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DEVELOPMENT OF SEDIMENT TRANSPORT MODEL AND ITS APPLICATION TO SONGKHLA LAKE BASIN

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ABSTRACT: In the past few decades, rapid population growth in Thailand has resulted in excess utilization of natural resources and degradation of environmental quality. Deforestation in sloping areas for agricultural and residential purposes has resulted in soil erosion from deforested areas with subsequent sediment transport and deposition in receiving water bodies. In order to evaluate the significance of this problem, a mathematical model is developed to simulate sediment transport phenomena in a receiving water body. In model development, total sediment transport is classified as bedload transport and suspended load transport, which result in two interrelated transport models. Three-dimensional mass balance equation is used as a basic governing equation for suspended load transport model, whereas two-dimensional mass balance equation is used as a basic governing equation for the bedload transport model. The finite element method is used to solve these governing equations. Since sediment grain size and specific gravity are important factors affecting sediment transport either in the form of bedload or suspended load, the simulation models are developed for each group of sediment grain size and specific gravity, and then the simulated sediment concentrations of various groups are combined to obtain spatial distribution of total sediment concentrations at each time step. The inflow sediment along the boundary of water body is classified into corresponding groups based on their grain size and specific gravity. The developed model is applied to simulate sediment transport pattern in Songkhla Lake which is one of the most important water resources in Southern Thailand.

Key words: Sediment transport, suspended load transport, bedload transport, Songkhla lake.

INTRODUCTION

Sediment transport occurs in two main modes: bedload and suspended load. The bedload transport is the transport of coarse material along the bottom of the water whereas the suspended load transport is the transport of sediment suspended in water column. Estimation of bedload sediment transport is mainly based on the concept of bottom shear stress (Meyer-Peter and Muller, 1984). Grain size and specific gravity are important parameters which affect sediment settling velocity in water column and also affect the rate of resuspension of bedload sediment from the water bed. These two parameters must be considered in detail in the development of sediment transport model.

OBJECTIVE

The objective of this study is to develop a sediment transport model consisting of suspended sediment transport model and bedload transport model which are linked to each other, taken into consideration the effects of sediment grain size and specific gravity. The developed model is applied to Songkhla lake, one of the most important water resources in the south of Thailand, as a case study.

MODEL FORMULATION

Modeling Concept

The formulated sediment transport model consists of 2 interrelated models, i.e. 1) suspended sediment transport (SST) model and 2) bedload transport (BLT) model. These two models must be run together using the rate of sediment settling as a sink term of the SST model and as a source term of the BLT model. On the other hand, the rate of bedload resuspension is considered as a sink term of the BLT model and as a source term of the SST model.

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As previously mentioned, the important factors affecting sediment transport are sediment settling velocity and resuspension rate, both of which depend on sediment grain size (*D*) and specific gravity (*s*). In this study, inflow sediment loads from various sources are divided into several groups based on their grain size (*D*) and specific gravity (*s*). The developed models are applied to estimate the transport of sediment in each group. Then, the total suspended sediment concentration and bedload sediment are computed from Eq. (1) and Eq. (2).

$$c_{total}(x,y,z,t) = \sum_j \sum_k c^{jk}(x,y,z,t) \quad (1)$$

$$c_{b,total}(x,y,t) = \sum_j \sum_k c_b^{jk}(x,y,t) \quad (2)$$

In which

$c_{total}(x,y,z,t)$ is total suspended sediment concentration at (x,y,z) at time t;

$c^{jk}(x,y,z,t)$ is concentration of suspended sediment with grain size D_j and specific gravity s_k at (x,y,z) at time t.

$c_{b,total}(x,y,z,t)$ is the amount of bedload sediment at (x,y) at time t;

$c_b^{jk}(x,y,t)$ is amount of bedload sediment with grain size D_j and specific gravity s_k at (x,y) at time t.

Suspended Sediment Transport Model

The governing equation for the SST model is the three-dimensional mass transport equation as Eq. (3).

$$\frac{\partial c^{jk}}{\partial t} + u\frac{\partial c^{jk}}{\partial x} + v\frac{\partial c^{jk}}{\partial y} + \left(w - v_s^{jk}\right)\frac{\partial c^{jk}}{\partial z} - \frac{\partial}{\partial x}\left(K_x \frac{\partial c^{jk}}{\partial x}\right) - \frac{\partial}{\partial y}\left(K_y \frac{\partial c^{jk}}{\partial y}\right) - \frac{\partial}{\partial z}\left(K_z \frac{\partial c^{jk}}{\partial z}\right) - R^{jk} + S_d^{jk} = 0 \quad (3)$$

where c^{jk} is suspended sediment concentration at (x,y,z) at time t; u, v, w are flow velocities at (x,y,z) in the x, y and z directions respectively at time t; v_s^{jk} is terminal settling velocity of sediment with grain size D_j and specific gravity s_k; K_x, K_y and K_z are dispersion coefficients in the x, y and z directions, respectively; R^{jk} is resuspension rate; and S_d^{jk} is suspended particles decaying rate.

In the weighted residual method the variable c^{jk} in the mass balance equation is replaced by an approximate function $\hat{c}^{jk}$ which is written in terms of the nodal concentrations as Eq. (4).

$$\hat{c}^{jk} = \sum_{i=1}^{n} N_i C_i^{jk} = \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{C}^{jk} \quad (4)$$

The error or residual which occurs from this approximation is multiplied with a weighting function w_f and the integral of the product over the whole study domain is set to zero. This results in the following weighted residual equation:

$$\iiint_\Omega w_f \left\{ \frac{\partial \hat{c}^{jk}}{\partial t} + u\frac{\partial \hat{c}^{jk}}{\partial x} + v\frac{\partial \hat{c}^{jk}}{\partial y} + \left(w - v_s^{jk}\right)\frac{\partial \hat{c}^{jk}}{\partial z} - \frac{\partial}{\partial x}\left(K_x \frac{\partial \hat{c}^{jk}}{\partial x}\right) - \frac{\partial}{\partial y}\left(K_y \frac{\partial \hat{c}^{jk}}{\partial y}\right) - \frac{\partial}{\partial z}\left(K_z \frac{\partial \hat{c}^{jk}}{\partial z}\right) - R^{jk} + S_d^{jk} \right\} dV = 0 \quad (5)$$

By integrating by part, the following equation is obtained Eq. (6).

$$\iiint_\Omega w_f \left\{ \frac{\partial \hat{c}^{jk}}{\partial t} + u\frac{\partial \hat{c}^{jk}}{\partial x} + v\frac{\partial \hat{c}^{jk}}{\partial y} + \left(w - v_s^{jk}\right)\frac{\partial \hat{c}^{jk}}{\partial z} - R^{jk} + S_d^{jk} \right\} dV + \iiint_\Omega \left\{ K_x \frac{\partial w_f}{\partial x}\frac{\partial \hat{c}^{jk}}{\partial x} + K_y \frac{\partial w_f}{\partial y}\frac{\partial \hat{c}^{jk}}{\partial y} + K_z \frac{\partial w_f}{\partial z}\frac{\partial \hat{c}^{jk}}{\partial z} \right\} dV - \iiint_\Omega \left\{ \frac{\partial}{\partial x}\left(w_f K_x \frac{\partial \hat{c}^{jk}}{\partial x}\right) + \frac{\partial}{\partial y}\left(w_f K_y \frac{\partial \hat{c}^{jk}}{\partial y}\right) + \frac{\partial}{\partial z}\left(w_f K_z \frac{\partial \hat{c}^{jk}}{\partial z}\right) \right\} dV = 0 \quad (6)$$

By using Divergence Theorem, the last volume integral in Eq. (6) can be replaced by surface integral as Eq. (7).

$$\iiint_\Omega \left\{ \frac{\partial}{\partial x}\left(w_f K_x \frac{\partial \hat{c}^{jk}}{\partial x}\right) + \frac{\partial}{\partial y}\left(w_f K_y \frac{\partial \hat{c}^{jk}}{\partial y}\right) + \frac{\partial}{\partial z}\left(w_f K_z \frac{\partial \hat{c}^{jk}}{\partial z}\right) \right\} dV = \iint_S w_f \left\{ \left(K_x \frac{\partial \hat{c}^{jk}}{\partial x}\right)\bar{i} + \left(K_y \frac{\partial \hat{c}^{jk}}{\partial y}\right)\bar{j} + \left(K_z \frac{\partial \hat{c}^{jk}}{\partial z}\right)\bar{k} \right\} . \boldsymbol{n} dA \quad (7)$$

in which

$\bar{i}, \bar{j}, \bar{k}$ are unit vectors in the x, y and z directions, respectively.

n is unit vector normal to the domain boundary with direction outward from the study domain.

The term $\left\{ \left(K_x \frac{\partial \hat{c}^{jk}}{\partial x}\right)\bar{i} + \left(K_y \frac{\partial \hat{c}^{jk}}{\partial y}\right)\bar{j} + \left(K_z \frac{\partial \hat{c}^{jk}}{\partial z}\right)\bar{k} \right\} . \boldsymbol{n}$ can be replaced by the sediment influx rate per unit area of the boundary. Then, Eq. (6) can be written as Eq. (8).

$$\iiint_\Omega w_f \left\{ \frac{\partial \hat{c}^{jk}}{\partial t} + u\frac{\partial \hat{c}^{jk}}{\partial x} + v\frac{\partial \hat{c}^{jk}}{\partial y} + \left(w - v_s^{jk}\right)\frac{\partial \hat{c}^{jk}}{\partial z} - R^{jk} + S_d^{jk} \right\} dV + \iiint_\Omega \left(K_x \frac{\partial w_f}{\partial x}\frac{\partial \hat{c}^{jk}}{\partial x} \right) + \left(K_y \frac{\partial w_f}{\partial y}\frac{\partial \hat{c}^{jk}}{\partial y} \right) + \left(K_z \frac{\partial w_f}{\partial z}\frac{\partial \hat{c}^{jk}}{\partial z} \right) dV - \iint_S w_f q_n^{jk} dA = 0 \quad (8)$$

where q_n^{jk} is the influx rate per unit area of sediment with grain size D_j and specific gravity s_k.

In the Galerkin's method, the shape functions or interpolation functions N_i (i = 1,2,..n) are used as weighting function w_f. This results in the following weighted residual equations:

$$\iiint_\Omega N_i\left\{\frac{\partial \hat{c}^{jk}}{\partial t}+u\frac{\partial \hat{c}^{jk}}{\partial x}+v\frac{\partial \hat{c}^{jk}}{\partial y}+\left(w-v_s^{jk}\right)\frac{\partial \hat{c}^{jk}}{\partial z}-R^{jk}+S^{jk}\right\}dV$$

$$+\iiint_\Omega\left\{K_x\frac{\partial N_i}{\partial x}\frac{\partial \hat{c}^{jk}}{\partial x}+K_y\frac{\partial N_i}{\partial y}\frac{\partial \hat{c}^{jk}}{\partial y}+K_z\frac{\partial N_i}{\partial z}\frac{\partial \hat{c}^{jk}}{\partial z}\right\}dV$$

$$-\iint_S N_i q_n^{jk}dA=0 \qquad (i=1,2,\dots n) \tag{9}$$

which can be written in matrix form as:

$$\iiint_\Omega \boldsymbol{N}\left\{\frac{\partial \hat{c}^{jk}}{\partial t}+u\frac{\partial \hat{c}^{jk}}{\partial x}+v\frac{\partial \hat{c}^{jk}}{\partial y}+\left(w-v_s^{jk}\right)\frac{\partial \hat{c}^{jk}}{\partial z}-R^{jk}+S_d^{jk}\right\}dV$$

$$+\iiint_\Omega\left\{K_x\frac{\partial \boldsymbol{N}}{\partial x}\frac{\partial \hat{c}^{jk}}{\partial x}+K_y\frac{\partial \boldsymbol{N}}{\partial y}\frac{\partial \hat{c}^{jk}}{\partial y}+K_z\frac{\partial \boldsymbol{N}}{\partial z}\frac{\partial \hat{c}^{jk}}{\partial z}\right\}dV$$

$$-\iint_S q_n^{jk}\boldsymbol{N}dA = \boldsymbol{0} \tag{10}$$

Besides function $\hat{c}^{jk}$, flow velocities u, v, w can be written in terms of their nodal values in the similar form. After substituting into Eq. (10), we obtain:

$$\iiint_\Omega \boldsymbol{N}\left\{\frac{\partial\left(\boldsymbol{N^T C}^{jk}\right)}{\partial t}+\boldsymbol{N^T U}\frac{\partial\left(\boldsymbol{N^T C}^{jk}\right)}{\partial x}+\boldsymbol{N^T V}\frac{\partial\left(\boldsymbol{N^T C}^{jk}\right)}{\partial y}\right.$$

$$\left.+\boldsymbol{N^T W}\frac{\partial\left(\boldsymbol{N^T C}^{\mathbf{jk}}\right)}{\partial z}-v_s^{jk}\frac{\partial\left(\boldsymbol{N^T C}^{jk}\right)}{\partial z}-R^{jk}+S_d^{jk}\right\}dV$$

$$+\iiint_\Omega\left\{K_x\frac{\partial \boldsymbol{N}}{\partial t}\frac{\partial\left(\boldsymbol{N^T C}^{jk}\right)}{\partial x}+K_y\frac{\partial \boldsymbol{N}}{\partial y}\frac{\partial\left(\boldsymbol{N^T C}^{jk}\right)}{\partial y}+K_z\frac{\partial \boldsymbol{N}}{\partial z}\frac{\partial\left(\boldsymbol{N^T C}^{jk}\right)}{\partial z}\right\}dV$$

$$-\iint_S q_n^{jk}\boldsymbol{N}dA = \boldsymbol{0} \tag{11}$$

which can be rearranged as:

$$\iiint_\Omega \boldsymbol{NN^T}dV\frac{\partial \boldsymbol{C}^{jk}}{\partial t}+\left[\iiint_\Omega \boldsymbol{NN^T U}\frac{\partial \boldsymbol{N^T}}{\partial x}dV+\iiint_\Omega \boldsymbol{NN^T V}\frac{\partial \boldsymbol{N^T}}{\partial y}dV\right.$$

$$+\iiint_\Omega \boldsymbol{NN^T W}\frac{\partial \boldsymbol{N^T}}{\partial z}dV-\iiint_\Omega v_s^{jk}\boldsymbol{N}\frac{\partial \boldsymbol{N^T}}{\partial z}dV+\iiint_\Omega K_x\frac{\partial \boldsymbol{N}}{\partial x}\frac{\partial \boldsymbol{N^T}}{\partial x}dV$$

$$\left.+\iiint_\Omega K_y\frac{\partial \boldsymbol{N}}{\partial y}\frac{\partial \boldsymbol{N^T}}{\partial y}dV+\iiint_\Omega K_z\frac{\partial \boldsymbol{N}}{\partial z}\frac{\partial \boldsymbol{N^T}}{\partial z}dV\right]\boldsymbol{C}^{jk}-\iiint_\Omega R^{jk}\boldsymbol{N}dV$$

$$+\iiint_\Omega S_d^{jk}\boldsymbol{N}dV-\iint_S q_n^{jk}\boldsymbol{N}dA = 0 \tag{12}$$

which can be written in a compact form as:

$$\boldsymbol{M}\frac{d\boldsymbol{C}^{jk}}{dt}+\boldsymbol{PC}^{jk}-\boldsymbol{Q}=\boldsymbol{0} \tag{13}$$

in which

$$\boldsymbol{M} = \iiint_\Omega \boldsymbol{NN^T}dV \tag{14}$$

$$\boldsymbol{P} = \iiint_\Omega \boldsymbol{NN^T U}\frac{\partial \boldsymbol{N^T}}{\partial x}dV+\iiint_\Omega \boldsymbol{NN^T V}\frac{\partial \boldsymbol{N^T}}{\partial y}dV+\iiint_\Omega \boldsymbol{NN^T W}\frac{\partial \boldsymbol{N^T}}{\partial z}dV$$

$$+\iiint_\Omega \boldsymbol{NN^T W}\frac{\partial \boldsymbol{N^T}}{\partial z}dV-\iiint_\Omega v_s^{jk}\boldsymbol{N}\frac{\partial \boldsymbol{N^T}}{\partial z}dV+\iiint_\Omega K_x\frac{\partial \boldsymbol{N}}{\partial x}\frac{\partial \boldsymbol{N^T}}{\partial x}dV$$

$$+\iiint_\Omega K_y\frac{\partial \boldsymbol{N}}{\partial y}\frac{\partial \boldsymbol{N^T}}{\partial y}dV+\iiint_\Omega K_z\frac{\partial \boldsymbol{N}}{\partial z}\frac{\partial \boldsymbol{N^T}}{\partial z}dV \tag{15}$$

$$\boldsymbol{Q} = \iiint_\Omega R^{jk}\boldsymbol{N}dV-\iiint_\Omega S_d^{jk}\boldsymbol{N}dV+\iint_S q_n^{jk}\boldsymbol{N}dA \tag{16}$$

Eq. (13) is a set of equations for computing distribution of suspended sediment at various nodal points in the study domain. This will be computed alternately with a set of equations for bedload transport which is described below:

Bedload Transport Model

Most studies concerning transport of bedload sediment transport utilize relationship between bedload movement and Shields parameter. In the past few decades, several bedload transport equations have been proposed (Reeve et al. 2004). A dimensionless parameter called "bedload transport rate factor" (Φ) has been used to relate the rate of bedload transport with particle grain size (D) and specific gravity (s). The relationship is as Eq. (17).

$$\Phi = \frac{q_b}{[g(s-1)D^3]^{1/2}} \tag{17}$$

in which q_b is the rate of bedload transport per unit width.

The parameter Φ can be computed from Shields parameter θ_s and critical Shields parameter θ_{cr} as follows (Neilson, 1992)

$$\Phi = 12\theta_s^{1/2}(\theta_s-\theta_{cr}) \tag{18}$$

Shields parameter θ_s and critical Shields parameter θ_{cr} are expressed by (Reeve et al, 2004):

$$\theta_s = \tau/(\rho_s-\rho)gD \tag{19}$$

and $$\theta_{cr} = \tau_{cr}/(\rho_s-\rho)gD \tag{20}$$

in which

τ is bed shear stress which is related to current velocity, wave action and water depth. τ_{cr} is critical bed shear stress over which bedload transport will occur.

ρ_s is particle density of sediment, its ranging from 2.60-2.75 g/cm^3;

ρ is density of water;

D is grain size diameter of sediment.

It can be seen that the grain size diameter and specific gravity of particles affect the value of bedload transport rate factor Φ and the rate of bedload transport rate per unit width q_b . Therefore, in this study the total bedload sediment is divided into groups based on their diameter and specific gravity. The model is developed to simulate transport of each group of bedload sediment. Then, the amount of various groups is combined to obtain the total bed load per unit area at an identified location.

For particles with grain size diameter D_j and specific gravity s_k, the mass balance equation for bedload transport can be written as Eq. (21).

$$\frac{\partial s_b^{jk}}{\partial t} + \frac{\partial Q_x^{jk}}{\partial x} + \frac{\partial Q_y^{jk}}{\partial y} - v_s^{jk} c_b^{jk} + q_r^{jk} = 0 \qquad (21)$$

in which

s_b^{jk} is the amount per unit area of bedload sediment with diameter D_j and specific gravity s_k

Q_x^{jk} and Q_y^{jk} are the rates of bedload transport per unit width in the x and y directions, respectively, of bedload sediment with diameter D_j and specific gravity s_k

v_s^{jk} is terminal settling velocity of particle with diameter D_j and specific gravity s_k

c_b^{jk} is concentration of suspended sediment with diameter D_j and specific gravity s_k just above the sea bed

q_r^{jk} is the resuspension rate of bedload sediment with diameter D_j and specific gravity s_k

The study domain of the BLT model is the bottom boundary of the SST model projected on the horizontal plain (Fig. 1), since the value of variable s_b^{jk} and parameters Q_x^{jk} , Q_y^{jk} and q_r^{jk} are the values on the horizontal planes.

In the weighted residual method, the variable s_b^{jk} is replaced by approximate function $\hat{s}_b^{jk}$ which is written in terms of its nodal values as Eq. (22).

$$\hat{s}_b^{jk} = \sum_{i=1}^{n_b} N_i S_{bi}^{jk} = \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{S}_b^{jk} \qquad (22)$$

in which

S_{bi}^{jk} is the amount per unit area at node i for bedload sediment with diameter D_j and specific gravity s_k

N_i is interpolation function

$\boldsymbol{S}_b^{jk}$ is matrix of S_{bi}^{jk}

$\boldsymbol{N}$ is matrix of N_i

n_b is total number of nodes in the study domain

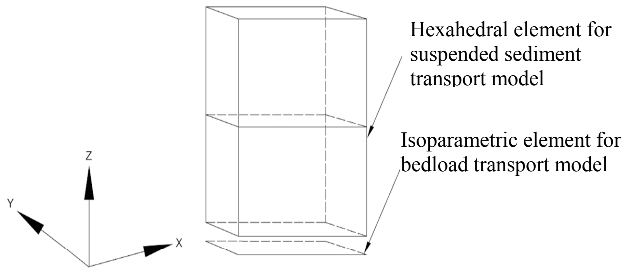

Fig. 1 Types of elements used in the developed sediment transport model

When s_b^{jk} in Eq. (21) is replaced by $\hat{s}_b^{jk}$, an error or residual will occur. This residual is multiplied with a weighting function w_s and the integral of their product over the whole study domain is set to zero. This results in the D_j following weighted residual equation:

$$\iint_A w_s \left\{ \frac{\partial \hat{s}_b^{jk}}{\partial t} + \frac{\partial Q_x^{jk}}{\partial x} + \frac{\partial Q_y^{jk}}{\partial y} - v_s^{jk} c_b^{jk} + q_r^{jk} \right\} dA = 0 \qquad (23)$$

In Galerkin's technique, the interpolation function N_i (i =1,2,..,n_b) are used as a weighting function. Therefore, the following set of weighted residual equations is obtained:

$$\iint_A N_i \left\{ \frac{\partial \hat{s}_b^{jk}}{\partial t} + \frac{\partial Q_x^{jk}}{\partial x} + \frac{\partial Q_y^{jk}}{\partial y} - v_s^{jk} c_b^{jk} + q_r^{jk} \right\} dA = 0 \qquad (i = 1,2,..,n_b) \quad (24)$$

which can be written in the matrix form as:

$$\iint_A \boldsymbol{N} \left\{ \frac{\partial \hat{s}_b^{jk}}{\partial t} + \frac{\partial Q_x^{jk}}{\partial x} + \frac{\partial Q_y^{jk}}{\partial y} - v_s^{jk} c_b^{jk} + q_r^{jk} \right\} dA = \boldsymbol{0} \qquad (25)$$

Besides $\hat{s}_b^{jk}$, the parameters Q_x^{jk} , Q_y^{jk} and c_b^{jk} can be expressed in terms of their nodal values in the similar form. When substituting into Eq. (25) we obtain:

$$\iint_A \boldsymbol{N} \left\{ \frac{\partial}{\partial t}\left(\boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{S}_b^{jk}\right) + \frac{\partial\left(\boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{Q}_x^{jk}\right)}{\partial x} + \frac{\partial\left(\boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{Q}_y^{jk}\right)}{\partial y} - v_s^{jk} \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{C}_b^{jk} + q_r^{jk} \right\} dA = \boldsymbol{0} \qquad (26)$$

which can be arranged as:

$$\iint_A \boldsymbol{N}\boldsymbol{N}^{\boldsymbol{T}} dA \frac{\partial \boldsymbol{S}_b^{jk}}{\partial t} + \iint_A \boldsymbol{N} \frac{\partial \boldsymbol{N}^{\boldsymbol{T}}}{\partial x} dA \boldsymbol{Q}_x^{jk} + \iint_A \boldsymbol{N} \frac{\partial \boldsymbol{N}^{\boldsymbol{T}}}{\partial y} dA \boldsymbol{Q}_y^{jk} - \iint_A v_s^{jk} \boldsymbol{N}\boldsymbol{N}^{\boldsymbol{T}} dA \boldsymbol{C}_b^{jk} + \iint_A q_r^{jk} \boldsymbol{N} dA = \boldsymbol{0} \qquad (27)$$

or in a compact form as:

$$\boldsymbol{M} \frac{d\boldsymbol{S}_b^{jk}}{dt} + \boldsymbol{M}_x \boldsymbol{Q}_x^{jk} + \boldsymbol{M}_y \boldsymbol{Q}_y^{jk} - \boldsymbol{M}_v^{jk} \boldsymbol{C}_b^{jk} + \boldsymbol{M}_b^{jk} = 0 \qquad (28)$$

in which

$$M = \iint_A N N^T dA \quad (29)$$

$$M_x = \iint_A N \frac{\partial N^T}{\partial x} dA \quad (30)$$

$$M_y = \iint_A N \frac{\partial N^T}{\partial y} dA \quad (31)$$

$$M_v^{jk} = \iint_A v_s^{jk} N N^T dA \quad (32)$$

$$M_b^{jk} = \iint_A q_r^{jk} N dA \quad (33)$$

In the finite element method, the study domain is divided into elements with nodal points. The variables and parameters at a point in each element are expressed in terms of the values at nodal points of that element. The matrices ***M, P*** and ***Q*** in Eq. (13) and the matrices ***M***, $\boldsymbol{M_x}$, $\boldsymbol{M_y}$, $\boldsymbol{M_v}$ and $\boldsymbol{M_b}$ in Eq. (28) can be determined for each element which result in element matrices. These element matrices are then assembled to form system matrices.

Computation Procedure

Computation of suspended sediment transport and bedload transport will be made alternately using Eq. (13) and Eq. (28), respectively. The computation procedure can be described as follows (Fig. 2):

1) From sediment sampling data, grain size of sediment particles and specific gravity are divided into groups. Let D_j and s_k be mean diameter and specific gravity of group *jk.* Total sediment load from watershed area is estimated. The load of each group is determined based on the grain size distribution data which are previously analyzed.

2) For each group of grain size and specific gravity, the terminal settling velocity is determined. Also, the sediment resuspension rate is estimated from some empirical formula. With these parameters together with flow velocity data, which are normally obtained from the hydrodynamic model, the matrices ***M, P*** and ***Q*** in Eq. (13) can be computed.

3) Given initial values of suspended sediment concentrations at all nodal points in the study domain, the nodal concentrations at the next time step can be determined from Eq. (13).

4) From flow velocity data, water depth, wind and wave data at each nodal point, bed shear stress can be estimated. Then, Shields parameter θ_s is computed from Eq. (19). Also, the critical bed shear stress τ_{cr} and the critical Shields parameter θ_{cr} can be determined from Eq. (20). Then, the bedload transport rate factor Φ is computed from Eq. (18) and the rate of bedload transport rate per unit width q_b^{jk} can be computed from Eq. (17).

5) At each nodal point, multiply q_b^{jk} with bedload sediment bulk density and gravitational acceleration *g* to obtain the bedload transport rate Q_n^{jk} in terms of weight per unit width per unit time.

6) Compute Q_x^{jk} and Q_y^{jk} from the following equations:

$$Q_x^{jk} = \frac{u}{|V|} Q_n^{jk} \quad (34)$$

and

$$Q_y^{jk} = \frac{v}{|V|} Q_n^{jk} \quad (35)$$

in which

u and *v* are current velocities in the *x* and *y* directions, respectively

$|V| = \sqrt{u^2 + v^2}$ is overall current velocity

7) Arrange the values of Q_x^{jk} and Q_y^{jk} at all nodal points on the bed to form matrices $\boldsymbol{Q_x^{jk}}$ and $\boldsymbol{Q_y^{jk}}$, respectively.

8) From the selected element configuration on the water bed, determine element matrices and then assembled to the system matrices ***M***, $\boldsymbol{M_x}$, $\boldsymbol{M_y}$, $\boldsymbol{M_v}$ and $\boldsymbol{M_b}$ by using Eqs. (29) – (33).

9) Given initial values of bedload sediment at all nodal points in the study domain, the nodal values at the next time step can be determined from Eq. (28).

10) Computation in the next time step is then repeated following the procedures from (2) to (9) described above.

MODEL APPLICATION

The developed model is applied to study sediment transport in Songkhla lake which is one of the most important water resources in the southern part of Thailand.

The Songkhla Lake

The Songkhla lake is a natural lake (lagoon) located in the south of Thailand. It has very unique characteristics, with 3 water ecosystems, i.e. fresh water, brackish water and saline water. The lake covers about 1,042 km^2 whereas the catchment area covers about 8,754 km^2 consisting of 13 sub-basins. Water depth in the lake varies in the range of 1-8 m.

Nowadays, large portion of its watershed area has been facing erosion problem. Human activities exert a profound influence on soil erosion. About 288,000 hectares (27.3% of the basin area) has an erosion rate exceeding 12.5 tons/hectare/year. Out of this, over 112,000 hectares has an erosion rate exceeding 93.75 tons/hectare/year which is rather severe (Songrakkiat, 2007). The main sources of sediment in the lake are from 3 sources, i.e. 1) sediment from eroded coastline surrounding the lake; 2) sediment from surface runoff; and 3) sediment from decomposition of humus in the lake. In some previous studies, researchers attempted to present many models for predicting the sedimentation rate in the lake. Chittrakarn (1996) determined the sedimentation rate in the lake by using isotope Cs-137 and reported that the sedimentation rate was 5-6 mm/year. VKI et al. (1999) reported that the sedimentation rate was in the range of 0.1-0.4 mm/year. ONEP (2006) estimated that the sedimentation rate was about 0.04-0.19 mm/year.

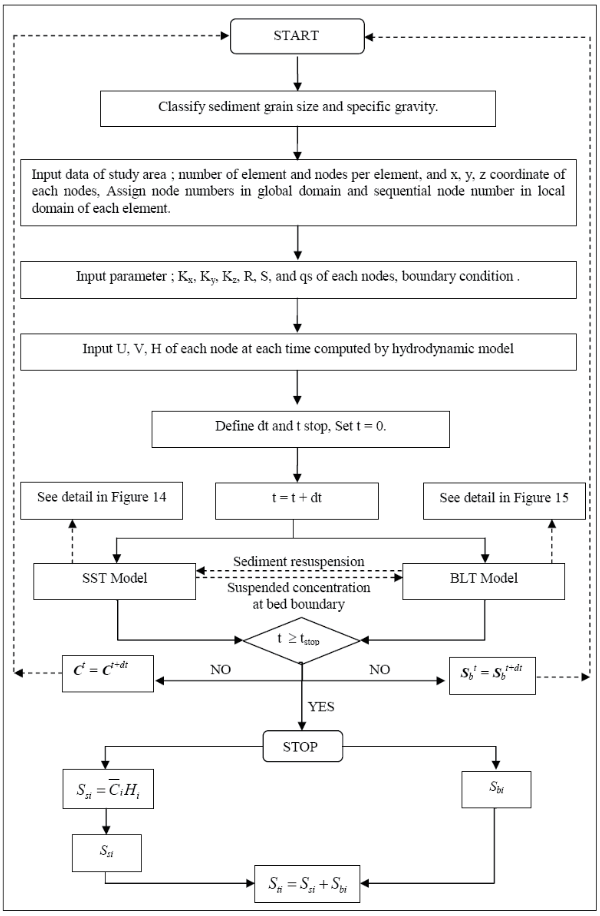

Fig. 2 Computation procedure for sediment transport model

Suspended Sediment Sampling and Analyses

In this study, water samples are collected at 41 stations which include 16 stations in canals from the sub-basins and 25 stations inside the lake (Fig. 3). Total suspended solid concentration and particle size distribution of the collected samples are determined. The obtained results are shown in Tables 1 and 2. It is found that the total suspended sediment concentrations in canal waters and in the lake is not so high, normally less than 100 mg/L. Seasonal variations in suspended sediment concentrations are not significant. Since there is no significant difference in specific gravity of sediment from various sources, the sediment load from each sub-basin is divided into 3 groups with average grain size of 20 μm, 35 μm and 100 μm.

Element Configuration

In this study, the hexahedral elements are used for the three-dimensional suspended sediment transport (SST) model and the isoparametric elements are used for the two-dimensional bedload transport (BLT) model. The total area of 1,042 km^2 of Songkhla lake is divided into 414 hexahedral elements with 880 nodal points. The bottom boundary of the lake projected on a horizontal plane is divided into 138 isoparametric elements with 220 nodal points as shown in Fig. 4.

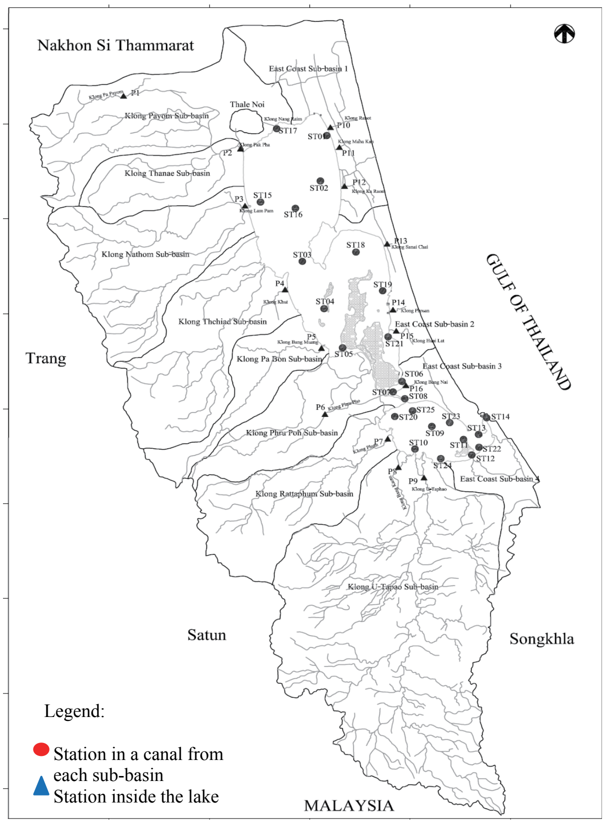

Fig. 3 Sampling stations in Songkhla Lake Basin

Table 1 Total suspended solids in Songkhla Lake

Station	Area	TSS (mg/L)		
		Sep 06	Dec 06	Feb 06
ST01	Northern Lake	6.25	29.50	41.50
ST02	Northern Lake	29.00	19.00	25.00
ST03	Northern Lake	101.50	26.50	108.00
ST04	Central Lake	1.00	61.00	87.00
ST05	Central Lake	12.50	17.00	41.00
ST06	Central Lake	114.50	14.00	31.67
ST07	Central Lake	20.25	15.00	51.00
ST08	Central Lake	13.25	13.25	92.00
ST09	Southern Lake	10.25	17.00	11.83
ST10	Southern Lake	25.50	12.50	66.50
ST11	Southern Lake	40.50	7.00	28.00
ST12	Southern Lake	7.50	14.00	13.00
ST13	Southern Lake	20.00	5.25	70.75
ST14	Southern Lake	19.75	21.50	11.25
ST15	Northern Lake	26.50	16.50	46.50
ST16	Northern Lake	47.00	29.00	31.00
ST17	Northern Lake	17.75	8.00	61.00
ST18	Central Lake	150.50	11.00	153.00
ST19	Central Lake	65.50	23.00	174.50
ST20	Southern Lake	11.50	22.00	58.50
ST21	Central Lake	17.50	46.50	86.00
ST22	Southern Lake	9.00	16.75	16.75
ST23	Southern Lake	8.75	2.25	20.50
ST24	Southern Lake	14.00	14.00	5.25
ST25	Southern Lake	12.50	19.25	49.50

Model Input Data

Besides coordinates of all nodal points in the study domain, the sediment transport model requires data on flow velocities at each nodal point, water depth, sediment load and sediment resuspension rate (for each group of grain size). Since the Songkhla lake is a shallow lake and most part of Songkhla Lake is approximately 1.5 m deep, the vertically average hydrodynamic model is used to simulate flow velocities u and v and water depth h at nodal points of the isoparametric elements on the bottom plane while the vertical component of velocity w is neglected. Besides, we calculated vertical velocity in term of settling velocity (v_s) of sediments. Regarding sediment load data, the results of soil erosion study in the Songkhla lake basin conducted by Songrakkiat (2007) are used. Data on suspended sediment concentrations obtained from field sampling are used as initial values of the SST model. For the initial value of bedload sediment, it is assumed that there is no bedload sediment at time t = 0. Therefore, the results obtained from the BLT model will reveal the locations where accumulation of bedload sediment occurs. In this study, the effect of wind driven wave is not considered due to the lack of reliable wind data.

Table 2 Total suspended solids in Songkhla lake sub-basin

Station	Sub-basin	TSS (mg/L)									
		June 06	July 06	Aug 06	Sep 06	Oct 06	Nov 06	Dec 06	Jan 06	Feb 06	Mar 06
P1	Klong Pa Payom Sub-basin	3.00	5.25	1.00	2.00	1.00	8.25	44.00	4.50	4.25	5.00
P2	Klong Thanae Sub-basin	51.0	45.00	22.50	12.00	12.50	14.00	19.75	13.50	31.00	92.00
P3	Klong Nathom Sub-basin	42.5	31.50	5.50	8.00	11.00	10.75	12.25	10.00	18.25	17.50
P4	Klong Tachiad Sub-basin	17.5	11.75	12.50	6.00	12.50	8.50	8.25	4.75	31.00	24.00
P5	Klong Pa Bon Sub-basin	13.0	24.00	2.50	4.00	8.50	13.50	10.75	6.00	23.25	4.25
P6	Klong Phru Poh Sub-basin	18.0	9.75	2.50	3.75	13.50	6.25	27.00	5.25	6.50	20.50
P7	Klong Rattaphum Sub-basin	42.5	32.50	7.50	8.75	15.00	25.00	58.50	33.25	20.00	7.83
P8	Klong Bang Klao (Bang Yee Tample)	27.5	16.00	5.00	12.00	12.00	14.50	15.00	15.25	18.50	8.50
P9	Klong Pa Bon Sub-basin	17.5	17.50	6.50	47.00	22.50	7.25	9.00	7.50	13.50	15.00
P10	Klong Ranot (East Coast 1)				4.25	74.50	37.50	51.50	34.50	46.50	24.50
P11	Klong Maha Kan (East Coast 1)				3.00	38.00	15.00	34.50	35.50	62.17	52.50
P12	Klong Ka Ram (East Coast 1)				17.00	53.00	20.00	22.00	59.0	49.00	57.00
P13	Klong Sanam Chai (East Coast 2)				8.25	24.75	34.00	26.50	13.75	30.00	27.00
P14	Klong Phruan (East Coast 2)				20.00	42.00	16.50	20.00	21.00	12.75	4.17
P15	Klong Huai Lat (East Coast 2)										23.50
P16	Klong Bang Nai (East Coast 3)										16.25

Fig. 4 Element configuration in Songkhla Lake

The mean depth of the Songkhla Lake is rather shallow, so the average depth model is reasonably applied to the lake. The dispersion and advection values are equal for all direction i.e., Kx = Ky = Kz = 100 m^2/s.

In addition, Songkhla lake is brackish water so the effect of variations in salinity on flocculation process is important. However, salinity in Songkhla lake not over 30 ppt as shown in Fig. 5. We considered salinity(S) is a factor for adjust settling velocity as Eq. (36) and Eq. (37) :

$$v(S) = v(0)\left(1 + \frac{k_1 S}{k_2 + S}\right) \quad (36)$$

$$v(0) = \frac{(s-1)gD^2}{18\upsilon} \quad (37)$$

Where k_1 and k_2 are determined from model verification, k_1=k_2=1 m^2/s.

The equation to calculate the parameter which use in model application is shown in Table 3.

Table 3 Equation for Parameter and variable in model application

Parameter	Equation
Bed shear stress (τ_b)[1]	$\tau_b = \rho C_D V_h^2$
Shield parameter (θ)[2]	$\theta = \frac{\tau_b}{(\rho_s - \rho)gD}$
Critical Shield parameter (θ_{cr})[2]	$\theta_{cr} = \frac{0.3}{1+1.2D^*} + 0.055\left[1 - \exp\left(-0.02D^*\right)\right]$
Critical shear stress (τ_{cr})[2]	$\theta_{cr} = \frac{\tau_{cr}}{(\rho_s - \rho)gD}$
bedload transport rate factor (Φ)[2]	$\Phi = \frac{q_b}{\left[g(s-1)D^3\right]^{\frac{1}{2}}}$
bedload transport rate factor (Φ)[3]	$\Phi = 12\theta_s^{\frac{1}{2}}(\theta_s - \theta_{cr})$
Terminal settling velocity (v)[4]	$v = \frac{(s-1)gD^2}{18\upsilon}$
Sediment resuspension rate[5]	$E_b = 0 \quad ; \tau_b < \tau_{cr}$ $E_b = M\left(\frac{\tau_b}{\tau_{ce}} - 1\right) ; \tau_b > \tau_{cr}$

Sources : [1] Reeve *et al.* (2004)
[2] Shields (1936)
[3] Nielsen (1992)
[4] Van Rijn (1993)
[5] Partheniades E (1965)

RESULTS AND DISCUSSION

The sediment transport in the Songkhla lake is simulated based on current flow data and sediment load data in November 2006 which is the rainy season in this region.

The value of suspended sediment concentration at each nodal point computed from the model is multiplied with average water depth at that point to obtain suspended sediment amount per unit area of lake bottom. The distribution pattern of this suspended sediment amount (in g/m^2) in November 2006 is shown in Fig. 6, whereas the accumulation of bedload sediment is shown in Fig. 7.

The suspended sediment per unit area of lake bottom varies from less than 150 g/m^2 to more than 1,000 g/m^2. The high amounts are found near the mounts of the canals draining sediment into the lake and also in the canal connecting the central Lake and the Southern Lake where current velocity is high. Sediment grain size and resuspension from the lake bottom play significant role in suspended sediment distribution.

The amount of bedload sediment computed from the model is mostly less than 50 g/m^2 since it is the accumulation over one month period of model simulation. However, a few locations are found to have high bedload sediment accumulation. These locations are associated with high suspended sediment amount too. So, the high accumulation of bedload sediment might be due to deposition of suspended sediment from water column in those areas

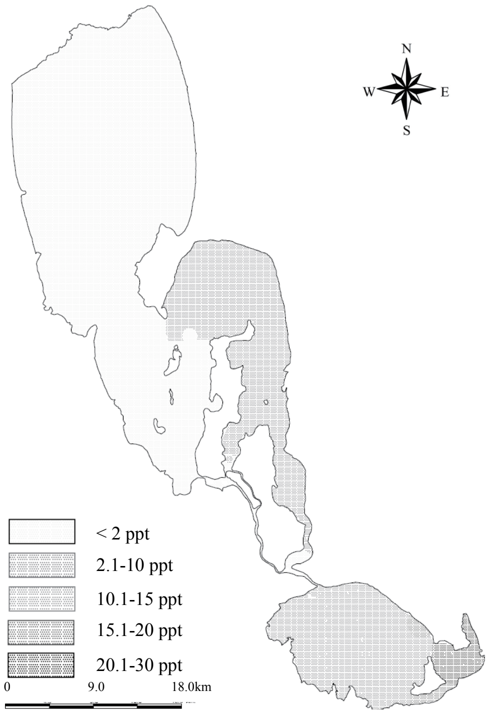

Fig. 5 Salinity (ppt) in Songkhla lake

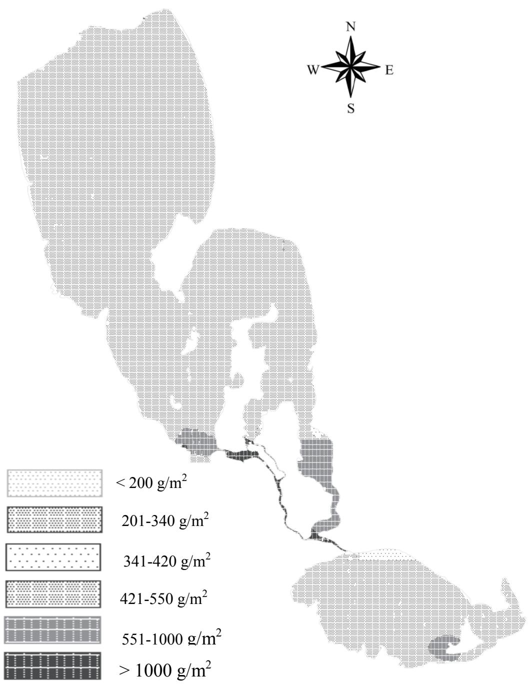

Fig. 6 Suspended sediment in Songkhla lake (g/m^2)

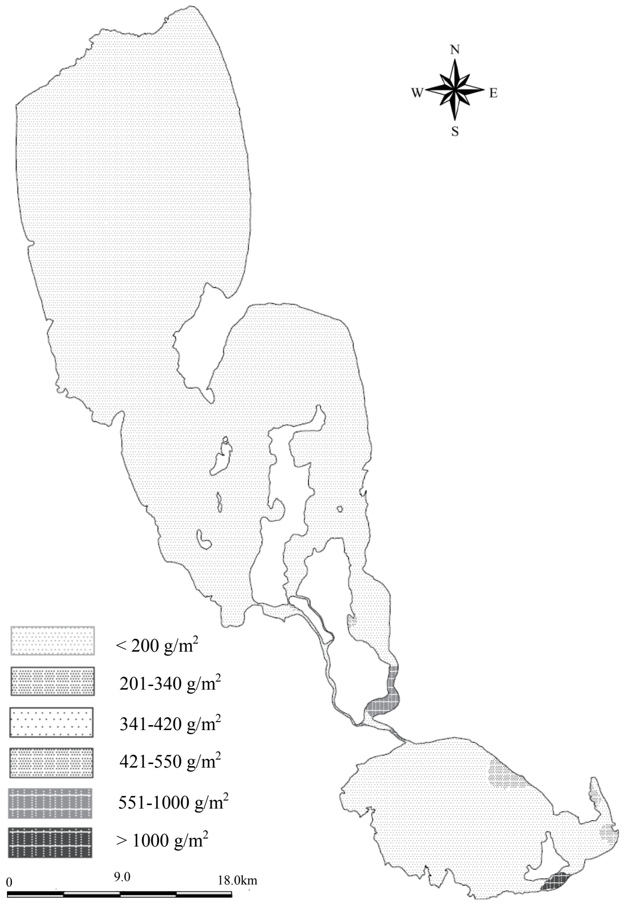

Fig. 7 Bedload sediment transport (g/m^2) in Songkhla lake

However, it is found that there is some significant difference between the simulated results and the field sampling data. This might be caused by significant difference in the resuspension rate used in the model and the rate actually occurred in the lake. Wind driven wave is likely to play an important role in sediment resuspension because water depth in the lake is very shallow. So, reliable wind data should be used in the simulation.

CONCLUSION

1) In this study, a sediment transport model is developed in which suspended sediment and bedload sediment are interrelated to each other. The developed sediment transport model is an effective tool for determine sediment transport in the Songkhla lake with unsteady flow and multiple discharge points. Moreover, the developed model is an effective tool for predicting sediment transport pattern in the future.

2) Rate of suspended sediment and resuspension of bedload sediment are significant source and sink terms

of the sediment transport model. Therefore, in this study the sediment load from various sources are divided into groups based on its grain size and specific gravity. Then, transport pattern of each group is calculated and combined together to get the total amount of suspended sediment and bedload sediment at each location (nodal point) in the study domain. With this technique, the transport pattern simulated from the model is expected to be more realistic.

3) The finite element method is used in model development in this study. Three-dimensional model is developed for the suspended sediment transport model whereas two-dimensional model is developed for the bedload transport model. The bottom boundary of the suspended load transport study domain projected on the horizontal plane is used as the study domain of the bedload transport model.

4) The sediment transport process which has occurred in the past affects distribution patterns of both suspended sediment and bedload sediment at present. In order to simulate the sediment transport patterns in the future, it is necessary that reliable initial values which are suspended sediment and bedload sediment distributions at present or at the beginning of the simulation time (t = 0) are given. Field sampling and analyses are needed to collect these data.

5) For shallow water body like the Songkhla lake, wind driven wave plays an important role on sediment transport. Therefore, detailed study on wind data and wave generation by wind is necessary so as to obtain a reliable sediment transport model.

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