

## EFFECT OF THICKNESS OF OVERLAYING CLAY LAYER OF LOWLAND REGION ON SENSITIVITY OF LATERAL DEFLECTION OF LONG PILES EMBEDDED IN NON-HOMOGENEOUS SOIL – PART IIA: NUMERICAL STUDY OF SAND PARAMETERS

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**ABSTRACT:** This part of the paper is a direct continuation of our first part paper. The first part of the paper presented the theoretical formulation of the sensitivity of the lateral head deflection of piles embedded in non-homogeneous soil consisting of clay overlying sand to changes in the design parameters. The current paper presents numerical sensitivity studies based on this theoretical formulation. The effect of variation in sand parameters on the variation of the lateral head deflection which is an important serviceability measure is studied. Both the clay and pile parameters will be investigated in part IIB of the paper. In lowland areas, the soil profiles starting from the soil surface contain soft clay layer of variable thickness. The effect of the thickness of the overlying clay layer on the sensitivity results is explored through comparative sensitivity assessment. Sensitivity results are given in the form of graphical presentations of sensitivity operators which show along the pile length where and how the change of each design parameter can affect the change of lateral pile-head deflection. It is shown that the thickness of the overlying clay has an effect on changing the magnitude and location of the maximum values of the sensitivity operators.

**Keywords:** Sensitivity, numerical analysis, sand parameters, clay thickness, non-homogeneous soil

### INTRODUCTION

Many engineering structures such as high rise buildings, offshore structure and bridge abutments are subject to lateral loading and require pile foundations to resist this lateral load. The lateral deflection of the pile head is an important measure of the serviceability of the superstructure that is supported by the laterally loaded piles. Therefore the variation of lateral deflection of the pile head due to any variation of the system parameters is of considerable importance both in the design stage and in the assessment of the aging infrastructure system.

The effect of the changes of the different system parameters on the variation of the lateral deflection of the pile can be studied using sensitivity analysis. The first part of this paper (Hafez and Budkowska, 2006a) presented the theoretical formulation of the sensitivity of the lateral pile-head deflection for piles embedded in non-homogeneous soil. The non-homogeneity is of layered type where a clay layer overlies a sand layer. Both layers are under water table and the pile is subjected to cyclic loading. The sensitivity formulation was derived based on the adjoint method of sensitivity analysis with distributed design variables (Kleiber, 1997). Other methods of sensitivity were reviewed in the first

part of the paper. The sensitivity formulation resulted in obtaining sensitivity operators for each parameter that can be graphically plotted along the pile length. This graphical presentation will allow the engineer to detect where along the pile length and how each parameter affects the lateral top deflection. The parameters investigated were those that define the pile structure and the adjacent  $p$ - $y$  clay and  $p$ - $y$  sand soil (where  $p$  stands for soil resistance and  $y$  for the pile deflection). The  $p$ - $y$  relations are the well known "Matlock and Reese"  $p$ - $y$  relations derived from full scale field results and they were described in part I (Hafez and Budkowska, 2006a).

In this part of the paper, the sand parameters will be numerically investigated, i.e. the effect of the variation of sand parameters on the variation of the lateral head deflection of the pile will be studied. The pile and clay parameters will be numerically investigated in part IIB of the paper (Hafez and Budkowska, 2006b). The sensitivity results for sand parameters are given in the form of graphical presentation of the sensitivity operators along the pile length.

In addition, the effect of the thickness of the overlying clay layer on the sensitivity results will be numerically investigated through comparative sensitivity assessment. Originally, the pile is embedded in

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homogeneous sand soil. This case is compared with other possible situations of embedment of laterally loaded pile in lowland areas in which the soil profiles starting from the soil surface contain soft clay layer of variable thickness. In the current study, the evolution of soft clay layer is developed up to the point of zero deflection of laterally loaded long pile. The development of soft clay layer is investigated in a discrete fashion as far as its thickness is concerned. The thickness of the clay layer overlying the sand layer will range from 0% of the pile length up to 50%.

Two approaches are undertaken to study the thickness effect. In the first one, different cases of the clay thickness are compared when the pile is subjected to the same load in the different cases. It is understood that a case is defined by the geological profile. For example, the geological profile that contains top clay layer of thickness equal to 10% of total pile length whereas the remaining soil in which the pile is embedded is sand is considered as a case. In the second approach, the comparison between the different profiles is performed when the pile is subjected to the same deflection.

The input data used for the numerical investigation is first presented. The numerical investigations are performed by the aid of the computer program COM624P (Wang and Reese 1993) used for laterally loaded piles. It is followed by the sensitivity results of the variation of lateral head deflection of piles embedded in different cases of non-homogeneous soil due to variations in the sand parameters. The effect of the thickness of the overlying clay layer on the sensitivity results is then presented. Finally the conclusions and final remarks on the work performed are drawn.

## INPUT DATA

### Pile Length and Soil Stratification

The behavior of a laterally loaded pile depends on whether it is a long or a short pile. A case of a long pile with a free head subjected to cyclic lateral load embedded in non-homogeneous soil is investigated in this paper. Evans and Duncan (1982) proposed a characteristic load method to classify laterally loaded piles, embedded in homogeneous soils, into long and short piles. This method depends on a relative stiffness factor  $T$  available for each type of soil, which is calculated as follows for a free head pile with pile bending stiffness  $EI$  and subjected to a lateral force  $P$ :

$$T = \sqrt[3]{\frac{y_t EI}{A_y P}} \quad (1)$$

where  $A_y$  is a constant depending on the pile head constraint and  $y_t$  is the lateral top deflection of the pile.

However, the case studied is a long pile embedded in a non-homogeneous soil. To deal with such a case, the relative stiffness factor was calculated for a homogeneous layer of clay  $T_c$  and a homogeneous layer of sand  $T_s$ , both layers below water table and subjected to cyclic loading. Numerical studies were performed using an average value  $T_{av}$  ( $T_{av} = (T_c + T_s)/2$ ). The pile was shown to behave as a typical long pile for all cases of non-homogeneous soil (ranging from the special case of a soil consisting of 100% clay and 0% sand to a case consisting of 0% clay and 100% sand) and for all the range of applied loads starting from a length equal to  $8T_{av}$ .

A long pile with length  $l$  equal to  $8T_{av}$  ( $l = 8T_{av} = 16\text{m}$ ) is used for the sensitivity investigation. The different cases of non-homogeneous soil range from 0% clay where the pile is embedded in sand only to 50% clay where the upper 8 m of the pile are embedded in soft clay and the lower 8 m are embedded in sand.

### Initial Values of Design Variables and Loading

Initial values of the design variables are needed to perform the numerical sensitivity investigations since the sensitivity operators are determined for constant initial values of the design variables. The design variables are given in the following vector:

$$\mathbf{d} = \{EI, b, \gamma'_c, c, \varepsilon_{50}, \gamma'_s, \phi, k\}^T \quad (2)$$

where  $EI$  is the pile bending stiffness,  $b$  is the pile diameter,  $\gamma'_c$  is the submerged unit weight of soft clay,  $c$  is the undrained cohesion and  $\varepsilon_{50}$  is the strain corresponding to one-half of the compressive strength of clay,  $\gamma'_s$  is the submerged unit weight of sand,  $\phi$  is the friction angle of sand, and  $k$  is a constant representing the modulus of subgrade reaction in the linear stage of sand behavior. The following typical initial values are used (Canadian Institute of Steel Construction 2000, Perloff and Baron 1976):  $EI = 55,400 \text{ kNm}^2$ ;  $b = 406 \text{ mm}$ ;  $\gamma'_c = 7.5 \text{ kN/m}^2$ ;  $c = 18 \text{ kN/m}^2$ ;  $\varepsilon_{50} = 0.02$ ;  $\gamma'_s = 10 \text{ kN/m}^2$ ;  $\phi = 33^\circ$ ; and  $k = 16,285.8 \text{ kN/m}^3$ .

The sensitivity analysis is developed in the vicinity of the applied load since we are dealing with a nonlinear behavior of the soil. Therefore the loading of the pile is applied in a discrete fashion. The loading  $P$  is applied at the pile head (where the pile head is at the ground surface) in increments of 25kN.

For each case of non-homogeneity, the pile was loaded up to a load that causes the lateral deflection of the pile head to exceed 0.305m at the ground surface. This amount of deflection causes the clay at the ground

surface to reach the plastic flow stage (refer to Fig. 1 in Part I (Hafez and Budkowska, 2006a)). Accordingly, the various stages of soil deformability can be investigated in the sensitivity analysis. For the case of 0% clay, the lateral deflection of 0.305m will fulfill that sand at the ground surface has also reached the plastic flow stage (where plastic flow in sand starts at deflection  $y = 3b/80 = 0.015\text{m}$  as shown in Fig. 2 in Part I (Hafez and Budkowska, 2006a) and repeated below in Fig. 1 in the current paper.)

Since there are different thicknesses of clay layer, then the maximum load reached for each case of non-homogeneous soil that gives the same deflection at the ground surface will be different. For each case of non-homogeneity, the clay thickness, the maximum load reached and the corresponding deflection at ground surface are given in Table 1.

## SENSITIVITY ANALYSIS AND RESULTS

### Sensitivity Analysis

Sensitivity analysis using the adjoint method involves the analysis of a primary structure and an adjoint one as explained in Part I of the paper (Hafez and Budkowska, 2006a). The numerical analysis of the primary and adjoint piles is performed by means of the nonlinear finite difference program COM624P (Wang and Reese, 1993). Using this program, the laterally loaded pile is solved as a beam on elastic foundation involving nonlinear modeling of the soil-pile interaction response ( $p$ - $y$  curves). The results from this program (soil resistance, deflections and bending moments of the primary and adjoint pile) are substituted in the expressions required for the sensitivity analysis.

The theoretical formulation of the sensitivity analysis resulted in the derivation of Eq. (22) in Part I of the paper (Hafez and Budkowska, 2006a) which takes into account the non-homogeneity of the soil. Equation (22) is repeated below for convenience:

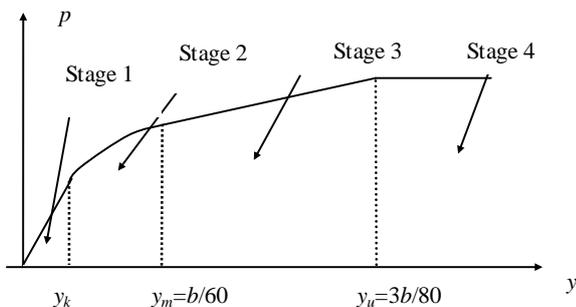


Fig. 1 Nonlinear behavior of sand subjected to cyclic loading after Reese et al. (1974)

Table 1. Maximum load P reached for each case of non-homogeneity

% clay	Thickness of clay layer (m)	Maximum load P (kN)	Corresponding deflection of pile head (m)
0%	0	500	.34
10%	1.6	450	.34
20%	3.2	325	.33
30%	4.8	225	.31
40%	6.4	175	.31
50%	8.0	160	.34

$$\begin{aligned}
 1_a \delta y_t = & \int_0^{H_1} (S_{EI})_c (\delta EI_N)_c dx + \int_0^{H_1} (S_b)_c (\delta b_N)_c dx + \\
 & \int_0^{H_1} (S_{\gamma'_c})_c (\delta \gamma'_{cN})_c dx + \int_0^{H_1} (S_c)_c (\delta c_N)_c dx + \\
 & \int_0^{H_1} (S_{\varepsilon_{50}})_c (\delta \varepsilon_{50N})_c dx + \int_{h_2}^{h_2+H_2} (S_{EI})_s (\delta EI_N)_s dx + \quad (3) \\
 & \int_{h_2}^{h_2+H_2} (S_b)_s (\delta b_N)_s dx + \int_{h_2}^{h_2+H_2} (S_{\gamma'_s})_s (\delta \gamma'_{sN})_s dx + \\
 & \int_{h_2}^{h_2+H_2} (S_{\phi})_s (\delta \phi_N)_s dx + \int_{h_2}^{h_2+H_2} (S_k)_s (\delta k_N)_s dx
 \end{aligned}$$

where  $1_a$  is the unit load applied to the adjoint pile at the pile head,  $\delta y_t$  is the variation in the top deflection in the pile,  $(S_{(\cdot)})_c$  and  $(S_{(\cdot)})_s$  denote the normalized sensitivity operators for clay and sand, respectively and the symbols  $(\delta(\cdot)_N)_c$  and  $(\delta(\cdot)_N)_s$  denote the normalized variations of design variables for clay and sand, respectively.

The current paper investigates the sensitivity of the lateral top deflection to changes in the sand parameters that are defined in the  $p$ - $y$  sand relations. These parameters are  $b$ ,  $\gamma'_s$ ,  $\phi$  and  $k$ . Since the pile diameter  $b$  is also a parameter of the  $p$ - $y$  soft clay relation (Matlock, 1970), it will be studied with the clay parameters in Part IIB of the paper (Hafez and Budkowska, 2006b) along with the pile's bending stiffness. The  $p$ - $y$  sand relation involves four stages of the soil behavior (Reese et al., 1974) that were discussed in Part I of the paper (Hafez and Budkowska, 2006a) and are shown again in Fig. 1.

Accordingly, the following normalized sensitivity operators are studied:  $(S_{\gamma'_s})_s$ ,  $(S_{\phi})_s$  and  $(S_k)_s$ . These sensitivity operators are calculated at discrete points along the pile length (in increments of 0.1m) by performing the differentiation operations required by Eq. (3) given above. These operators are graphically represented and discussed for discrete lateral forces  $P$

and the different cases of the clay thickness in the following subsection.

Sensitivity Results

The graphical presentation of the sensitivity operators allows us to examine the effect of the change of the design variables on the change of the pile-head lateral deflection  $\delta y$ , along the pile length. Thus the most and least effective locations of the change of the design variables on the pile's lateral head deflection can be observed. Due to the normalization process, the units of all the sensitivity operators are in kN.

The sensitivity operator  $(S_{\gamma'_s})_s$  that detects along the pile length the effect of the variation of the submerged unit weight of sand on the variation of the pile-head lateral deflection is plotted for each case of thickness of clay layer in Figs 2 to 7. In each figure the operator is plotted for different loads applied in increments of 25 kN as explained above. However for clear outputs only increments of 50 kN are shown in some figures.

As seen in the figures, the sand operators start with the beginning of the sand layer. For a given case of clay

thickness, as the load  $P$  increases the sensitivity of the pile head deflection to change in the submerged unit weight of sand increases and the maximum value is obtained at a greater depth from ground surface.

The maximum value of the operator indicates that this is the critical location at which a change of  $\gamma'_s$  has a maximum effect on the change of the lateral head deflection. The negative sign of the operator confirms the known fact that an increase of the value of unit weight decreases the deflection of the soil.

The jumps in the operator's values observed in the figures are associated with the change in the soil stage. The sudden drop in the operators is due to the change of the soil behavior from stage to stage. The parameter  $\gamma'_s$  affects the soil behavior when the soil is in stages 2 and 3 only, i.e. when deflection  $y$  of the pile is  $y_k < y < y_m$  and  $y_m < y < y_u$  as shown in Fig. 1. As we move downward along the pile length the deflection of the pile decreases and the soil changes its behavior from stage 3 to stage 2. This change causes the sudden drop in the value of the sensitivity operator. As the deflection decreases to  $y < y_k$ , the value of the sensitivity operator  $(S_{\gamma'_s})_s$  becomes equal to zero.

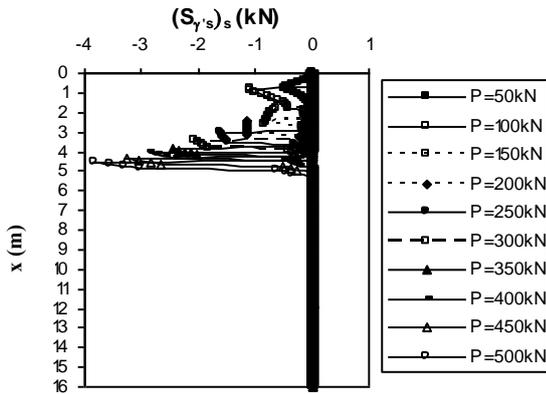


Fig. 2 Distribution of  $(S_{\gamma'_s})_s$  for 0% clay

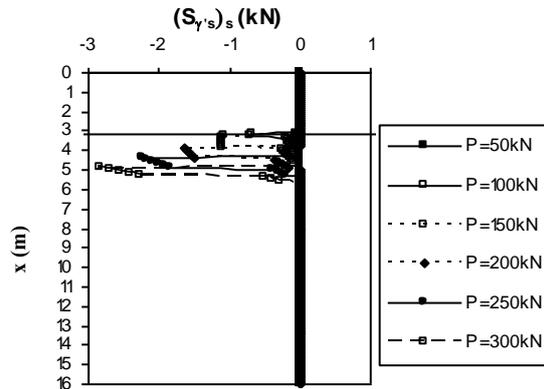


Fig. 4 Distribution of  $(S_{\gamma'_s})_s$  for 20% clay

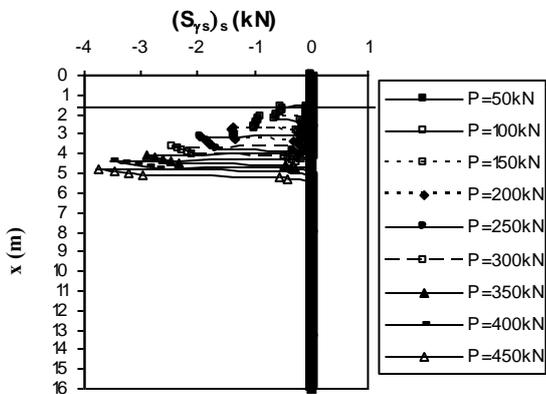


Fig. 3 Distribution of  $(S_{\gamma'_s})_s$  for 10% clay

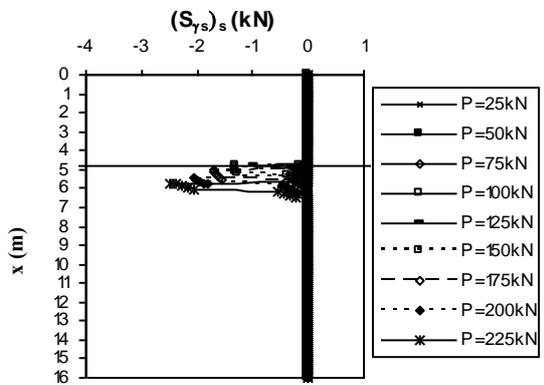


Fig. 5 Distribution of  $(S_{\gamma'_s})_s$  for 30% clay

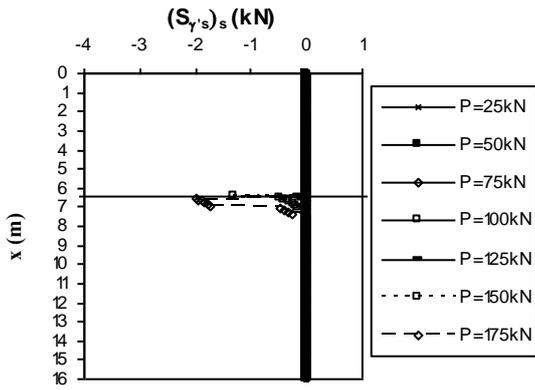


Fig. 6 Distribution of  $(S_{\gamma'_s})_s$  for 40% clay

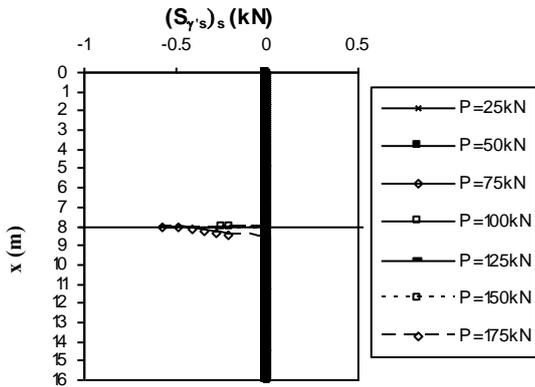


Fig. 7 Distribution of  $(S_{\gamma'_s})_s$  for 50% clay

As the thickness of clay increases (Figs. 2 through 7), the sand starts at a deeper level and the operators are not developed for low loads that cause the sand deflection to be less than  $y_k$ .

It can be added in conclusion, that for the homogeneous sand layer (case 1 = 0% clay) the sensitivity results at the top of the sand layer are different from the traditional behavior (consistent results explained above) observed for the non-homogeneous cases (cases 2 to 6 = 10%clay to 50%clay). A minimum thickness of clay was required to receive consistent results due to the feature of the  $p$ - $y$  model used.

The sensitivity operator  $(S_{\phi})_s$  that detects along the pile length the effect of the variation of the angle of friction of sand on the variation of the pile-head lateral deflection is plotted for each case of thickness of clay layer in Figs. 8 to 13. The distribution of  $(S_{\phi})_s$  along the pile has exactly the same shape as the distribution of  $(S_{\gamma'_s})_s$ . The same comments given above on  $(S_{\gamma'_s})_s$  apply to  $(S_{\phi})_s$ . The only difference is in the numerical values. It is observed that the numerical values of  $(S_{\phi})_s$  are much higher than those of  $(S_{\gamma'_s})_s$ . This indicates that the effect of the change of the angle of friction  $\phi$  on the lateral head deflection is in general higher than that of the submerged unit weight  $\gamma'_s$ .

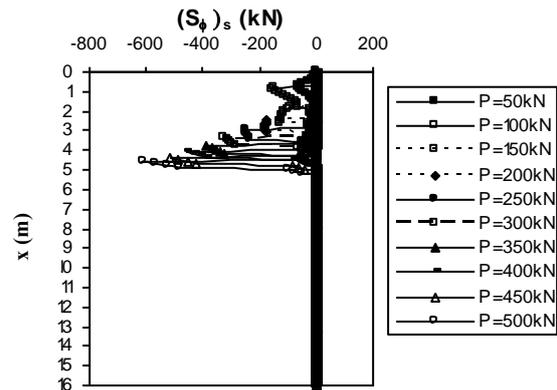


Fig. 8 Distribution of  $(S_{\phi})_s$  for 0% clay

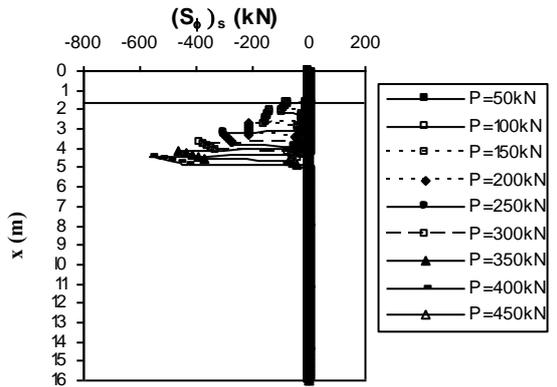


Fig. 9 Distribution of  $(S_{\phi})_s$  for 10% clay

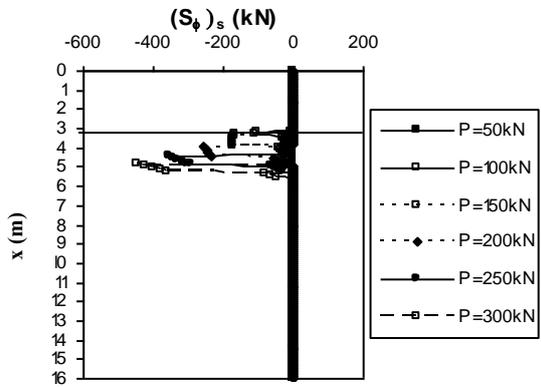


Fig. 10 Distribution of  $(S_{\phi})_s$  for 20% clay

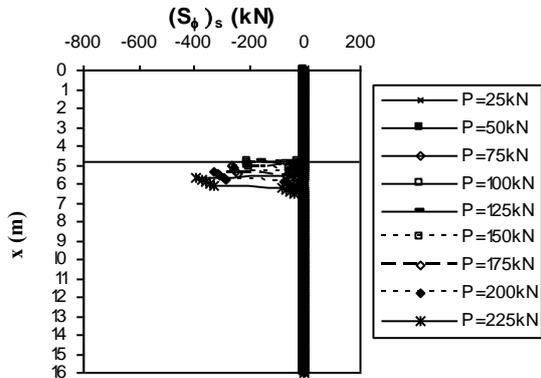


Fig. 11 Distribution of  $(S_{\phi})_s$  for 30% clay

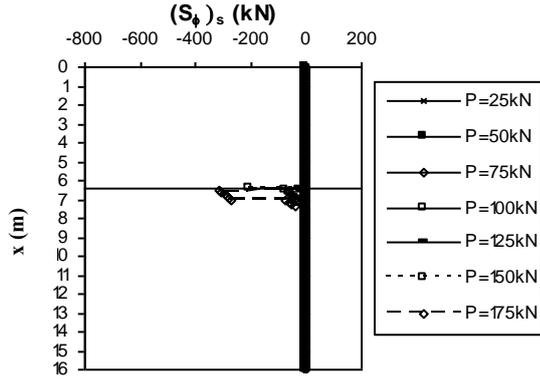


Fig. 12 Distribution of  $(S_{\phi})_s$  for 40% clay

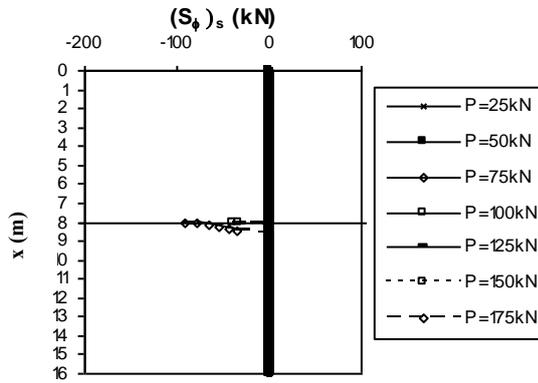


Fig. 13 Distribution of  $(S_{\phi})_s$  for 50% clay

The results of sensitivity of head deflection to  $k$  are plotted in Figs 14 to 19 for each case of clay thickness. The value of  $k$  affects only the behavior of the pile soil system when the sand is in the linear elastic stage (stage 1 in  $p$ - $y$  curves for sand). Therefore the operator  $(S_k)_s$  will develop only when the deflection causes the sand to be in the linear elastic stage when  $y < y_k$ . Accordingly, for a given load and a given case of clay thickness, the operators will develop at deeper levels than those for  $(S_{\gamma'_s})_s$  and  $(S_{\phi})_s$ , since the deflection decreases as we move downward along the pile's length. This means that the effect of  $k$  is in the lower part of the pile relative to the zone of effect of  $\gamma'_s$  and  $\phi$ .

For a given case of clay thickness, as the load increases the deflection  $y$  increases and the elastic stage starts at a greater depth from the top of sand layer. This explains why  $(S_k)_s$  curves start at a greater depth as the load increases.

In addition, for each load the distribution of  $(S_k)_s$  shows two negative peak values and a positive one. A positive value means that as  $k$  increases the lateral deflection increases and the negative value means that an increase in  $k$  will cause a decrease in the lateral deflection. These peaks can be explained in view of the equation used to calculate  $(S_k)_s$  given as follows:

$$(S_k)_s = -x_2 y y_a \tag{4}$$

where  $x_2$  is the local coordinate of sand,  $y$  and  $y_a$  are the deflections of the primary pile and adjoint pile respectively at the point where  $(S_k)_s$  is calculated.

For more clarification,  $(S_k)_s$  for the case of clay thickness = 20% and load  $P = 200\text{kN}$  is plotted on a separate figure along with the corresponding deflection of the primary pile  $y$  and the adjoint pile  $y_a$  in Figs 20, 21 and 22, respectively.

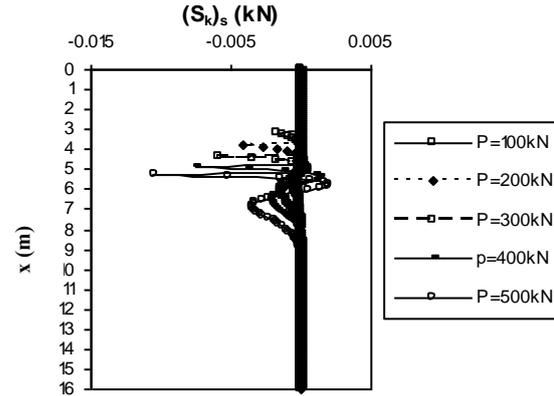


Fig. 14 Distribution of  $(S_k)_s$  for 0% clay

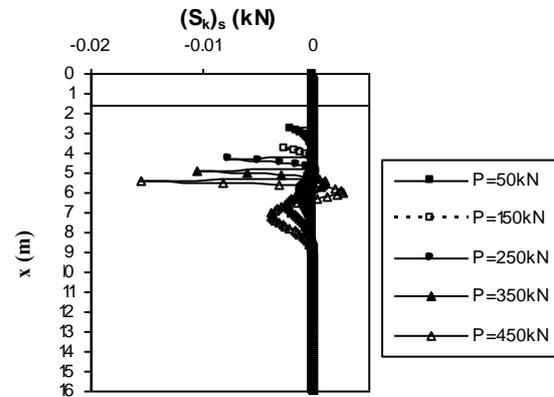


Fig. 15 Distribution of  $(S_k)_s$  for 10% clay

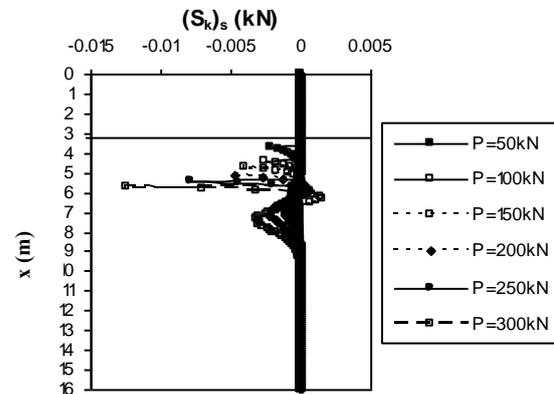


Fig. 16 Distribution of  $(S_k)_s$  for 20% clay

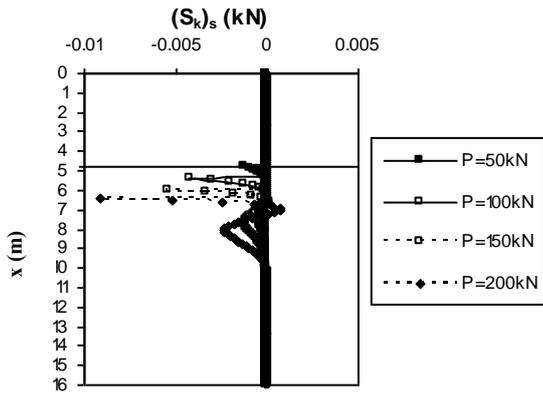


Fig. 17 Distribution of  $(S_k)_s$  for 30% clay

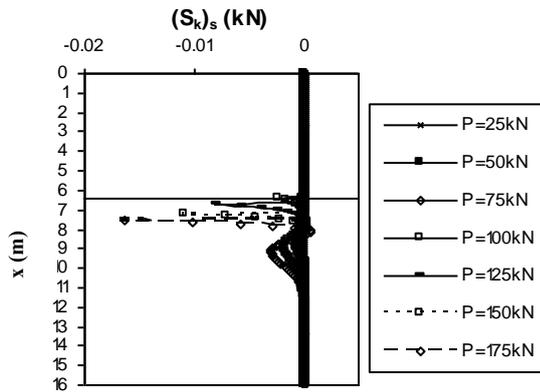


Fig. 18 Distribution of  $(S_k)_s$  for 40% clay

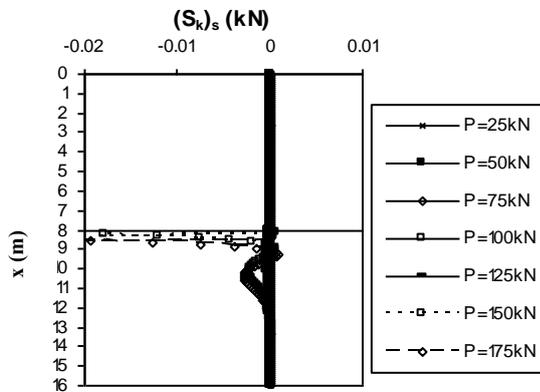


Fig. 19 Distribution of  $(S_k)_s$  for 50% clay

The deflection lines  $y$  and  $y_a$  (Figs 21 and 22) developed in the elastic soil stage have similar distributions along the pile length. However, these two deflection lines are not the exactly reduced copies of each other that differ only in terms of numerical values. They can intersect the  $y=0$  at different depths. It is worth noting that both  $y$  and  $y_a$  twice intersect  $y=0$ . Comparative analysis of both deflection lines shows that they can tangle along  $y=0$ . Bearing in mind that  $(S_k)_s$  is the result of multiplicative combination of  $y$  and  $y_a$ , therefore there is a possibility to obtain two negative peaks or two negative and one positive peak.

In Fig. 20, the first negative peak corresponds to the maximum positive deflections ( $y$  and  $y_a$ ) the pile experiences when the soil state starts to be in the linear elastic stage (for  $P=200\text{kN}$ , the elastic stage starts at depth  $x = 5.1\text{m}$  where the first peak occurs, i.e.  $y < y_k$  starts at  $x = 5.1\text{m}$ ). The second peak corresponds to the maximum negative deflections ( $y$  and  $y_a$ ) that the long pile experiences as seen in the deflection patterns of primary and adjoint piles in Figs 21 and 22, respectively. These peaks physically reflect the nature of the deflection pattern associated with the long piles.

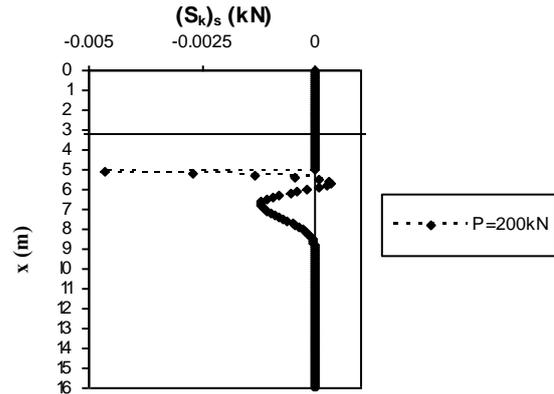


Fig. 20 Distribution of  $(S_k)_s$  for 20% clay

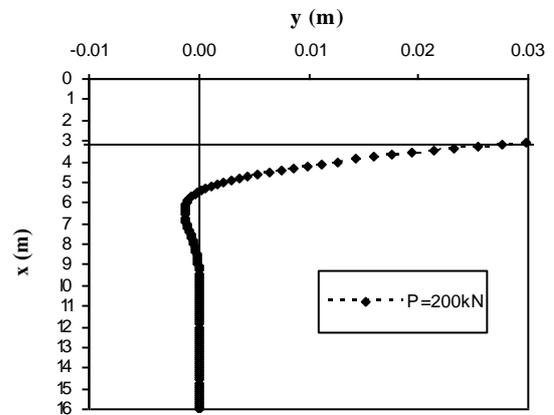


Fig. 21 Deflection of primary pile for 20% clay

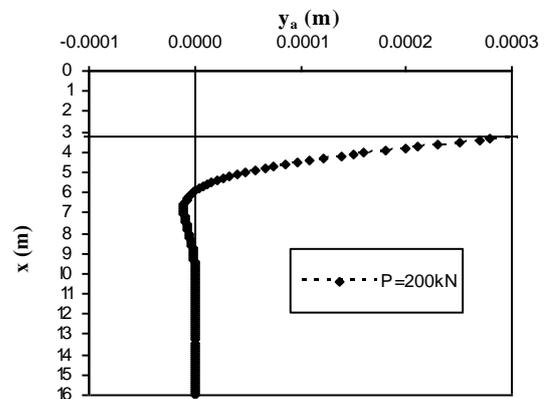


Fig. 22 Deflection of adjoint pile for 20% clay

EFFECT OF THICKNESS OF OVERLYING CLAY ON SENSITIVITY RESULTS

To study the effect of thickness of clay layer on the sensitivity results two approaches are taken. The first approach is to compare the sensitivity results for different thicknesses of clay while the deflection of the pile at the ground surface is kept constant, i.e. deflection-based comparison. The second approach is to compare the sensitivity results for different thicknesses while the load is kept constant, i.e. load-based comparison.

Deflection-based Approach

For the first approach, the sensitivity operators  $(S_{\gamma'_s})_s$ ,  $(S_{\phi})_s$  and  $(S_k)_s$  are plotted in Figs 23 to 25 for different thicknesses of clay while the deflection at the pile head is almost equal for the different cases. In other words, the operators are plotted at loads that give almost same deflection at the pile head. These loads and corresponding deflections are given in Table 1.

From Figs 23 and 24, it is observed that as the thickness of the clay layer increases, the sensitivity of the lateral deflection to changes in  $\gamma'_s$  and  $\phi$  decreases. As the thickness of the clay layer increases, the deflection should increase due to the increase of the weakness of the supporting soil. However, in this approach we are comparing the results that cause almost the same deflection of the pile at the pile head. Therefore, as the thickness of the clay layer increases, the load P that causes the same deflection at the pile head decreases.

Globally the deflection of the pile in all cases is almost the same. However, locally, concerning the analysis of the sand layer, the deflection of the pile decreases as the thickness of clay increases. Two factors cause the pile deflection to decrease with regards to the analysis of the sand layer. The first factor is that as the thickness of clay increases, the sand starts at a deeper level from ground surface and the deflection decreases along the pile length with depth.

The second and more important factor is the effect of non-homogeneity. For the analysis of the sand, the non-homogeneity is accounted for by replacing the clay layer by an equivalent sand layer that gives the same resultant of ultimate resistance at the interface between the two layers (Georgiadis, 1983). The sand is then analyzed as a homogeneous soil with its local coordinates as shown in Fig. 26.

As the thickness of clay  $H_1$  increases, the corresponding equivalent sand layer  $h_2$  relatively decreases. In Table 2, each thickness of clay and its corresponding equivalent thickness of the sand layer and the percent of that equivalent sand thickness compared to the original clay thickness are given.

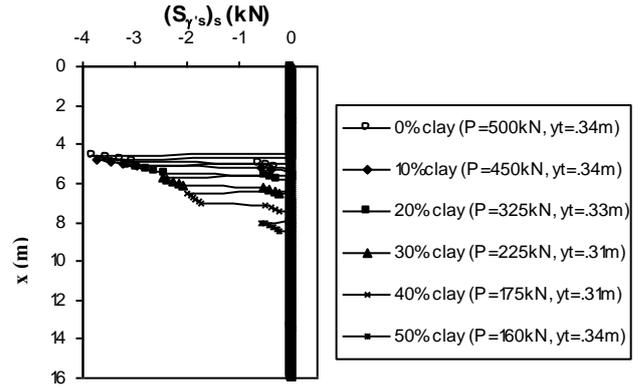


Fig. 23 Effect of clay thickness on  $(S_{\gamma'_s})_s$  for piles having almost same deflection  $y_t$  at ground surface

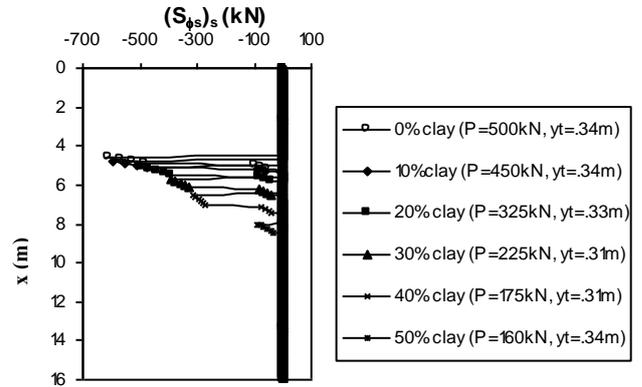


Fig. 24 Effect of clay thickness on  $(S_{\phi})_s$  for piles having almost same deflection  $y_t$  at ground surface

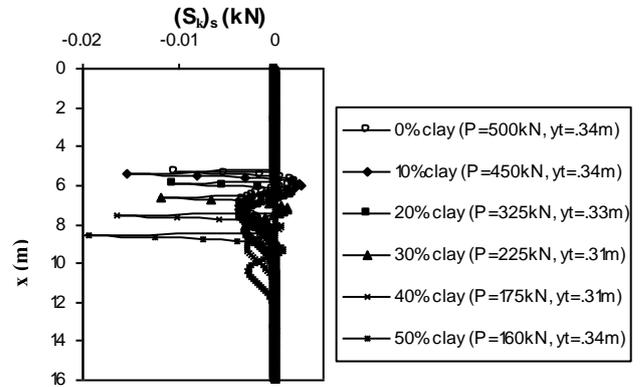


Fig. 25 Effect of clay thickness on  $(S_k)_s$  for piles having almost same deflection  $y_t$  at ground surface

Table 2 Equivalent thickness of sand compared to original clay thickness

% clay	0%	10%	20%	30%	40%	50%
$H_1$ (m)	0	1.6	3.2	4.8	6.4	8
$h_2$ (m)	0	1.6	2.28	2.82	3.22	3.54
$\%h_2/H_1$	100	100	71	58	50	44

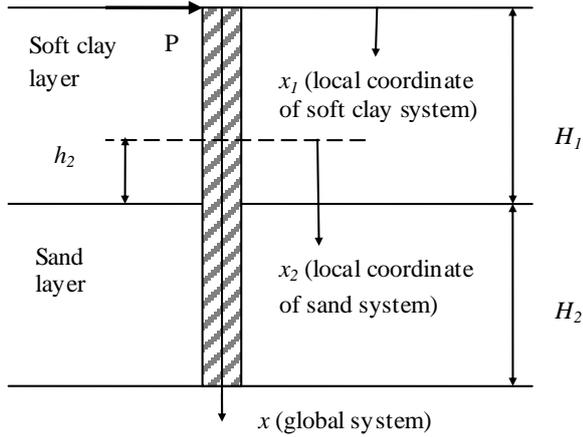


Fig. 26 The coordinate systems of a pile embedded in a non-homogeneous soil

Therefore, although at a certain depth, the deflection of the pile globally is almost equal, the local depth  $x_2$  used for sand decreases as the thickness of the clay layer increases. As an example if the deflection at depth of 8m from ground surface is almost equal for all cases of non-homogeneity, this deflection according to the analysis of the sand layer is achieved for 10% clay at depth 8 m from the local coordinate  $x_2$  of the sand while it is achieved at a depth of 3.54 m from the local coordinate  $x_2$  for the 50% clay. Therefore, this implies that as the thickness increases the pile deflection decreases with regards to sand analysis.

Therefore for deflection-based comparison, as the thickness of clay increases,  $(S_{\gamma's})_s$  and  $(S_{\phi})_s$  decrease since the deflection with regards to sand analysis decreases. For 50% clay, the values of  $(S_{\gamma's})_s$  and  $(S_{\phi})_s$  experience a significant decrease since the decrease in deflection due to the above two factors causes the deflection in sand to start at the second soil stage, i.e.  $y_k < y < y_m$  at which the values of  $(S_{\gamma's})_s$  and  $(S_{\phi})_s$  decrease significantly as explained in the previous section. In addition, the values of  $(S_{\gamma's})_s$  and  $(S_{\phi})_s$  are very close for the case of 0% and 10% clay because according to the sand layer the analysis is very similar (Table 2).

From Fig. 25, it is observed that as thickness of clay increases the value of  $(S_k)_s$  increases in general. The value of  $(S_k)_s$  is calculated from Eq. (4) given above. The effect of the parameter  $k$  starts when the deflection of the pile becomes less than  $y_k$  and soil experiences a linear elastic stage. As thickness of clay increases, both  $x_2$  and  $y$  decrease as explained above. However,  $y_a$  increases due to the fact that as the thickness of clay increases, the soil becomes weaker and the load required to obtain the same deflection at ground surface decreases. The deflection of the adjoint pile  $y_a$  is obtained by applying a unit load to the adjoint pile which is in the state of deformation of the primary pile subjected to a given load.

The value of  $y_a$  is obtained numerically by subtracting the deflection of a pile subjected to load  $P+1$  from the deflection of a pile subjected to load  $P$ . Therefore as  $P$  decreases  $y_a$  increases (for example a difference between the deflection of a pile subjected to load 2001kN and 2000 kN will be less than the difference between piles subjected to loads 2kN and 1kN). Although  $x_2$  and  $y$  decrease as clay thickness increases, the effect of  $y_a$  is dominating on the behavior of  $(S_k)_s$ . Thus the value of  $(S_k)_s$  increases with the increase of thickness of clay.

#### Load-based Approach

The second approach taken to study the effect of the thickness of overlaying clay layer on the sensitivity results is to investigate this effect while the pile is subjected to a constant load. The sensitivity operators are compared for the different thicknesses of clay layer when the pile is subjected to loads  $P = 100\text{kN}$ ,  $P = 200\text{kN}$  and  $P = 300\text{kN}$ . The sensitivity operator  $(S_{\gamma's})_s$  is plotted for the three constant loads in Figs 27 to 29.

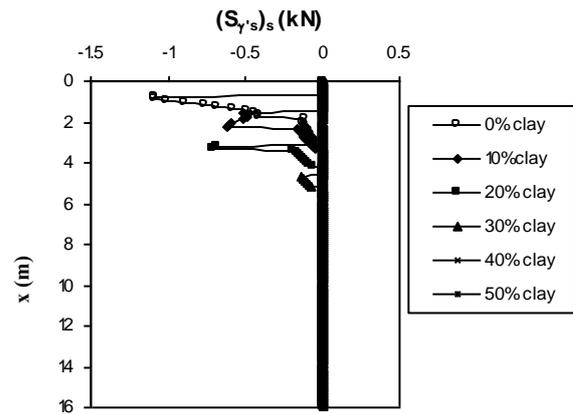


Fig. 27 Effect of clay thickness on  $(S_{\gamma's})_s$  for piles subjected to load  $P = 100\text{kN}$

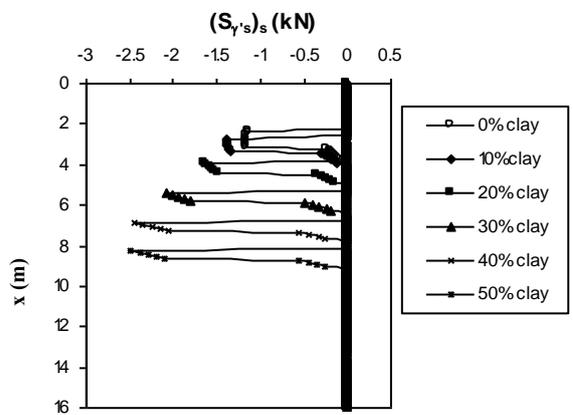


Fig. 28 Effect of clay thickness on  $(S_{\gamma's})_s$  for piles subjected to load  $P = 200\text{kN}$

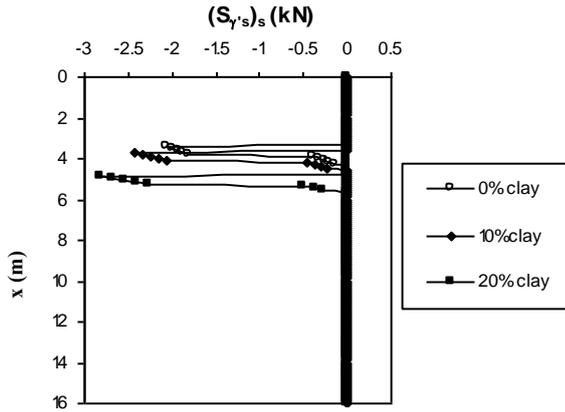


Fig. 29 Effect of clay thickness on  $(S_{\gamma's})_s$  for piles subjected to load  $P = 300\text{kN}$

For a constant load, as the thickness of clay increases, the deflection of the pile increases because the soil becomes weaker and accordingly the sensitivity increases in general. However, since sand starts at a deeper level as clay thickness increases, the sensitivity value can decrease if the load is not high enough to cause the sand to experience the third stage of the sand behavior. Accordingly, the sensitivity decreases if deflection in sand starts at  $y_k < y < y_m$  (as the case of load  $P=100\text{kN}$  and clay thickness=30%) or completely vanishes if  $y < y_k$  (as seen for  $P=100\text{kN}$  and thickness equal to 40% and 50%). The exceptionally higher values of  $(S_{\gamma's})_s$  and  $(S_{\phi})_s$  for the homogeneous layer of sand (0%clay at  $P=100\text{kN}$ ) was explained in the previous section.

The same above comments apply to the sensitivity operator  $(S_{\phi})_s$  which is plotted in Figs 30 to 32 for the three different constant loads. The only difference between the two operators  $(S_{\gamma's})_s$  and  $(S_{\phi})_s$  is that the magnitude of the operator  $(S_{\phi})_s$  is higher than that of  $(S_{\gamma's})_s$ .

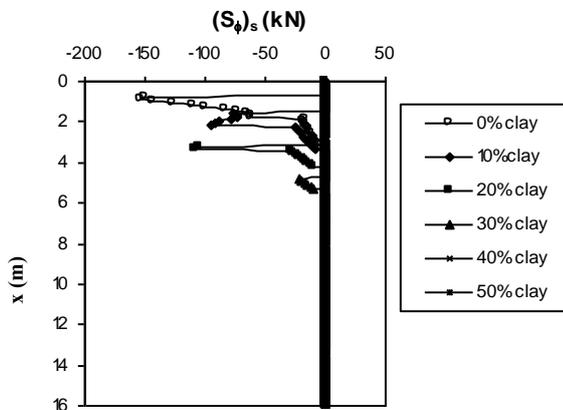


Fig. 30 Effect of clay thickness on  $(S_{\phi})_s$  for piles subjected to load  $P = 100\text{kN}$

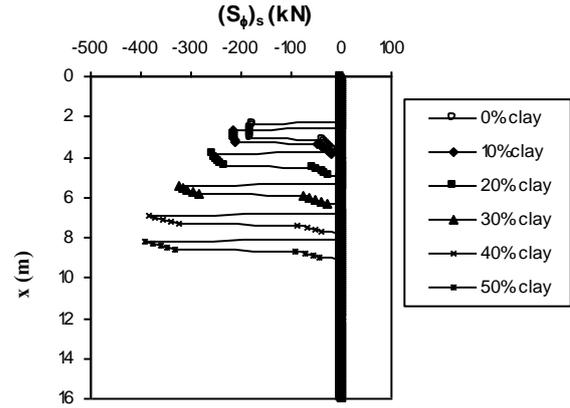


Fig. 31 Effect of clay thickness on  $(S_{\phi})_s$  for piles subjected to load  $P = 200\text{kN}$

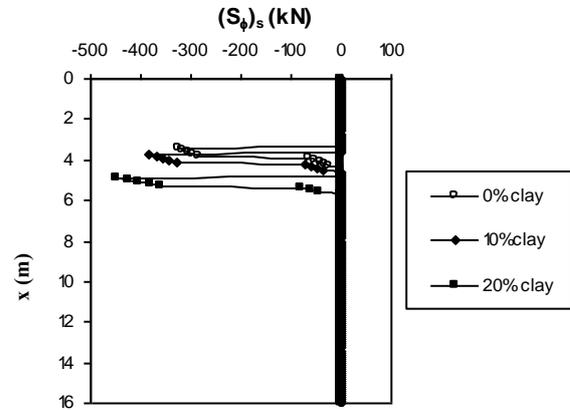


Fig. 32 Effect of clay thickness on  $(S_{\phi})_s$  for piles subjected to load  $P = 300\text{kN}$

The effect of the thickness of clay layer on the sensitivity operator  $(S_k)_s$  is shown in Figs 33 to 35.  $(S_k)_s$  is calculated using Eq. (4). The maximum value of the sensitivity operator increases as the thickness of clay increases because both the deflections  $y$  and  $y_a$  increase. The deflection  $y$  of pile increases because the clay thickness increases. The adjoint deflection  $y_a$  increases as load  $P$  increases as explained in the previous section. However, the load is constant for this load-based comparison. The other factor that affects  $y_a$  in this case is that it increases as the deflection  $y$  increases.

For 40% and 50% clay, the sensitivity studies covered up to loading equal to 175kN (Table 1), however the sensitivity operators were calculated for  $P = 200\text{kN}$  for 40% and 50% clay for the sake of comparison of a broader spectrum in the case of loading  $P = 200\text{kN}$  (Figs 28, 31 and 34).

SUMMARY AND CONCLUSIONS

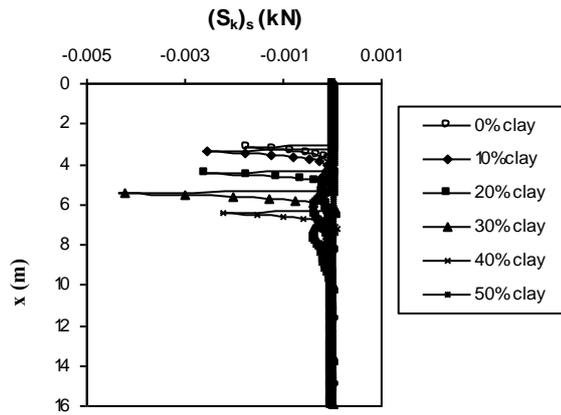


Fig. 33 Effect of clay thickness on  $(S_k)_s$  for piles subjected to load  $P = 100\text{kN}$

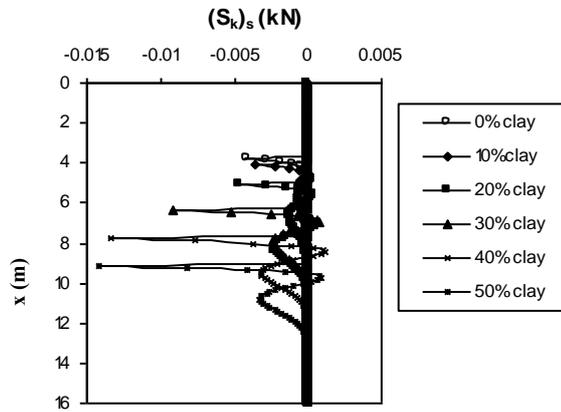


Fig. 34 Effect of clay thickness on  $(S_k)_s$  for piles subjected to load  $P = 200\text{kN}$

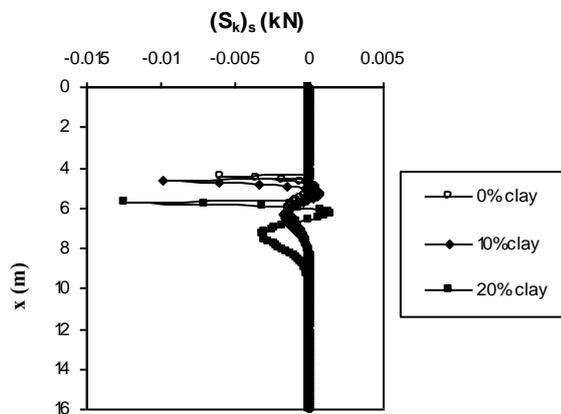


Fig. 35 Effect of clay thickness on  $(S_k)_s$  for piles subjected to load  $P = 300\text{kN}$

Numerical sensitivity studies were performed in this part of the paper based on the theoretical formulation of sensitivity analysis derived in Part I of the paper (Hafez and Budkowska, 2006a). The paper covered the study of the effect of the changes of sand parameters on the maximum lateral deflection of long piles subjected to cyclic loading and embedded in non-homogeneous soil consisting of soft clay overlying sand. The sensitivity results were given in the form of graphical presentation of sensitivity operators that allows the engineer to detect where and how each design variable affects the lateral pile-head deflection. The three design variables studied were the submerged unit weight  $\gamma'_s$ , the friction angle of sand  $\phi$ , and  $k$  (a constant representing the modulus of subgrade reaction in the linear stage of sand behavior).

In addition, the effect of thickness of the overlying clay layer on the sensitivity results was explored based on two approaches. The first approach addresses engineers interested in the assessment of the effect of increase in the overlying clay layer on the sensitivity results while the pile is maintained at constant deflection by decreasing the load applied as clay thickness increases (i.e. deflection-based comparison). The second approach is addressed to engineers interested in exploring the effect of evolution of the overlying clay thickness while the pile is subjected to a constant load (i.e load-based comparison). Based on these approaches, the following conclusions can be drawn;

1. For deflection-based comparison, the sensitivity of lateral head deflection to changes in  $\gamma'_s$  and  $\phi$  decreases as the thickness of clay layer increases while the sensitivity to changes in  $k$  increases.
2. For load-based comparison, the sensitivity of lateral head deflection to changes in  $\gamma'_s$  and  $\phi$  as well as  $k$  increases as the thickness of clay layer increases although the sand layer becomes further away from the subjected load.
3. The study of the effect of non-homogeneity of soil presented is of significant importance to areas in which the overlaying soft clay layer is of different thicknesses such as in lowland areas.
4. For a given case of thickness of clay layer, as the applied load increases, the sensitivity of the lateral deflection to changes in the three sand parameters increases.
5. The graphical presentations of the sensitivity operators are presented as a guide to engineers to detect critical locations of sensitivity of pile performance to sand parameters and to complement the understanding of the pile-system behavior.

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