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EFFECT OF THICKNESS OF OVERLAYING CLAY LAYER OF LOWLAND REGION ON SENSITIVITY OF LATERAL DEFLECTION OF LONG PILES EMBEDDED IN NON-HOMOGENEOUS SOIL - PART I: THEORETICAL FORMULATION

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ABSTRACT: The importance of sensitivity analysis of soil-structure interaction system is embodied in two facts, that is, the soil is a nature-made material that affects the system's performance and the sensitivity theory constitutes an inherent part of the behavior of the system. Moreover, the access to the supporting system is limited. Therefore, it is essential to have a reliable basis for the assessment of how changes of the parameters of the system affect its performance. The paper presents the study of sensitivity of laterally loaded piles using the distributed parameter sensitivity method. Other available sensitivity methods are briefly described in the paper. The theoretical formulation of the sensitivity of the lateral head deflection of piles embedded in non-homogeneous soil to changes in the design parameters is derived. The non-homogeneous soil consists of soft clay overlying sand and the design parameters are those that define the pile and the adjacent clay and sand. The formulation resulted in obtaining sensitivity operators that can show along the pile length where and how the change of each parameter affects the change of lateral pile-head deflection. The formulation provides the basis for studying the effect of the thickness of the overlying clay on the sensitivity results.

Keywords: Sensitivity, distributed-parameter method, non-homogeneity, laterally loaded piles

INTRODUCTION

Civil engineering structures such as high rise buildings, bridge piers and abutments, transmission towers, sport facilities (stadiums), retaining walls, overhead signs on highways, noise barrier walls, and offshore platforms for oil production are often subjected to horizontal loads. To support these structures, a number of piles that are designed to carry horizontal loads may be needed. The pile-head deflection is a crucial quantity for the behavior of the superstructure and an important serviceability measure. Good and safe performance of the pile system and superstructure will be achieved if certain measures of deflections are satisfied.

Traditionally, infrastructure design practices have considered initial conditions, loads and material properties as primary input variables for structural design without taking into account the effect of environmental and material degradation over time. Such an approach does not adequately assess the actual service life of the structure. The performance of an infrastructure element or system is considered good if it performs as developed and provides an acceptable level of service over its intended life.

Therefore, it is important to assess how the spatially distributed changes of material properties affect the changes of the performance of the pile-soil infrastructure system described by the lateral pile-head deflection. The answer to this question is of vital importance to the safety as well as the assessment of the deterioration of the system. It was not possible to give answer to this inquiry at the time of the design and construction of soil-structure infrastructure systems (being now in the advanced ageing process) since the sensitivity theory of distributed parameters was in its early infancy stage.

The distributed parameter sensitivity theory considers the material properties (in general terms) as spatial functions. This means that sensitivity theory postulates that the changes of the performance of the system are attributed to the changes of material parameters of the system. More importantly, the sensitivity theory, through the fact of making the material parameters the spatially dependent functions, is able to show where and how these material changes are distributed in the system investigated. The spatial functions determined in the scope of sensitivity analysis that allow to assess and to

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localize those parameters that affect the performance of the system are called the sensitivity operators. Physically, they represent the potential material change that contributes to the changes of the performance of the system.

The sensitivity theory of linear systems employed in explorations of deep foundations subjected to various types of load (torsional, axial and buckling) has been presented in a number of publications (Budkowska and Szymczak 1993a, 1993b and 1996). Previous sensitivity studies were applied to laterally loaded piles embedded in homogeneous soil (Budkowska and Priyanto, 2003a and 2003b).

The paper presents the theoretical formulation for sensitivity of lateral head deflection of laterally loaded piles embedded in non-homogeneous soil. The presented formulation in this paper extends the sensitivity formulation derived by Budkowska and Priyanto (2003a) for homogeneous soils to non-homogeneous soils. An arbitrary number of soil layers are incorporated in the formulation rather than a single homogeneous soil layer. Moreover, the current formulation provides a basis for analysis of non-homogeneous geological profiles that contain layers of soil with design variables varying in arbitrary fashion along the depth.

The non-homogeneous soil stratification is a more general case especially in lowland areas where the soil profile starts with clay layers of variable thicknesses. The investigated non-homogeneity is of layered type in which the soil profile consists of soft clay overlying sand. Both layers are below water table and the pile is subjected to cyclic loading.

The behavior of the pile-soil system is mathematically modeled for the sensitivity analysis using the *p-y* approach. The original idea of *p-y* models (where *p* stands for soil reaction and *y* means the lateral deflection) was established based on an extensive program conducted on the natural scale piles embedded in various types of homogeneous soils subjected to static and cyclic loadings (Matlock 1970, Reese et al. 1974). Since then research on *p-y* models continues providing new recommendations for *p-y* models taking some effects into account like the pile installation effect (Kim et al., 2004). However, the original *p-y* models still enjoy high popularity being commonly used for the design purpose all over the world. The *p-y* models are highly nonlinear and require the involvement of various soil strength parameters as well as pile structure parameters. The models serve basically for the determination of the kinematic and strength performance of the laterally loaded pile-soil systems.

The incorporation of those homogeneous *p-y* models into the studied non-homogeneous media of layered type is based on the equivalent thickness approach which considers the criterion of continuity of soil's ultimate lateral resistance force along the pile's axis (Georgiadis, 1983, Reese et al., 2001). The investigations of non-homogeneous media of layered type make the explorations more realistic and comprehensive. They also broaden the spectrum of physical parameters required to be involved in analysis.

The following section gives a brief review on the different sensitivity methods based on which the choice of the applied distributed parameter sensitivity method is justified. The mathematical *p-y* models required for the sensitivity formulation which incorporate the non-homogeneity are then presented. The theoretical formulation for the sensitivity of the lateral pile-head deflection to changes in the system parameters is then derived in detail for piles embedded in non-homogeneous soil. This formulation furnishes the basis for studying the effect of the thickness of overlying clay layer on the sensitivity results through the non-homogeneity of the soil considered.

REVIEW OF SENSITIVITY METHODS

The behavior of the structural system can be described in terms of the relationships between a cause and an effect. In case of civil engineering structures, a cause is usually associated with various forces of static or dynamic type as well as thermal, environmental and hazardous loads. The results of these actions affect the behavior, that is, the state of the system that is measured by means of state variables. Another cause of behavioral changes of the structural system can be related to other factors that are intrinsically involved in the behavior of the system such as properties of the materials, geometry of the system, conditions of an environment and other factors. Thus, the behavior of the system depends not only on the external causes but also on internal ones. The paper is focused on investigating effects of inherent causes on the behavior of the structural system.

In general, the sensitivity theory deals with analysis of how inherent causes called the design variables affect the behavior of the structural system. Knowing the effects the changes of the design variables have on the performance of the system, it is possible to employ this knowledge in various areas of engineering activities such as design, improvement, rehabilitation, etc. Sensitivity theory has direct application since it refers to arbitrary, however existing structural models. The framework of sensitivity theory provides several methods of sensitivity analysis that depend on the type of the design variables and the descriptions of the behavior of the systems

(Haug et al., 1986, Kleiber et al., 1997). Some of the most common analytical sensitivity methods are:

1. Direct differentiation method (DDM) with discrete design variables
2. Direct differentiation method (DDM) with distributed design variables
3. Adjoint system method (ASM) with discrete design variables
4. Adjoint system method (ASM) with distributed design variables

Method 1: DDM with Discrete Design Variables

The direct sensitivity method of analysis with discrete design variables employs the description of the structural behavior formulated in the framework of the finite element method (FEM). This approach is based on the direct differentiation of all quantities entering the equilibrium of the system with respect to the design variables including the unknown displacement vector. The shortcomings of this method are: the necessity of using the inverted stiffness matrix and discrete differentiation of the stiffness matrix with respect to design variables.

Method 2: DDM with Distributed Design Variables

There are two main advantages for the distributed parameter approach (continuous/distributed design variables). First, a rigourous mathematical theory is obtained, without the uncertainity associated with the discretized approximation error. Second, explicit relations for design sensitivity are obtained in terms of physical quantities, rather than in terms of sum of derivatives of element matrices. The main disadvantage of the distributed parameter approach is the higher level of mathematical sophistication associated with this method from an engineers point of view.

In addition, DDM in general is insensitive to the number of performance functions and sensitive to the number of design variables studied since calculations should be performed for each design parameter independent of the others. So the method is costly when the number of design variables is large and the ASM is preferred in that case.

Method 3: ASM with Discrete Design Variables

The considerable improvement in comparison to direct differentiation sensitivity method is achieved when the direct differentiation sensitivity equation is reshaped in such a fashion that it can be solved with the aid of an auxiliary structure called the adjoint structure that is subjected to a suitable load. The crucial feature of this concept is embodied in its physical interpretation. Namely, the analysis of the adjoint system shows that it represents a virtual system that is employed in the virtual work principle. More precisely, it employs the virtual load method.

The connection of the adjoint system of virtual work principle substantially enhances the range of applications. This is done due to the fact that the virtual work principle is the most general principle that can be applied to any material (Malvern, 1969). Thus it provides a powerful and effective tool of analysis. The virtual system requires to be deformed from the actual state of deformation of the original system called the primary system. Accordingly, the notion of virtual work principle implies a self explaining procedure and essence of the method. Interestingly, FEM itself is also based on the virtual work principle.

It is worth noting that ASM is, opposite to DDM, insensitive to the number of design variables. It is sensitive to the number of performance functions.

The adjoint system sensitivity method in discrete formulation (with discrete design variables) provides a final value (in numerical terms) of the sensitivity of the performance of a chosen point when the design variables change in known places by a given amount.

Method 4: ASM with Distributed Design Variables

The adjoint system method with distributed (continuous) design variables provides a new way of investigation. Namely, it allows conducting a sensitivity investigation of a performance of the system in such a way that, besides final numerical value of sensitivity result , it also gives an opportunity to show precisely in spatial and numerical terms how and where the design variables affect the performance of a system the physical model of which is known. These features of the adjoint system method of distributed parameters turn out to have a significant appeal to all sorts of civil engineering applications. They provide a valuable knowledge on the behavior of the system if employed at the design stage or in an aging process and in general, in decision making process. It also gives basis for the quantification of the importance of the distributed design variables on the performance of the system.

The specified advantages of the adjoint system method with distributed design variables of sensitivity analysis are the result of an opportunity to conduct all investigations explicitly under spatial integral, before integrating them with respect to the spatial variable. On the other hand, the engagement of finite element analysis in the adjoint system method of discrete design variable employs already integrated results through the fact that it uses the stiffness matrix that requires spatial integration of the investigated problem.

The distributed parameter sensitivity analysis is conducted under a spatial integral, therefore, the sensitivity operators are called the sensitivity integrands. They are spatial functions that can be visualized graphically. This feature of sensitivity analysis of distributed parameters is of substantial importance as far as the engineering applications are concerned. They are also critical in physical interpretation of sensitivity spatial changes when explanations are related to the physical model of the primary system. Finally, it is worth adding that integration of each sensitivity integrand with respect to spatial variables gives the numerical value of the change of quantity of interest due to the change of the corresponding design variables.

Conclusion

It is concluded that sensitivity analysis is uniquely connected with the type of an investigated system and with the physical model employed in analysis of structural system. It is clear from the above brief presentation of the different sensitivity methods that the most appropriate method for fulfilling the current research purpose is the adjoint system method ASM with distributed design variables (method 4). The current research investigates the sensitivity of the performance of the pile to the different design variables. The performance of the pile is described by the lateral pile-head deflection and the design variables are the pile and soil parameters (8 parameters). Accordingly, ASM is preferred over DDM since the number of design variables are large compared to the performance function investigated. The distributed parameter approach is used since it allows for the spatial graphical presentation of the distribution of sensitivity operators along the pile length which shows how and where each design variable affects the system's performance as explained above.

SOIL MODELS CONTRIBUTING TO NON-HOMOGENEOUS SOIL OF LAYERED TYPE

The lateral loading besides an axial load is one of the most frequently applied loads the pile foundations have to resist. The significance of lateral loads acting on deep foundations motivated the thorough field research in the period of offshore energy explorations. The in situ piles embedded in various types of homogeneous soils were examined. Consequently, it prompted ingenuity to development of a special soil-pile interaction system known as *p-y* models. The primary objective of the *p-y* models is to furnish an adequate representation of homogeneous soil that can be used in engineering applications.

The preponderance of *p-y* soil models over laboratory simulated models is that the former have been on real scale piles embedded in natural soil conditions. Thus, the usual shortcomings of laboratory models with regard to idealistic conditions and scaling do not apply to in situ models. The methodology developed using *p-y* soil models for laterally loaded piles continues to be attractive to many researches through decades. Every passing year provides new papers on recent developments to laterally loaded piles giving new recommendations for the *p-y* models that take some effects into account like the effect of various installation techniques on the behavior of *p-y* pile-soil system (Bransby, 1999 and Kim et al., 2004).

The *p-y* soil models represent all suitable analog to the three dimensional real pile-soil behavior. The existing *p-y* soil models are spatially continuous and mechanically discrete. The latter term means that deformations of the soil model when subjected to a loading are local which means they do not provide continuity of deformation. However, when combined with the pile to which the load is applied, deformations and internal forces of the pile-soil system are continuous.

The original *p-y* soil models were developed for homogeneous soils. To accommodate them to non-homogeneous soils of layered type, further research was required to provide a rationale for adequate performance of the pile-soil system. The work of Georgiadis (1983) is commonly adopted for this purpose. He proposed an equivalent thickness method which is used in the current study. In the current research, the laterally loaded pile is embedded in non-homogeneous soil of layered type and subjected to cyclic loading. In particular, a layer of *p-y* soft clay below water table overlays a layer of *p-y* sand. Both *p-y* models for homogeneous clay and sand are briefly described followed by the soil model used for the non-homogeneous soil.

The *p-y* Model for Soft Clay

The *p-y* model proposed by Matlock (1970) is used to describe the pile-soil system behavior. The typical soft clay *p-y* graphical presentation is shown in Fig. 1. The nomenclature employed in the *p-y* soil model is defined in spatial coordinate system *x-y* attached to the soil surface. The *x* axis is directed downward while *y* axis is pointed to the right.

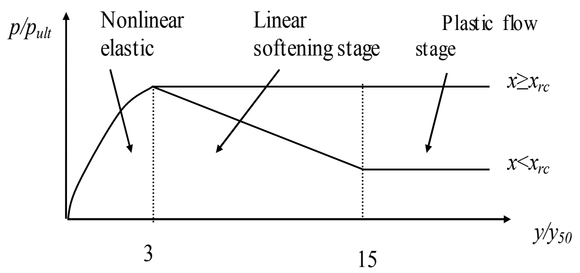

Fig. 1 Cyclic behavior of soft clay after Matlock (1970)

The *p-y* models employ the notion of ultimate soil resistance p_{ult} that enter the *p-y* soil relationships. The ultimate soil resistance, p_{ult}, is given as:

$$p_{ult} = \left(3 + \frac{\gamma'_c}{c}x + \frac{J}{b}x\right)cb \qquad \text{for } x \le x_{rc} \tag{1}$$

$$p_{ult} = 9cb \qquad \text{for } x \ge x_{rc} \tag{2}$$

where γ'_c is the submerged unit weight of soft clay, c is the undrained cohesion, J is a model constant, x is a spatial variable starting from the soil surface and directed downwards along the pile axis and x_{rc} is the depth of reduced resistance after which p_{ult} becomes constant (not a function of x) as clear from Eqs. (1) and (2). The value of x_{rc} is calculated by equating the two values of p_{ult} given in the above two equations.

For a depth less than x_{rc}, three stages define the soil behavior which are given in Table 1. In the first stage, the soil behaves in a nonlinear elastic manner. In the second stage the soil experiences softening while the third stage is a plastic flow stage. For a depth greater than x_{rc}, the soil experiences a nonlinear elastic behavior described by the same equation given in Table 1 (stage 1) followed directly by a plastic flow stage (given as $p = 0.72\, p_{ult}$). The relationships in Table 1 are given in terms of the deflection at one-half of that ultimate soil resistance, y_{50}. The deflection y_{50} is given as:

Table 1 Soft clay *p-y* relationships for the different soil stages

Soil Stage	*p-y* Relationships
$0 < y < 3y_{50}$	$\frac{p}{p_{ult}} = 0.5\left(\frac{y}{y_{50}}\right)^{1/3}$
$3y_{50}<y<15y_{50}$	$\frac{p}{p_{ult}} = \left(0.9 - 0.18\frac{x}{x_{rc}}\right) - 0.06\frac{y}{y_{50}}\left(1 - \frac{x}{x_{rc}}\right)$
$y > 15y_{50}$	$\frac{p}{p_{ult}} = 0.72\frac{x}{x_{rc}}$

$$y_{50} = 2.5\varepsilon_{50}b \tag{3}$$

where ε_{50} is the strain corresponding to one-half of the compressive strength of clay.

The *p-y* Model for Sand

The sand behavior proposed by Reese el al. (1974) is employed in the analysis and is shown in Fig. 2. The different stages of the sand behavior depend on the deflection. There are four stages depending on the deflection: a linear elastic stage, a parabolic nonlinear stage, a linear hardening stage and a plastic flow stage. The deflections differentiating between the different stages are y_k, y_m and y_u, where y_k is calculated by the intersection of the curves of the two stages while the values of y_m and y_u are shown in Fig. 2. Their corresponding soil reactions p_m and p_u are given as:

$$p_m = B_c p_s \ ; \qquad p_u = A_c p_s \tag{4}$$

where A_c and B_c are non-dimensional coefficients which are given in graphs as functions of (x/b) and p_s is the ultimate soil resistance per unit length of the pile. The resistance p_s is denoted as p_{st} for the upper part of the pile-soil system above the depth of reduced resistance for sand, x_{rs} ($x \le x_{rs}$) and is given in Eq. (5). It is denoted as p_{sd} for the lower part of the pile-soil system ($x \ge x_{rs}$) and is given in Eq. (6).

$$p_{st} = \gamma'_s x \left[\begin{array}{l} \dfrac{0.4x\tan\phi\sin\beta}{\tan(\beta-\phi)\cos\alpha} + \dfrac{\tan\beta}{\tan(\beta-\phi)} \\ (b + x\tan\beta\tan\alpha) \\ + 0.4x\tan\beta(\tan\phi\sin\beta - \tan\alpha) - K_a b \end{array}\right] \tag{5}$$

$$p_{sd} = K_a b\gamma'_s x(\tan^8\beta - 1) + (0.4b\gamma_s x\tan\phi\tan^4\beta) \tag{6}$$

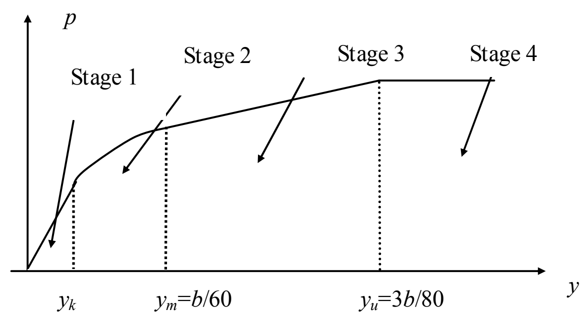

Fig. 2 Nonlinear behavior of sand subjected to cyclic loading after Reese et al. (1974)

where ϕ is the friction angle of sand, $\alpha = \phi/2$, $\beta = 45 + \phi/2$; K_a=$\tan^2(45+\phi/2)$ and γ_s' is the submerged unit weight of sand. The value of x_{rs} is calculated by the intersection of Eqs. (5) and (6). The *p-y* relationships are shown in Table 2. In Table 2, k is a constant representing the modulus of subgrade reaction in the linear stage in the curve.

Table 2 Sand *p-y* relationships for the different soil stages

Soil Stage	*p-y* Relationships
$0 < y < y_k$	$p = (kx)y$
$y_k < y < y_m$	$p = \left(\frac{60}{b}y\right)^{0.8\left(\frac{A_c}{B_c}-1\right)} p_m$
$y_m < y < y_u$	$p = p_s\left[B_c + \frac{48}{b}\left(y - \frac{b}{60}\right)(A_c - B_c)\right]$
$y > y_u$	$p = p_u$

The *p-y* Models for Non-homogeneous Soils

The homogeneous soil profile is rather a special case of natural soil deposit. The development of a methodology that allows the connection of two different *p-y* models requires formulation of a criterion for reliable analysis. The *p-y* relationships for homogeneous soils are described by means of ultimate soil resistance p_{ult} or p_s as explained above. The conducted experimental studies show that when combining homogeneous soils into non-homogenous medium of layered type, the continuity of the resultant of the ultimate resistance constitutes the necessary condition that is required to be satisfied. Consequently, this condition allows determining an imaginary soil surface of the underlying soil where the *x, y* coordinate system of underlying soil is attached (Georgiadis, 1983). This concept is shown in Fig. 3.

The determination of the resultant of ultimate soil resistance F_1 and F_2 of the overlying layer and the underlying layer, respectively, is conducted in corresponding local coordinate systems (x_1,y) and (x_2,y) respectively. Thus,

$$F_1 = \int_0^{H_1} p_{ult}\,dx_1 \qquad F_2 = \int_0^{h_2} p_s\,dx_2 \tag{7}$$

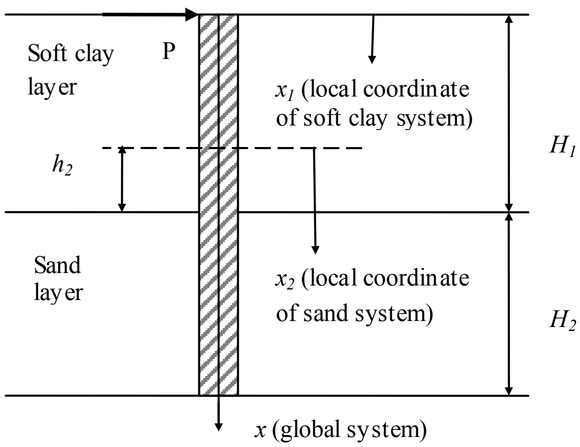

Fig. 3 The coordinate systems of a pile embedded in a non-homogeneous soil

The condition of continuity of F that allows determining an imaginary surface of the underlying layer is the following:

$$F_1 = F_2 \tag{8}$$

It is obvious that the thickness h_2 that defines the location of imaginary soil surface of underlying layer depends on the types of both *p-y* soil models contributing to the non-homogeneous soil of layered type and the thickness H_1. It can be located within the overlaying soil or above it.

It is worth noting that Eq. (7) can contain p_{ult} formula for ($x \le x_{rc}$) or for both ($x \le x_{rc}$ and $x \ge x_{rc}$), depending on the thickness of the overlying layer. Similarly, the underlying layer can have p_s for ($x \le x_{rs}$) or for both ($x \le x_{rs}$ and $x \ge x_{rs}$).

THEORETICAL FORMULATION

The investigated free head pile subjected to lateral force P is embedded in non-homogeneous soil of layered type. The lateral deflection of the system when subjected to constant load P is affected by the design variables gathered in the following vector of design variables:

$$\boldsymbol{d} = \{EI, b, \gamma_c', c, \varepsilon_{50}, \gamma_s', \phi, k\}^T \tag{9}$$

where EI is the pile's bending stiffness and $\{\}^T$ denotes the transpose of the vector.

According to Eq. (9), the explored system forms an eight parameter sensitivity system. The maximum value of the lateral deflection is connected with the pile head thus; the maximum deflection occurs at the top of the pile and is denoted as y_t. It is considered as an indicator of serviceability of the supporting system that affects the serviceability of the superstructure. The critical notion that is postulated in this analysis is that change of deflection is not associated with the change of load but with other factors that affect the deflection of the system in similar fashion as deformability when subjected to load.

As discussed previously, the sensitivity analysis of distributed parameter type can be explored by means of an adjoint system method. The virtual displacement of the adjoint system, by definition, is measured from the equilibrium configuration of the primary structure (Malvern, 1969). It is worth noting that while the virtual displacement and associated strains and rotations are infinitesimal, no restrictions are placed on the magnitudes of the actual displacements from any reference configuration to the equilibrium configuration. The principle can therefore be used in finite-displacement problems.

The determination of the change of maximum deflection δy_t can be obtained based on the virtual work principle employed in the virtual load method. Thus, it requires introducing the adjoint structure that defines state of deformation of the original structure, called the primary structure. Both structures are shown in Fig. 4.

Accordingly, the change of lateral top deflection δy_t caused by the change of the design variables $\delta \boldsymbol{d}$ is obtained by subjecting the adjoint pile to a unit lateral load, 1_a, at the top of the pile. Using the virtual load principle, δy_t is given as:

$$1_a \delta y_t = -\int_0^l M_a \delta y'' dx + \int_0^l p_a \delta y dx \quad (10)$$

where M_a , p_a are the internal forces (moment and soil reaction) of the adjoint structure subjected to the unit load, and $\delta y''$, δy are the changes of angle of flexural rotation and deflection of primary structure caused by changes in the design variables, respectively.

The interaction of two substructural materials such as the pile structure and the supporting soil system are defined in general terms as:

$$M = -EIy'' = M(y, \boldsymbol{d}) \quad (11)$$

$$p = p(y, \boldsymbol{d}) \quad (12)$$

The increments of angle of flexural rotation $\delta y''$, and deflection δy of primary structure in the presence of constant load can be referred to increments of suitable internal forces of the primary structure as;

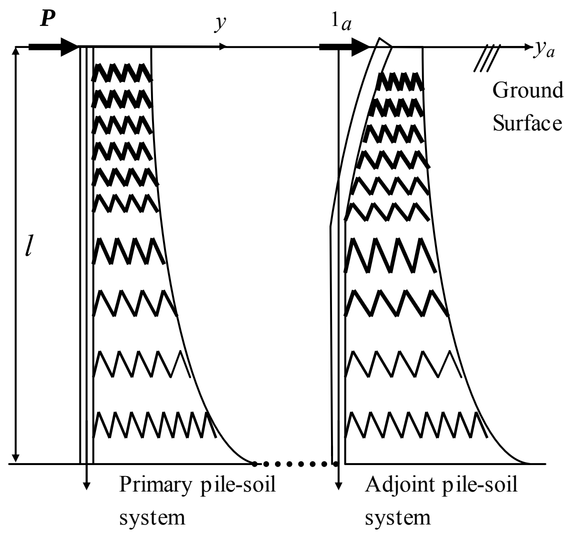

Fig. 4 Primary and adjoint pile-soil systems showing the reaction of the soil modeled by nonlinear springs

$$\delta M = \frac{\partial M}{\partial y''} \delta y'' + \frac{\partial M}{\partial \boldsymbol{d}} \delta \boldsymbol{d} \quad (13)$$

$$\delta p = \frac{\partial p}{\partial y} \delta y + \frac{\partial p}{\partial \boldsymbol{d}} \delta \boldsymbol{d} \quad (14)$$

It is worth noting that in Eqs. (11 and 13) the design variables vector $\boldsymbol{d}$ contains only one component that affects the pile material, that is EI, whereas Eqs. (12 and 14) have the remaining components of the design variables vector $\boldsymbol{d}$ that are associated with non-homogeneous soil.

Moreover, Eqs. (13 and 14) are characteristic equations of sensitivity analysis by the fact that they include the internal force-generalized displacement relationship and also the variability of the material parameters. This is considered as an intrinsic feature of distributed parameter sensitivity theory. Equations (13 and 14) are given in explicit fashion, which demonstrates simplicity of analysis and applicability to engineering practice.

Since the investigated primary system subjected to external load is in static equilibrium, therefore the increment of internal forces is not allowed to develop, thus they must vanish. This means that:

$$0 = \frac{\partial M}{\partial y''} \delta y'' + \frac{\partial M}{\partial \boldsymbol{d}} \delta \boldsymbol{d} \quad (15)$$

$$0 = \frac{\partial p}{\partial y} \delta y + \frac{\partial p}{\partial \boldsymbol{d}} \delta \boldsymbol{d} \quad (16)$$

Equations (15) and (16) provide basis for determination of the sought variation $\delta y''$ and δy. Thus,

$$\delta y'' = -\frac{\partial y''}{\partial M}\left(\frac{\partial M}{\partial \boldsymbol{d}}\right)\delta \boldsymbol{d} = -\left(\frac{\partial \boldsymbol{M}}{\partial y''}\right)^{-1}\frac{\partial M}{\partial d}\delta d \qquad (17)$$

$$\delta y = -\left(\frac{\partial p}{\partial y}\right)^{-1}\left(\frac{\partial p}{\partial \boldsymbol{d}}\right)\delta \boldsymbol{d} \qquad (18)$$

The substitution of Eqs. (17 and 18) into Eq. (10) gives:

$$1_a \delta y_t = \int_0^L M_a[\left(\frac{\partial M}{\partial y''}\right)^{-1}\frac{\partial M}{\partial d}\delta d]dx + \int_0^L p_a[-\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial d}\delta d]dx \qquad (19)$$

Employing the suitable components of the design variables vector $\boldsymbol{d}$ together with corresponding soil components which are appropriately assigned with respect to spatial variables using the local coordinates of each layer, it is arrived at:

$$1_a \delta y_t = \int_0^{H_1} M_a\left(\frac{\partial M}{\partial y''}\right)^{-1}\frac{\partial M}{\partial EI}\delta EI dx - \int_0^{H_1} p_a\left(\frac{\partial p}{\partial y}\right)^{-1}[\frac{\partial p}{\partial b}\delta b + \frac{\partial p}{\partial \gamma'_c}\delta\gamma'_c + \frac{\partial p}{\partial \varepsilon_{50}}\delta\varepsilon_{50} + \frac{\partial p}{\partial c}\delta c]dx - \int_{h_2}^{h_2+H_2} M_a\left(\frac{\partial M}{\partial y''}\right)^{-1}\frac{\partial M}{\partial EI}\delta EI dx - \int_{h_2}^{h_2+H_2} p_a\left(\frac{\partial p}{\partial y}\right)^{-1}[\frac{\partial p}{\partial b}\delta b + \frac{\partial p}{\partial \gamma'_s}\delta\gamma'_s + \frac{\partial p}{\partial \phi}\delta\phi + \frac{\partial p}{\partial k}\delta k]dx \qquad (20)$$

The review of Eq. (20) shows that terms under integrals associated with different design variables have different units. This fact makes assessment of sensitivity integrands inconvenient when comparison of various terms is required. The high nonlinearity of the system does not allow for normalization of changes of the performance with respect to the load values. Therefore, the applied load is considered in a discrete fashion and a normalization process for the variations of the design variables with respect to their initial values is performed. The normalization of variations of the design variables with respect to their initial values has two advantages. First, it allows expressing all the variations of the design variables in unitless form. Second, all resulting integrands have the same unit, which is unit of force.

Performing the normalization process and using Eq. (11), Eq. (20) can be written as:

$$1_a \delta y_t = \int_0^{H_1}[-y''_a y'' EI]\left[\frac{\delta EI}{EI}\right]dx + \int_0^{H_1}[-p_a\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial b}b]\left[\frac{\delta b}{b}\right]dx + \int_0^{H_1}[-p_a\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial \gamma'_c}\gamma'_c]\left[\frac{\delta\gamma'_c}{\gamma'_c}\right]dx + \int_0^{H_1}[-p_a\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial c}c]\left[\frac{\delta c}{c}\right]dx + \int_0^{H_1}[-p_a\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial \varepsilon_{50}}\varepsilon_{50}]\left[\frac{\delta\varepsilon_{50}}{\varepsilon_{50}}\right]dx + \int_{h_2}^{h_2+H_2}[-y''_a y'' EI]\left[\frac{\delta EI}{EI}\right]dx + \int_{h_2}^{h_2+H_2}[-p_a\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial b}b]\left[\frac{\delta b}{b}\right]dx + \int_{h_2}^{h_2+H_2}[-p_a\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial \gamma'_s}\gamma'_s]\left[\frac{\delta\gamma'_s}{\gamma'_s}\right]dx + \int_{h_2}^{h_2+H_2}[-p_a\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial \phi}\phi]\left[\frac{\delta\phi}{\phi}\right]dx + \int_{h2}^{h_2+H_2}[-p_a\left(\frac{\partial p}{\partial y}\right)^{-1}\frac{\partial p}{\partial k}k]\left[\frac{\delta k}{k}\right]dx \qquad (21)$$

The expressions in small square brackets result in sensitivity operators $S_{(..)}$ carrying units of force whereas the variations of design variables divided by their initial values give normalized variations of the design variables $\delta\,(..)_N$.

Accordingly, Equation (21) can be written as:

$$1_a \delta y_t = \int_0^{H_1}(S_{EI})_c(\delta EI_N)_c dx + \int_0^{H_1}(S_b)_c(\delta b_N)_c dx + \int_0^{H_1}(S_{\gamma'_c})_c(\delta\gamma'_{cN})_c dx + \int_0^{H_1}(S_c)_c(\delta c_N)_c dx + \int_0^{H_1}(S_{\varepsilon_{50}})_c(\delta\varepsilon_{50N})_c dx + \int_{h_2}^{h_2+H_2}(S_{EI})_s(\delta EI_N)_s dx + \int_{h_2}^{h_2+H_2}(S_b)_s(\delta b_N)_s dx + \int_{h_2}^{h_2+H_2}(S_{\gamma'_s})_s(\delta\gamma'_{sN})_s dx + \int_{h_2}^{h_2+H_2}(S_\phi)_s(\delta\phi_N)_s dx + \int_{h_2}^{h_2+H_2}(S_k)_s(\delta k_N)_s dx \qquad (22)$$

where $(S_{(..)})_c$ and $(S_{(..)})_s$ denote the normalized sensitivity operators for clay and sand, respectively, corresponding to each design variable $_{(..)}$ which are given between [ ] in Eq. (21). The symbols $(\delta(..)_N)_c$ and $(\delta(..)_N)_s$ denote the normalized variations of design variables for clay and sand, respectively, corresponding to each design variable (..), which are given between $\left[-\right]$ in Eq. (21).

The operators $S_{(..)}$ are spatial functions which when being integrated with respect to dx give final results in unit being product of force and length. (eg. kNm). Consequently, the left hand side of the equation has unit of work (i.e kNm) as the right hand side. The operators $S_{(..)}$ represent also the influence line of the change of pile head lateral deflection caused by the moving variation of the design variable $\delta\ (..)_N$ equal to unit. The graphical presentation of these operators associated with each parameter along the pile length allows the engineer to detect the locations of maximum and minimum influence the change of that parameter has on the variation of the lateral pile-head deflection.

CLOSING NOTES

The paper presented the application of the sensitivity analysis of distributed parameters to laterally loaded piles. It investigated the effect of changes of the design variables on the performance of laterally loaded piles. The performance of the pile is presented by its lateral pile-head deflection which is considered an important serviceability measure for the superstructure carried by the system of piles. The theoretical formulation of sensitivity of lateral head deflection of piles subjected to cyclic loading embedded in non-homogeneous soil consisting of sand overlain by clay was derived. It furnishes the basis for assessment of the effect of the clay thickness on the sensitivity of lateral pile-head deflection when the pile supporting system is embedded in non-homogeneous soil of layered type. In particular, the soil profile that consists of sand overlain by soft clay of variable thickness simulates the process of accumulation of soft clay layer in geological history. It can be considered as the simulation of an evolution of geological soil deposit for various areas such as lowland areas.

The review of sensitivity methods provided basis for understanding of the possibilities and associated specifics each method of sensitivity furnishes. The paper highlighted advantages and shortcomings attributed with each method.

The basis for accommodation of *p-y* homogeneous soil models to non-homogeneous medium was explained. The parameters investigated in the sensitivity analysis were those defining these *p-y* models for clay and sand and the pile’s bending stiffness.

The theoretical formulation of sensitivity analysis of distributed parameters resulted in obtaining sensitivity operators associated with each design variable. These operators are spatial functions and they are physically interpreted as influence lines of the first variation of the performance quantity due to moving variation of the design variable equal to one. The graphical presentation of the sensitivity operator of each parameter allows for the detection of how and where the change of this parameter affects the change of the maximum lateral deflection at the pile top. Numerical studies will be applied to obtain these operators in parts II of the paper (IIA for sand parameters and IIB for clay and pile parameters) (Hafez and Budkowska, 2006a and 2006b).

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