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UNCERTAINTY OF EMPIRICAL PREDICTION MODEL FOR WALL DEFLECTION OF DEEP EXCAVATION IN SHANGHAI SOILS

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ABSTRACT: Empirical and semiempirical methods are simple models for estimating the maximum wall deflection induced by an excavation by practicing engineers for preliminary design. Various factors, such as excavation geometry, wall stiffness, strut spacing, ground condition, dewatering, etc, may affect deformation behavior of an excavation. It is impossible and not practical to incorporate all these factors in a prediction model for excavation-induced wall deflection. Hence, the prediction model of wall deflection is subject to model uncertainty, which is necessary to be quantified. In this paper, a database of 25 well-documented case histories of braced excavations in Shanghai is established. The model uncertainties of two semiempirical models for wall deflection, i.e., the KJHH model (Kung et al. 2007) and the C&O method (Clough and O'Rourke 1990) are quantified using the Bayesian updating approach. A model bias factor is defined as the ratio of the observed maximum wall deflection over the estimated value by the prediction model. With the information of the case histories, the uncertainty of the model bias factor is reduced. It is found that the posterior mean of the bias factor of the KJHH model is closer to 1.0 than that of C&O method and the uncertainty of the KJHH model is smaller than that of C&O method.

Keywords: excavation, empirical model, Bayesian method, model uncertainty

INTRODUCTION

Most of the deep excavations in Shanghai are conducted in soft ground conditions. In downtown area, buildings and facilities are often in close proximity of deep excavations. In order to protect the adjacent buildings and facilities, engineers should estimate the wall deflection and ground movement induced by excavation and modify design schemes if necessary to avoid potential damage during construction.

The displacements induced by excavations can be predicted with various methods such as empirical models, analytical methods, numerical models and etc. Among them, empirical and semiempirical methods (Peck 1969; Bowles 1988; Clough and O'Rourke 1990; Ou et al. 1993; Hsieh and Ou 1998; Long 2001; Yoo 2001; Moormann 2004; Leung and Ng 2007; Kung et al. 2007) are simple for practicing engineers in preliminary design. In a complicate geotechnical system such as a braced excavation, various factors including excavation geometry, wall stiffness, strut spacing, ground condition, dewatering and etc, may affect deformation behavior of an excavation. It is impossible and not practical to incorporate all these factors in a prediction model for excavation-induced displacement. Hence, the prediction model is subject to model uncertainty, which is necessary to be quantified.

In this paper, a database of 25 well-documented case histories of braced excavations in Shanghai is established. The model uncertainties of two empirical models for wall deflection, i.e., the KJHH model (Kung et al. 2007) and the C&O method (Clough and O'Rourke 1990), are quantified using the Bayesian updating approach. The effects of multiple observations and prior information on the model bias factor are discussed. The model bias factors of the two empirical models are compared.

EMPIRICAL MODEL FOR WALL DEFLECTION

Clough & O'Rourke Method

Clough and O'Rourke (1990) recognized that in soft and medium clays basal stability may be an issue and proposed a semi-empirical design chart (see Fig. 1) for

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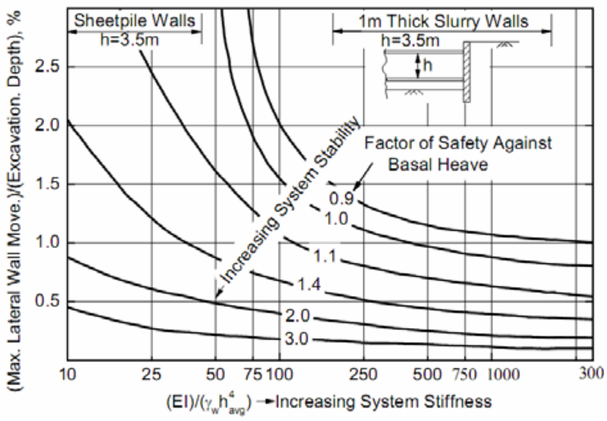


Fig. 1 Design chart for estimating maximum lateral wall movement in soft to medium clays (Clough and O'Rourke 1990)

predicting maximum lateral wall movements in terms of the factor of safety against basal heave *FS* and the system stiffness ($EI/\gamma_w h^4_{avg}$, where EI = wall stiffness, γ_w = unit weight of water, and h_{avg} = average support spacing). The Clough and O'Rourke Method (denoted as C&O method in this paper) is perhaps the most widely used method by practicing engineers for preliminary estimation of the maximum lateral wall deflection. The method can be used in circumstances where movements are primarily due to the excavation and support process.

KJHH (Kung et al. 2007) Model

Kung et al. (2007) recognized that accurate prediction of maximum wall deflection is difficult to achieve without using accurate representation of small-strain nonlinearity in soil model within finite element method. A simplified semiempirical model (denoted as KJHH model in this paper) for estimating maximum wall deflection, maximum surface settlement, and surface settlement profile was proposed for soft to medium clays. The model was developed based on a large number of FEM analyses of selected hypothetical excavation cases considering the nonlinear, stress–strain behavior of soils at small strain levels (Hsieh et al. 2003). The proposed model was then validated using well-documented case histories of braced excavations.

The KJHH Model consists of three component models, among which Model A is used in estimation of maximum wall deflection induced by excavations. Six parameters, which are excavation depth H_e, system stiffness $EI/\gamma_w h^4_{avg}$, excavation width B, ratio of the average undrained shear strength over the vertical effective stress s_u/σ'_v, ratio of the average initial Young's modulus over the vertical effective stress E_i/σ'_v, and ratio of the depth to hard stratum measured from the current excavation level over the excavation width T/B, are considered essential in the prediction of maximum wall deflection. The maximum wall deflection can be calculated using the regression equation shown below.

$$\begin{aligned}\delta = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 \\ + b_6X_1X_2 + b_7X_1X_3 + b_7X_1X_5\end{aligned} \tag{1}$$

where δ is maximum wall deflection; X_i are transformation variables of the five input variables x_i [H_e, $EI/\gamma_w h^4_{avg}$, $B/2$, s_u/σ'_v, and E_i/σ'_v]; b_0 to b_8 are regression coefficients. In Eq. (1), the regression coefficients b_0=-13.41973; b_1=-0.49351; b_2=-0.09872; b_3=0.06025; b_4=0.23766; b_5=-0.15406; b_6=0.00093; b_7=0.00285; and b_8=0.00198.

The transformation function is as follows:

$$X = t(x) = a_1x^2 + a_2x + a_3 \tag{2}$$

where the coefficients a_1, a_2, and a_3 for each variable are obtained through error minimization using the artificial data generated from FEM analyses. Details of KJHH Model and values of a_1, a_2, and a_3 for each variable can be referred to Kung et al. (2007).

BAYESIAN CALIBRATION OF MODEL ERROR

Definition of Model Bias Factor

Model calibration involves a comparison of the predicted performance by a prediction model to the observed performance. In this paper, a model bias factor is adopted to quantify the model error:

$$BF = \delta_m / \delta_e \tag{3}$$

where δ_m represents the measured maximum wall deflection and δ_e represents the estimated maximum wall deflection by a prediction model.

Updating Distribution of Model Bias Factor

According to the Bayesian theory, the posterior distribution is the combination of the prior distribution and the likelihood function. The posterior distribution of the model bias factor is expressed below:

$$f''(BF) = k \cdot lk(\varepsilon \mid BF) \cdot f'(BF) \tag{4}$$

where k is the normalized constant, $f'(BF)$ is the prior distribution of BF, $f''(BF)$ is the posterior distribution of BF and $lk(\varepsilon|BF)$ is the likelihood function which defines the probability of observing ε at given value of BF. Here the observed information is the measured maximum wall deflection is δ_m. Hence, the likelihood function can be written as follows:

$$lk(\varepsilon \mid BF) = P(\delta_m \mid BF) = f_{\delta_e}(\delta_e = \frac{\delta_m}{BF})d\delta_e \quad (5)$$

where f_{δ_e} is the probability density function of δ_e.

If there are multiple observations from n independent sites, the posterior distribution of the model bias factor is:

$$f''(BF) = k \cdot \prod_{i=1}^{n} f_{\delta_{ei}}(\delta_{ei} = \frac{\delta_{mi}}{BF}) \cdot f'(BF) \quad (6)$$

COLLECTION OF EXCAVATION CASE HISTORIES

The most commonly used retaining walls for deep excavations in Shanghai are diaphragm walls (DW) and contiguous pile walls (CPW). Diaphragm wall has relatively high stiffness and provides effective water tightness. It is usually adopted as retaining wall as well as outside wall of basement. Contiguous pile wall is usually adopted as temporary wall in Shanghai. The advantages of contiguous pile wall are lower cost and higher construction speed comparing to diaphragm walls.

Xu (2007) collected about 300 case histories of excavation in Shanghai and established a database. Among these case histories, fifteen well-documented case histories of excavations with contiguous pile walls and ten case histories of excavations with diaphragm walls are selected for model calibration in this study. The final excavation depth of the excavations with CPW ranges from 5.5 m to 14.7 m and the ratio of maximum wall deflection over excavation depth ranges from 0.16 to 0.67. Characteristics of the case histories with CPW are summarized in Table 1. The excavation depth of the excavations with DW ranges from 12.3 m to 19.5 m and the ratio of maximum wall deflection over excavation depth ranges from 0.30 to 0.74. Characteristics of the case histories with DW are summarized in Table 2.

The case histories used in this study have the typical Shanghai soil profiles (see Fig. 2). The top soil layer is backfill with a thickness less than 2.0 m in general. The groundwater table is generally 0.5 m to 1.0 m below the ground surface. The second layer is yellowish dark brown organic clay with medium plasticity and medium compressibility. The thickness of this layer ranges from 2.0 m to 4.0 m. The third layer is very soft silty clay with thickness of 5 m to 10 m. This layer has medium plasticity and high compressibility. The fourth layer is 5 m to 10 m in thickness, with largest void ratio and compressibility. The shear strength and coefficient of permeability for this layer are lowest among the soil layers. The fifth layer is grayish silty clay, which is low to medium plastic, with a thickness of 5-17m. Beneath this layer is a layer of dark green stiff clay with thickness of 2~6 m, which is low to medium plastic. The seventh layer is fine to very fine sand.

Table 1 Summary of excavations with contiguous pile walls

Case No.	Case name	Excavation depth (m)	Excavation width (m)	Wall length (m)	Pile diameter (m)	EI (MN·m^2/m)	δ_{hm}/H_e (%)
1	Jiangshan	11.8	36.0	30.5	1.00	1,472	0.49
2	Ganghui	14.7	167.2	30.0	1.00	1,472	0.67
3	Dongnan	9.1	61.4	23.2	0.85	699	0.46
4	Huaan	11.1	34.7	59.5	0.90	920	0.28
5	Lianhe	11.3	81.0	25.8	1.00	1,280	0.36
6	Meiluo	8.1	60.0	16.0	0.90	920	0.34
7	Renmin	10.6	38.0	21.6	0.90	1,017	0.47
8	PDMGC	9.7	45.5	15.6	0.85	768	0.50
9	Baiteng	10.7	36.0	27.0	1.00	1,227	0.16
10	DHH	13.4	93.3	25.4	1.10	1,796	0.56
11	SJYC	5.5	60.5	8.7	0.60	254	0.61
12	LW117	6.9	94.1	17.7	0.80	603	0.45
13	Gongshang	8.4	62.0	12.0	0.80	669	0.30
14	Yidong	8.9	78.0	19.2	0.85	768	0.26
15	Jiefang	7.5	40.7	13.0	0.60	254	0.22

Table 2 Summary of excavations with diaphragm walls

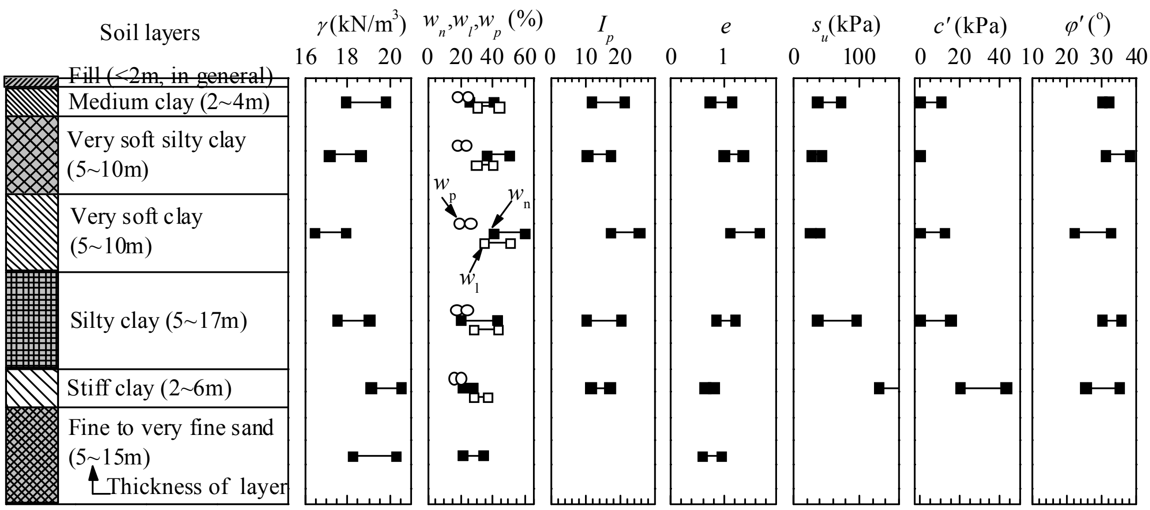
Case No.	Case name	Excavation depth (m)	Excavation width (m)	Wall length (m)	Wall thickness (m)	EI ($MN \cdot m^2/m$)	δ_{hm}/H_e (%)
16	Lansheng	13.2	40.0	26.0	0.80	1,280	0.30
17	Zhidi	13.7	42.2	28.0	0.80	1,280	0.31
18	Subway-R2	15.0	21.6	31.0	0.60	540	0.47
19	Huangpu	19.5	22.8	37.0	0.80	1,280	0.74
20	R1-HS	14.7	22.0	29.2	0.60	540	0.40
21	Zhongshan	16.0	26.0	26.0	0.60	540	0.54
22	R2-SM	15.0	19.6	28.5	0.80	1,280	0.43
23	R1-CB	12.3	15.0	21.5	0.65	687	0.42
24	R2-ZY	15.0	19.6	26.7	0.60	540	0.33
25	M8-YJ	14.4	19.2	26.0	0.80	1,280	0.49

The overall performance of deep excavations is mainly influenced by properties of the second, third and fourth soil layers. As the undrained shear strength of the three soil layers are less than 72 kPa and SPT N value of these soil layers are less than 5. The case histories in this study can be classified as deep excavations in 'soft' soil according to Moormann (2004).

MODEL CALIBRATION USING CASE HISTORIES

Methodology of Model Calibration

As shown in Eq. (5), the distribution of estimated maximum wall deflection δ_e should be obtained in order to determine the likelihood function. In this paper, Monte Carlo simulation is adopted to obtain the distribution of δ_e. The random variables for the C&O method are undrained shear strength s_u and system stiffness EI. For the KJHH model, the considered random variables are EI, s_u and E_i. s_u is modeled as a lognormal random variable with a COV equal to 20% (Phoon and Kulhawy 1999). As s_u and E_i are averaged values within the excavation depth, the mean values for s_u and E_i are assumed to be 40kPa and 75MPa, respectively. E_i is assumed to be lognormal with a COV equal to 30%. EI is modeled as lognormal random variable with COV of 20%. The EI values in Tables 1 and 2 are taken as the mean value.



Note: γ=unit weight, w_n=water content, w_p=plastic limit, w_l=liquid limit, I_p=plasticity index, e=void ratio, s_u=field vane shear strength, c'=effective cohesion, φ'=effective angle of internal friction

Fig. 2 Soil profile and geotechnical parameter ranges in Shanghai soft ground (Wang et al. 2010)

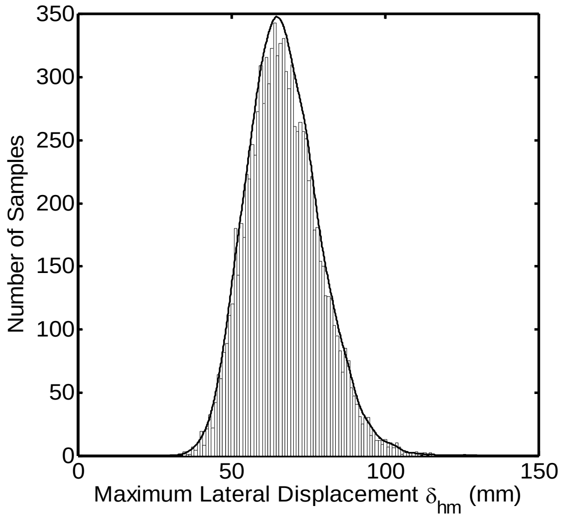


Fig. 3 Histogram of estimated maximum wall deflection using KJHH model (Case No. 5)

Fig. 3 shows the histogram of estimated maximum wall deflection using the KJHH model for case No. 5. The observed maximum wall deflection is 41 mm. With this observation, the posterior distribution can be obtained using Eq. (4). Kung et al. (2007) verified the developed KJHH model with 33 cases from various sites around the world. It is found that the mean value of the bias factor of the model is 1.0 and the COV is equal to 25%. Therefore, the prior distribution is assumed to be a normal distribution, with a mean value of 1.0 and a COV of 25%. Fig. 4 shows the prior distribution, the likelihood function and the posterior distribution of model bias factor for case No. 5 using KJHH Model. It shows that the uncertainty of the model bias factor is significantly reduced because of the increased knowledge about the prediction model with the information of the case history.

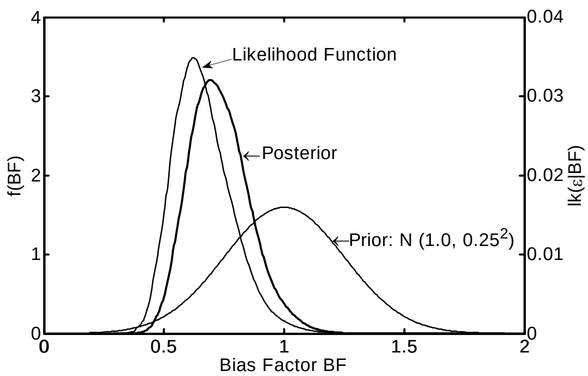


Fig. 4 Likelihood function and PDFs of bias factor (KJHH-Case No.5)

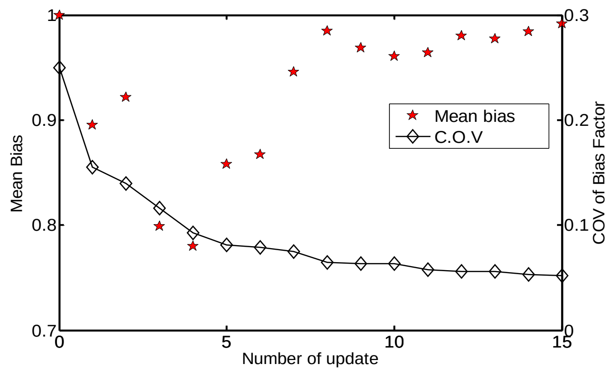


Fig. 5 Posterior mean and COV of bias factor with multiple CPW case histories (KJHH model)

Effect of Multiple Case Histories

Fig. 5 shows the effect of multiple case histories on updating the distribution of the bias factor for the KJHH model. The posterior mean bias factor fluctuates between 0.7 and 1.0 when the number of case histories is less than 10. When more case histories are used to update the distribution of bias factor, the mean bias factor varies only in a small range. With all the information of the 15 case histories is adopted, the mean bias factor is 0.99. The COV of bias factor decreases with field observations. With all the information of the 15 case histories, the COV value is decreased from 25% to 5.1%. It shows with multiple case histories, the model uncertainty can be reduced gradually.

Effect of Prior Mean and Prior COV

Different values of mean and COV of the prior distribution are assumed for the prior distribution, i.e., N(1.0, 0.25), N(1.0, 0.20^2), N(1.0, 0. 50^2), N(0.8, 0.25^2), N(1.2, 0.25^2), which are referred to as Prior-1, Prior-2, Prior-3, Piror-4 and Piror-5, respectively in Table 3. It shows that with the information from all fifteen case histories, the mean bias is about 0.99 and COV is around 5%. It means when enough observation is available, the mean and COV value of the prior distribution do not affect the posterior statistics and the information of observed measurements dominates the updated distribution of model bias factor.

Effect of Type of Prior Distribution

In the previous sections, the prior distribution is assumed to be normal. When no information is available, we can also assume a diffuse prior or a uniform prior distribution. Table 4 summarized the posterior statistics of the model bias factor for the KJHH model assuming

Table 3 Posterior statistics with different prior mean or prior COV (KJHH model, 15 CPW case histories)

	Prior-1	Prior-2	Prior-3	Prior-4	Prior-5
Mean	0.9935	0.9847	0.9894	0.9793	0.9958
COV	0.0513	0.0543	0.0496	0.0475	0.0489

Table 4 Posterior statistics with different types of prior distribution (KJHH model, 15 CPW case histories)

	Prior-1	Prior-6	Prior-7
Mean	0.9935	0.9949	0.9895
COV	0.0513	0.0489	0.0497

the prior distribution is uniform. In the case of Prior-6, the boundary of the bias factor is [0, 2]. For Prior-7, the boundary of the distribution is [0.2, 1.8]. It shows that the type of prior distribution does not affect the posterior mean and COV of bias factor when all fifteen case histories are used to update the distribution of bias factor.

Comparison of Prediction Models

Fig. 6 shows the updated distributions for the C&O Method and KJHH model. Here, the prior distribution is N (1.0, 0.25^2). The mean bias of C&O Method for contiguous pile walls is 0.93 and its COV is 6.9%. The mean bias of C&O Method for diaphragm walls is 0.48 and its COV is 13.9%. The mean bias of KJHH model for CPW is 0.99 and its COV is 5.1%. The mean bias of KJHH model for diaphragm walls (DW) is 0.84 and its COV is 7.2%. It shows that for both CPW and DW, the mean bias of KJHH model is closer to 1.0 than that of C&O method and the COV of the bias factor for the

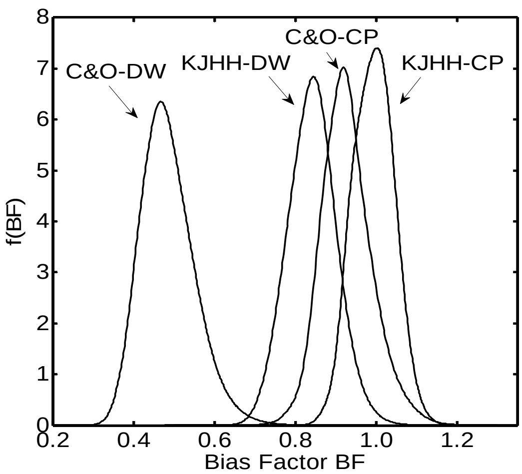


Fig. 6 Comparison of the posterior distribution for the C&O method and the KJHH model

KJHH model is smaller than that of C&O method. This is because the KJHH model was developed based on FEM analyses of excavation cases considering the nonlinear, stress–strain behavior of soils at small strain levels, which is more appropriate soil model for soil deformation in excavations.

CONCLUSIONS

In this paper, two empirical models for wall deflection, i.e., the KJHH model and the C&O method, are calibrated using the Bayesian updating approach based on a database of 25 well-documented case histories of braced excavations in Shanghai. The major findings are made as follows:

1. With information from multiple case histories, the model uncertainty can be reduced gradually.

2. When enough observation is available, the prior distribution does not affect the posterior statistics significantly and the information of observed measurements dominates the updated distribution of model bias factor.

3. The mean bias of C&O method for CPW and DW are 0.93 and 0.48, respectively. The COV values are 6.9% and 13.9%, respectively.

4. The mean bias of KJHH model for CPW is 0.98 and its COV is 5.1%. The mean bias of KJHH model for diaphragm walls is 0.84 and its COV is 7.2%.

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