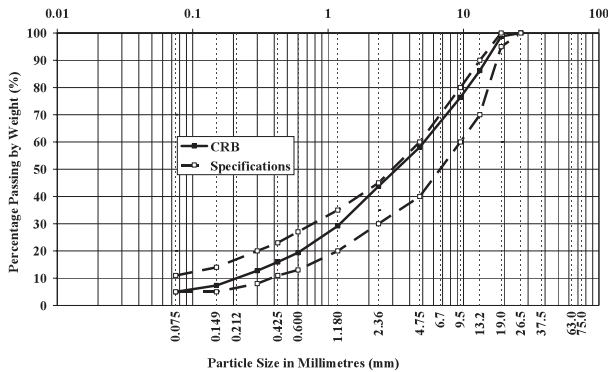


Fig. 1 CRB grading curves compared with WA Main Roads specifications. (Jitsangiam and Nikraz 2007)

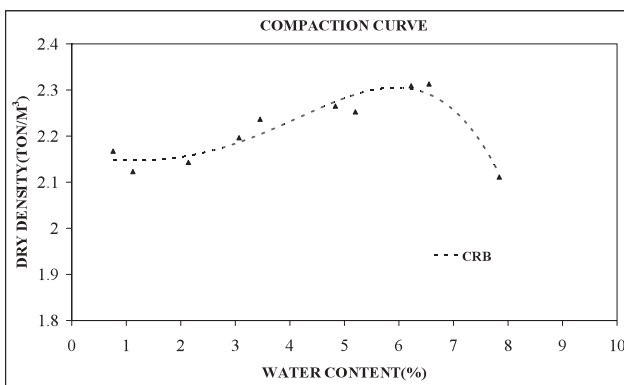


Fig. 2 The repeated loads triaxial apparatus

crushed rock samples were prepared (see Fig.1 for the grading curve) at 100% of maximum dry density (MDD) of 2.27 ton/m<sup>3</sup> and optimum moisture content (OMC) of that 5.5% as shown in Fig. 2. Material properties achieve base course specifications (Main Roads Western Australia 2003) and were tested at various moisture

content of 100% of OMC, 80% and 60% of OMC to observe the effect of moisture content in term of resilient modulus response. Figure 1, shows the grading curve of the crushed rock in this study achieves the upper and lower bound of the base course specifications. Significant comparisons of basic properties with specifications were made as shown in Table 1.

## LABORATORY PROGRAM AND TESTING

### Specimen Preparation

Sample preparations were carried out by using a standard cylinder mould 100 mm in diameter and 200 mm in height by the modified compaction method (Main Roads Western Australia 2007). Compaction was accomplished on 8 layers with 25 blows of a 4.9 kg rammer at a 450 mm drop height each layer. Fully bonding conduction between the layers of each layer had to be scarified to a depth of 6 mm before for the next layer was compacted. After compaction, the basic properties of each specimen were determined after which it was carefully carried to the base platen set of the chamber triaxial cell. A crosshead and stone disc were placed on the specimen and it wrapped in two platens by a rubber membrane and finally sealed with o-rings at both ends.

### REPEATED CYCLIC LOAD TRIAXIAL TESTS

The tests were carried out with a cyclic triaxial apparatus consisting of main set containing the load

actuator and a removable chamber cell. The specimens were placed in the triaxial cell between the base platen and crosshead of the testing machine as Fig. 3 shows. Controllers were used to manage the chamber, as well as the air pressure. The analogical signals detected by the transducers and load cell are received by a module where they are transformed to digital signals. A computer converts modules of the digital signals sent from the system. The system is located in the main set and facilitates the transmission of the orders to the actuator controller. User and the triaxial apparatus communication are controlled by a computer which uses convenient and precise software. This makes it possible to select the type of test to be performed as well as all the parameters, stress levels, data to be stored. The load cell, the confining pressure and the externally linear variable differential transducer (LVDT) on the top of the triaxial cell, used to measure deformations over the entire length of the specimens were measured by the control and data acquisition system (CDAS) which provided the control signals, signal conditioning, data acquisition. The CDAS was networked with the computer which provided the interfacing with the testing software and stored the raw test data. These enabled the resultant stress and strain in the sample to be determined.

This apparatus however, is limited to laboratory samples with a maximum diameter of 100 mm and a height of 200 mm based on the standard method of Austroads APRG 00/33-2000 (Voung and Brimble 2000). Moreover, the apparatus allows the laboratory sample to be subject to cyclic axial deviator stresses but it is not feasible to vary the confining radial stresses at the same time. Confining pressure was generated air to simulate the lateral pressure acting on the surrounding materials as occurs in a pavement layer.

The pressure was applied and stresses were found at different points in the granular material. The results were expressed in terms of deviator stress  $q = \sigma_1 - \sigma_3$ , mean normal stress  $p = (\sigma_1 + 2\sigma_3)/3$  and the confining pressure was simulated from the thickness of pavement base course layer that common use in Western Australia. For this reason, it was decided to subject the laboratory samples to 11 different stress levels and the particular confining pressure was level of 40 kPa. After the confining pressure had been applied, additional dynamic vertical stress was applied. Triaxial tests were carried out with axial stress pulses reaching stress ratios of  $\sigma_1/\sigma_3 = 5-15$ . The dynamic axial stress came from a high pressure air actuator capable of accurately applying a stress pulse following the stress level. In this test, there was haversine waveform frequency of 1 Hz over a period of 1.0 sec and a load pulse of 0.1 sec duration, as illustrated in Fig. 4.



Fig. 3 The repeated loads triaxial apparatus

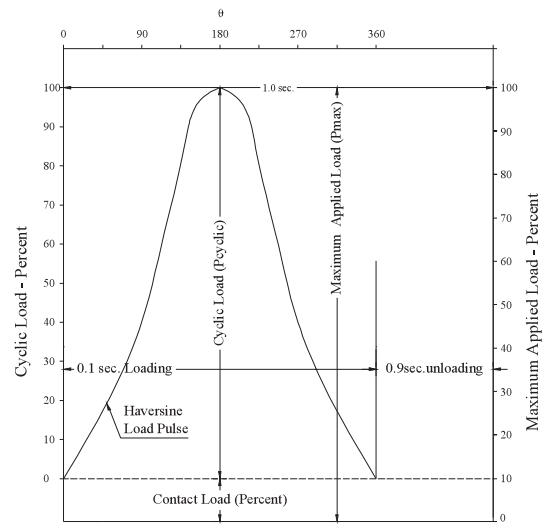


Fig. 4 The vertical loading waveform

## RESILIENT MODULUS TESTS AND PERMANENT DEFORMATION TESTS

The standard method of Austroads APRG 00/33-2000 (Voung and Brimble 2000) for RLT Test Method was followed for the resilient modulus tests and the permanent deformation tests. The UTM-14P digital servo control testing machine in the Geomechanics Laboratory, Department of Civil Engineering, Curtin University of Technology was used.

New specimens were prepared as described in the previous section. Permanent deformation testing was performed during which, the specimens were loaded with three stress stages at the ratios of the dynamic deviator stress ( $\sigma_d$ ) with frequency of 0.33 Hz to the static confining stress ( $\sigma_3$ ), each involving 10,000 cycles for each particular stress condition. After permanent deformation tests, in accordance with this standard (Voung and Brimble 2000), the same specimens were applied sequentially by the difference of the 65 stress stages straightaway to conduct the resilient modulus test to check the elastic condition of each specimen

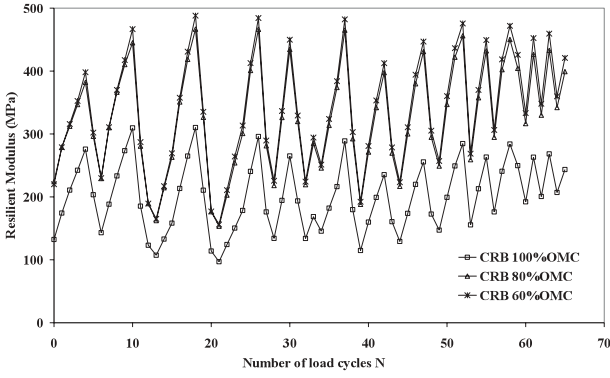


Fig. 5 Resilient modulus at various OMC

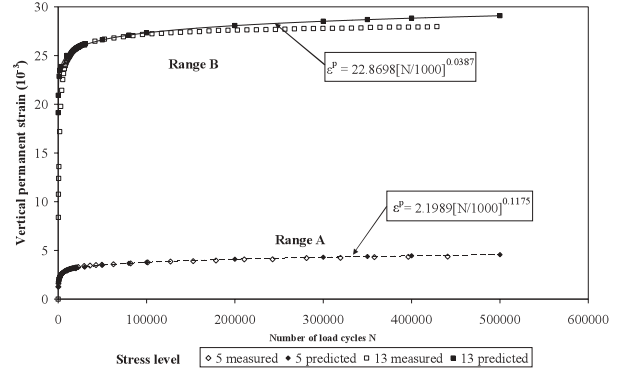


Fig. 9 Range A and B vertical permanent strain compared with the strain model

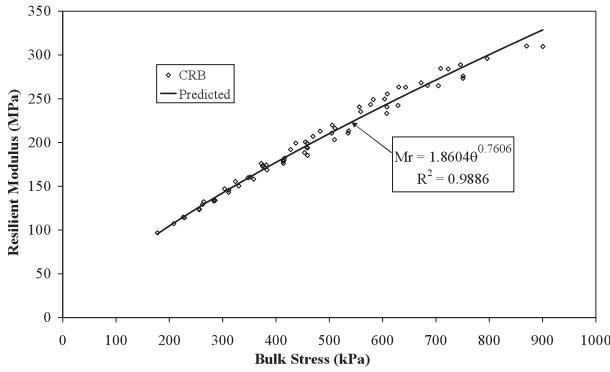


Fig. 6 The resilient modulus predictions

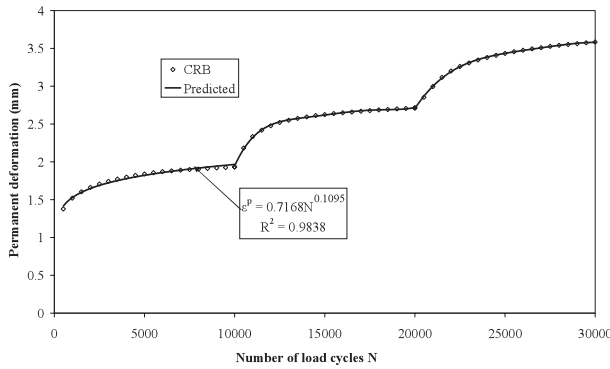


Fig. 7 The permanent deformation predictions

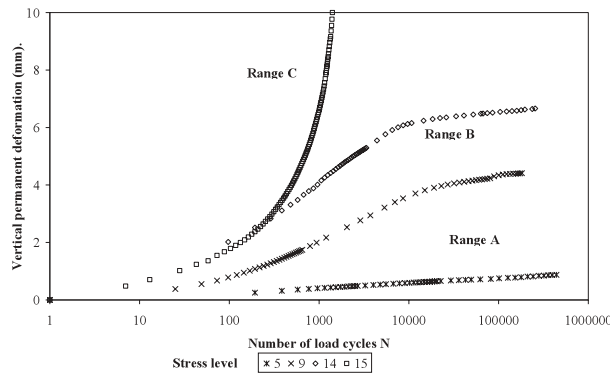


Fig. 8 Vertical permanent deformation versus number of load cycles ( $N$ )

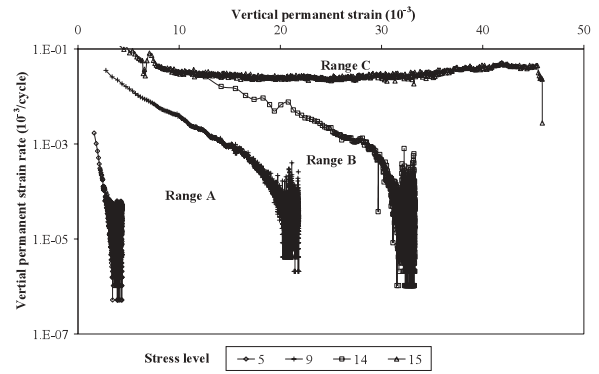


Fig. 10 Vertical permanent strain rate versus vertical permanent strain

throughout the multiple loading stress stages. This process simulates complicated traffic loading acting on pavement. Two hundred loading cycles of each stress stage were applied to the specimens.

## RESULTS AND DISCUSSION

### Resilient Modulus Tests and Permanent Deformation Tests

The resilient modulus determined from the RLT test is defined as the ratio of the repeated deviator stress to the recoverable or resilient axial strain:

$$M_r = \frac{\sigma_d}{\epsilon_r} \quad (1)$$

where  $M_r$  is the resilient modulus,  $\sigma_d$  is the repeated deviator stress (cyclic stress in excess of confining pressure), and  $\epsilon_r$  is the recoverable strain in a vertical direction. Based on the specification of CRB, the results of CRB in the condition of 100% MDD at 100%, 80% and 60% OMC are represented to show its characteristics as shown in Fig. 5 and to determine suitable mathematical models of resilient modulus and permanent

deformation of CRB. Figure 5 shows that the resilient modulus responses were improved relatively large after changing its moisture to 80% and 60% OMC. However, there is a slight different improvement between 80% and 60% OMC.

Figure 6 shows the results of the resilient modulus test which are plotted versus the bulk stress ( $\sigma_1 + \sigma_2 + \sigma_3$ ). Generally, they are non-linear with respect to the magnitude of applied stresses. Figure 6 also shows the results of resilient modulus of CRB can be modeled reasonably by using The  $K$ -Theta ( $K$ - $\theta$ ) model (Hick and Monosmith 1971). The representative  $K$ - $\theta$  model of CRB is exhibited in Eqn (2).

$$M_r = k_1 \cdot \theta^{k_2} = 1.8604 \theta^{0.7606} \quad (2)$$

where  $M_r$  is resilient modulus in MPa;  $\theta$  is bulk stress ( $\sigma_1 + \sigma_2 + \sigma_3$ ) where ( $\sigma_2 = \sigma_3$ );  $\sigma_1$  is major principal stress (vertical axial stress);  $\sigma_3$  is minor principal stress (confining stress);  $k_1 = 1.8601$  and  $k_2 = 0.7606$  are regression coefficients as shown in Fig. 6.

Figure 7 shows the typical results of the permanent deformation tests in terms of the relationship between permanent deformation and loading cycles for CRB and exhibits the comparison of the measured and permanent deformation values and the predicted values for a proposed permanent deformation model of CRB. Figure 7 also indicates that the permanent deformation can be modeled quite reasonably for CRB by using the model suggested by Sweere, G.T.H from SAMARIS (SAMARIS 2004). Sweere suggested for the long-term deformation behavior of unbound granular materials (UGMs) under a large number of load cycles an approach should be employed as the proposed permanent deformation model of CRB as shown in Eqn (3).

$$\varepsilon^p = A \cdot N^B = 0.7168 \cdot N^{0.1095} \quad (3)$$

where  $\varepsilon^p$  is permanent deformation in Millimeters;  $A=0.7168$  and  $B=0.1095$  are regression constants; and  $N$  is the number of loading cycles.

#### Shakedown Behavior

The permanent deformation accumulations were observed as shown in Fig. 8. As test results present CRB response always produce permanent deformation during cyclic loading, hence it can describe no purely elastic behavior under repeated cyclic loads in course base materials (Werkmeister, Dawson et al. 2001) and the multi-layer linear elastic theory is not enough to analyze the UGM layer. Permanent deformation behavior is described on the basis of internal friction between grains,

particle shape, compaction, consolidation, distortion, etc and test results can be separated into three ranges (Ranges A, B and C) based on the shakedown concept.

#### Range A - Plastic Shakedown Range

The lower lines (Stress levels 5-9) in Fig. 8 indicate the response of Range A. The behavior is entirely plastic for a number of cyclic load cycles although when it reaches a stable state after the post-compaction period, the response becomes completely resilient and no further vertical permanent displacement occur as Figs. 8 and 10. In Fig. 10, the vertical permanent strain rate decreases rapidly until it reaches a state of equilibrium compared with the strain model and model coefficients as shown in Fig. 9. For this range of material, the response amount of vertical displacement accumulation depends upon the stress level. Observation of each stress level shows the number of cycles required before a stable state is achieved. CRB behavior in these stress levels would become stable after post-compaction under service load. Also, Range A of the shakedown behavior is allowed in the pavement, an adequately small accumulated displacement, as acceptable permanent deformation would be seen in a course base layer and this would terminate after a set number of load cycles. CRB does not reach failure.

#### Range B- Plastic Creep

Figures 10 show an intermediate response of Range B (Stress levels 10-14). At the beginning of the load cycles, the level of permanent strain rate decreases rapidly but is less than Range A at the same time at a lower rate. The number of load cycles may define the end of post-compaction. A slow increase of the permanent strain rate occurred after 80,000 load cycles (Fig. 8). Test results were observed that although the deformation is not completely resilient, permanent deformation is acceptable for the first period of the cycles. In Fig. 8, the vertical permanent strains are compared with the strain model and model coefficients as shown in Fig. 9. A great number of failures could occur if the condition does not change and if it is maintained long enough, it deteriorate at the end as Range C.

#### Range C – Incremental Collapse

In Fig. 10, stress level 15 indicates Range C behavior and the permanent strain rate decreases during the first period of load cycles then becomes lower, nearly constant. Failure occurs with a relatively small number of load cycles when the cumulative permanent strain rate increases very rapidly after which the strain rate does not decrease again. CRB does not reach a stable state. Range

C behavior in CRB would be result in the failure of the pavement by shear deformation in the base layer experienced as rutting at the road pavement surface. This range should not develop in a designed pavement standard.

In Fig. 10 there is a complete distinction between behavior of Ranges A, B and C. The different vertical strain responses under the number of cycles with no cessation of the strain accumulation responses in Range C has to be separated from Ranges A and B. These can also be distinguished on the basis of plastic strain rate behavior. With Range A the permanent strain rate decreases rapidly and does not reach the constant level throughout the duration of testing. In Fig. 9, the vertical permanent strains of ranges A, B are compared with the strain model.

It has been shown that the use of the shakedown concept application to UGMs in the pavement analysis is possible. The limit ranges defined in this study, Ranges A, B, and C, occurs in CRB. CRB under fixed stress level conditions shows a relationship between permanent strain and stress level. When a cyclic loading is applied, the sample responds by changing its permanent strain. In a continuous and gradual increase of the loading amplitude  $\Delta\sigma$ , the material will start by trying to change the mechanical behavior. The possibility of purely elastic approach in pavement analysis is also discarded as no purely elastic response is found in the CRB during repeated cyclic loading.

The Range A limit (plastic shakedown limit) can be used to predict whether or not stable state occurs in the UGM layer of the road structure. The plastic shakedown limit of CRB should be used in the Western Australian pavement design guidelines. It can be shown that the maximum stresses occurring in the pavement UGM are within Range A. Based on pavement design guidelines, the approximate working stress of Western Australian road was level 11 at the CRB layer indicating that Range B behavior and possibly deterioration at a number of load repetitions. The new approach has been partially validated by the data from which those guidelines should be derived.

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