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STOCHASTIC CELLULAR MODEL FOR LOWLAND URBAN DEVELOPMENT

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ABSTRACT: An urban growth model using stochastic cellular simulation was developed with motivation to understand the consequence of zone management policies in lowland cities. The model could integrate the growth, decline, spread, intensification, and protected areas of the urban growth into a single generalization of both the Eden and the p-models. Calibration strategy was demonstrated using historical aerial photographs of Saga city, Japan.

Keywords: Stochastic cellular automata, aerial pictures, urban growth model, Eden model, calibration

INTRODUCTION

A special well known characteristic of lowland cities is its sensitivity from the fluctuating water levels. Flood and storm water are commonly regarded as the most frequent and widespread natural hazard for such places. In connection with urban development, the improvement regulation of zone management is one of the most comprehensive and long-term solutions for hazard mitigation. The overall aim is to reduce the risks involved in the present occupation of flood-prone land and to deter further invasion of such area (Smith and Ward, 1998). To make such policy of zone management effective, an urban development model is needed. Since real field experiments in urban development is impossible, numerical experiment using computer simulation can be utilized to comprehend the effect of zone management policies and to predict the long term effect of several urban development scenarios.

In the last decades, urban development modeling has attracted many researchers in urban planning fields because it may be used as laboratories for exploring ideas about how cities work and change over time (Torrens and O'Sullivan, 2001). (Clarke et al, 1997) proposed an urban development model using simple growth that an occupied cell has at least three neighbors will become a new developed cell if it can pass constraints of repeating spread and slope. This growth has analogy to the spread of fire in the forest. New growth location is always selected at a random location that can pass some constraint. Spontaneous growth is growth wherein a new seed can be put at a random location that has at least one neighbor and quite flat. However, the Clarke model relies heavily on ad hoc solution through combinations of many unrelated and independent sub models such as road, slope, seed cells, and protected areas. (Batty, 1991) uses the Diffusion Limited Aggregation (DLA) model to analog urban growth. The unconstrained real city growth however, is more similar to a circle rather than the tentacles-shaped DLA. (White and Engelen, 1993) uses fractal land-use structure and calibrates the model by simply matching the fractal dimension of simulated city with the map of the real city. (Benguigui, 1995) proposed to model the city as binary value of developed or undeveloped cell. The growth rule is similar to a simple cellular growth of Eden model with additional parameter number of visit p, before the cell is developed. A higher p value tends to make dispersion or unconnected development clusters. The dispersion phenomenon is related to the spread of the city growth toward scattered clusters rather than aggregated clusters. This model, however, do not incorporate the intensity of the development. The model could not distinguish which part of the city has more development than the other. The literature of cellular growth model can be traced back to 1961 where Murray Eden proposed a very simple stochastic aggregation model to describe the formation of a cell colony such as bacteria or germ cells. The process begins with a single seed and a new black cell is added at each step at a randomly chosen position adjacent to the existing cluster of black cells. The shape obtained is ultimately an almost perfect circle. (Teknomo et al, 2005) has proposed to extend the Eden model into a city growth model in lowland cities. The model, however, was only working for hypothetical unconstrained cities.

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In this paper, we describe an urban spatial model based on our previous work that has been improved and developed further with motivation to understand the consequence of zone management policies in lowland cities. The model is a simplified form of a city in which the focus is on the physical development of city blocks. The model is general enough to be applied for any lowland cities. It consists of space, time and development values. Space is the location of developments, time corresponds to the development stages and value indicates the constraint, opportunity of development and the value of the space over time. The model itself can also be seen as a multi agent cellular automata simulation with many developers as agents and users play as the government that derives certain policies over time.

In contrast to the existing urban spatial model (i.e. Clarke et al, 1997) that rely on ad hoc solutions through an assortment of many unrelated and independent sub models our approach is more comprehensive. Our model could integrate the growth, decline, spread, intensification, and protected areas of the urban growth into a single general model. In fact, we show in this paper that we have generalized the Eden and the p-model. In the simplest form, our model will be equal to Eden model (Eden, 1961). Setting some minimum constraint on the development index, our model will approach the p-model of (Benguigui, 1998).

This paper is organized as follows. The stochastic cellular model is explained in the next section. Then, calibration strategy of the model is described using an example of Saga city in Japan. Before the concluding remarks, theoretical characteristics from the simulations are discussed.

STOCHASTIC CELLULAR MODEL

The urban spatial model that we have developed is quite simple and applicable to many cities. The parameters of the model consist of one set of neighborhood probabilities $\mathbf{N} = p_i, i = 1..9$, maximum distance (layers) of the neighborhood, d_m , additional development index λ and maximum number of developments per year, e_m . The first two parameters influence the shape of the city and overall *spread* of the city. The third and fourth parameters influence the degree of development *intensity*. Neighborhood probability indicates urban growth characteristic, neighborhood's maximum distance control the dispersion of the city shape. Additional development index and maximum number of developments per year are parameters that represent the overall economic condition of the city. They measure how fast a development can go at a particular year.

The initial conditions consist of two sets: constrained development and seeds of development. The constraint is signified by a constraint matrix $\mathbf{C}$ where the value represents the probability that a cell is allowed to be developed. Physical constraints such as river, sea, forest, and mountain are valued with a zero probability while parcels nearer to the road will have higher probability to be developed. This probability values can also be set as threshold to represent government policy such as zone prohibition from development (e.g. flood prone area), minimum and maximum development index. The minimum development index may be interpreted as the developer's evaluation value to rather wait from the time to buy the land until a certain time to actually implement the decision to build it. The maximum development index is interpreted as restriction usually set by local government to keep some zone to be built for cultural or environmental reason (e.g. green belt, cultural heritage, maximum floor to area ratio, etc.)

The computational paradigm of the model uses a multi agent cellular automata simulation with many developers as agents. Users of the simulation play as the government that derives certain policies over time. The city is represented by a city matrix $\mathbf{S}$, where each cell represents a block space in the city with occupied cells as developed block and unoccupied cells represent barren land. Each occupied cell has some value to characterize further the other dimension of the development that we call development index.

The underlying phenomena or fact below are the basic assumptions of our model. The model considers how a developer chooses a location in the city and how the development typically occurs in a city. The process in the simulation imitates the rational actions of a real

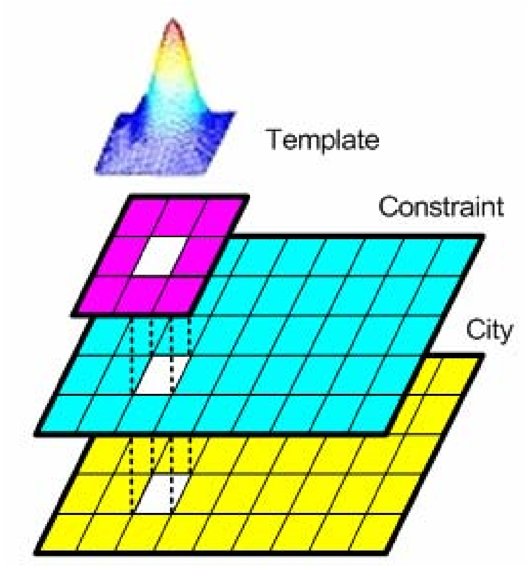

Fig. 1 Diagram of CA Transformation Rule

world developer.

1. Developers tend to choose an area or a neighborhood of interest that have certain types of development rather than in the empty places in the middle of nowhere. This fact can be seen generally from the land value, number of floors in the buildings, or population density. Neighborhoods that are more developed have a tendency to have higher land value due to higher demand.
2. Most developers choose a neighborhood of interest as a whole area, and then they tend to find the particular available places inside or surrounding the center of the neighborhood of interest. As if looking at the map and encircling a particular *neighborhood* first rather than a specific *point* in the map. After that, they find the more specific locations (point) as candidates in that chosen neighborhood.
3. In finding the specific point on the selected neighborhood, developers tend to have a higher preference for location that is near to the center of the neighborhood of interest and have very low interest to the location farther away from the center of the neighborhood of interest. The preference diminishes by distance.
4. Once the developers have selected the particular locations as candidates, they start the process of examining the availability of the land due to city regulations and land prices. This examination imposes some constraint to the developers.
5. A few developers start from a particular location (normally they already have the land available) and build on it.
6. Economic situation and taxation of the city or country may also affect the amount of development to be implemented.

To model the assumption above, any new candidate of development must be selected from the existing occupied cells in the city matrix. This is because the occupied cells represent developed places. Each developed cells has the equal chance to be selected. The assumptions above never imply that the city center has more chance to be developed. Building far from the city center may be less expensive and may attract more people. In fact, the model put *uniform* probability for all developed location to be chosen for further development. The results of the simulations, however, produce self-organization phenomena that the highest development will always occur around the city center.

3	3	3	3	3	3	3
3	2	2	2	2	2	3
3	2	1	1	1	2	3
3	2	1	0	1	2	3
3	2	1	1	1	2	3
3	2	2	2	2	2	3
3	3	3	3	3	3	3

Fig. 2 Chessboard distance measurement in neighborhood matrix

To represent the preference to select a particular location surrounding the area that has been selected, we introduce a template matrix $\mathbf{N}$ that consists of a neighborhood probability. In general, this template should represent the urban growth characteristic of the city. In latter section of Calibration Strategy we discuss how to obtain this template matrix from the aerial photographs of a city.

The urban growth model is a transformation of the city matrix at a certain time $\mathbf{S}(t)$ using a template matrix called Neighborhood probability matrix $\mathbf{N}$ subject to a constraint matrix $\mathbf{C}$. Let us denote that transformation as $T_{\mathbf{N}} : \mathbf{S} \rightarrow \mathbf{S}$, we have $\mathbf{S}(t) = T_{\mathbf{N}}(\mathbf{S}(t-1))$. Figure 1 shows these transformation rules in a diagrammatic manner. The transformation $T_{\mathbf{N}} : \mathbf{S} \rightarrow \mathbf{S}$ is defined with the following rules:

1. Select a location in the city matrix $\mathbf{S}$ at random, based on the existing occupied cells as the center of template matrix $\mathbf{N}$.
2. Select a position in the template according to the random distribution of the template and project this cell to the city matrix and to the constraint matrix $\mathbf{C}$.
3. Modify the value of the city matrix at the projection cell if and only if the projection cell can pass all the constraints.

The three rules of transformation above are put into operation into four steps of the simulation model

1. **Location Selection**: inside the neighborhood, randomly select a direction to point to the specific location of development plan
2. **Neighborhood Searching**: among the developed cells, randomly select a cell location of the center of a neighborhood and the size of the neighborhood
3. **Development Plan Evaluation**: examine all the constraints of the specific location of development plan. If the specific location pass all the constraint, go to stage 4, otherwise reconsider stage 2

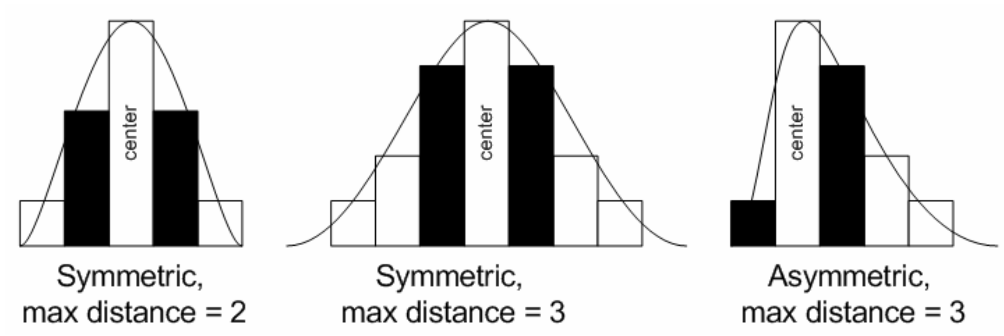

Fig. 3 Using immediate neighbor probability (black bar) to compute outer cells probability

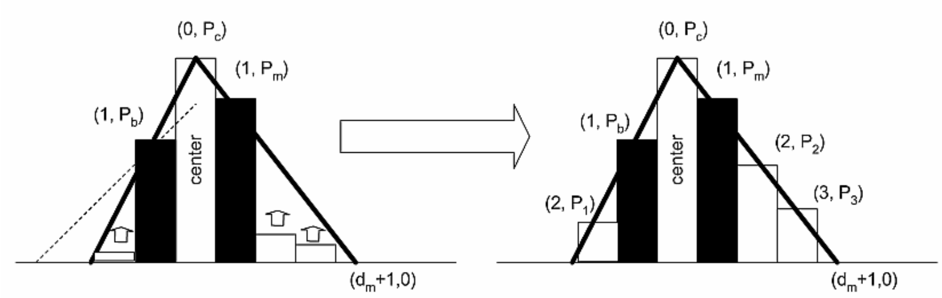

Fig. 4 Redistribution neighborhood probability using linear model

4. **Implementation Development Plan**: increase or decrease (by nature as developers) the development index value of the chosen cell.

The connectivity between cells is modeled using Moore neighborhood with 8 neighbor's connectivity. Distance between cells is computed as chessboard distance with zero in the center of the neighborhood matrix $\mathbf{N}$.

Using chessboard distance as the basic distance computation, one can use the distance as layer from the center. One novel idea of this model is the integration of maximum distance or maximum layer from the center of the neighborhood as one of the parameter of the model. This parameter influences the dispersion of the city shape. It represents a degree on how far the developer-agents are willing to develop from the developed area. Setting a wider neighborhood from the immediate layer to d_m will make development on the perimeter of the city go beyond the connected component of the cells. The idea of using maximum layer as parameter is structurally inside the model. This is in contrast to the simple sparse probability as introduced in Clarke model (Clarke et al 1997). Clarke model simply adds sparse probability as an independent sub model to control the dispersion, while our approach is to integrate the dispersion as an internal part of the model. The problem with using neighborhood layer is neighborhood probability determination of the layer. For a 3 by 3 neighborhood matrix, the user of the simulation can determine the probability of the 8 cells. However, for

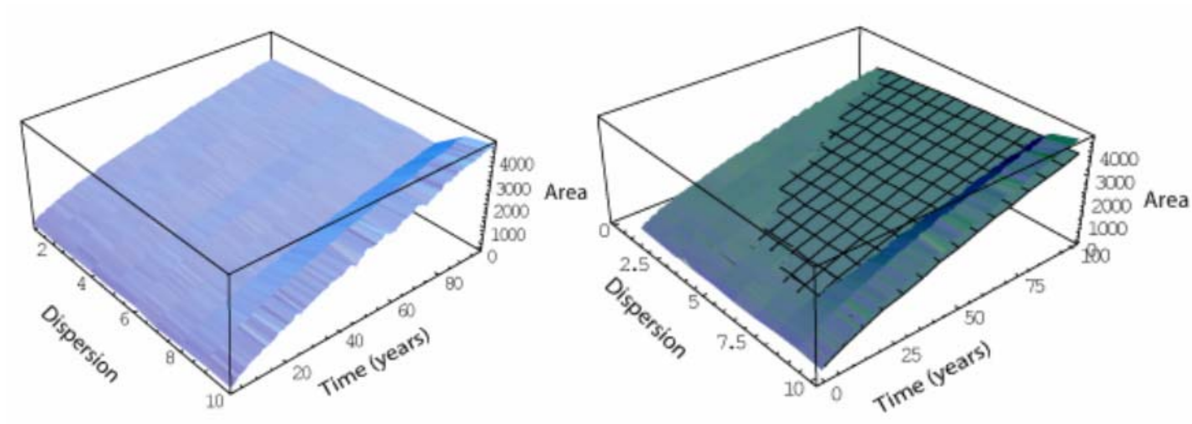

Fig. 5 Relationship of Area, time and dispersion parameter from simulation (left) and both simulation and approximation function (right)

example, for 7 by 7 neighborhoods, determining the neighborhood probability is quite problematic for the user. Instead of inputting all of those probabilities, a better but similar idea is to introduce only one maximum distance parameter and let the computer do the normalization probability values based on a probabilistic model. It is a probability function which shows that the farther the cell is from the center the lesser attention it should get compared to the nearer cell. Suppose the immediate neighbors has probability as black bars as in Fig. 3, then we can use the maximum distance parameter to calculate the neighborhood probabilities of the other cells up to the maximum distance from the center.

Though we can assume some theoretical 2 dimensional probability distribution model of neighborhood probability (such as Beta or Binomial), expanding it unsymmetrical into different directions will cause not only the change of the parameters but also change in the model itself. This is a disadvantage rather than advantage. In fact, as can be seen in the Calibration Strategy section, the neighborhood probability does not follow any theoretical distribution but is based on the aerial photographs of the city. Thus, we use another method to greatly simplify the redistribution through linearization of the smooth probability distribution. The error due to linearization is relatively small when the distance is small. The principle behind maximum distance parameter is to redistribute the neighborhood probability until the maximum distance from the center.

To calculate the other neighborhood cells, we assume the central probability as dummy probability and it is always the highest, regardless of what the user inputs as the center probability. If the maximum layer is one, no further calculation of the other neighborhood cells is needed. The following is an illustration of a simplified idea to redistribute the probability using a linear model. The purpose is to compute the value of neighborhood probabilities in each cell in that neighborhood up to the maximum distance.

1. Among the immediate neighborhood, we find the maximum probability, name it P_m
2. Let the maximum distance from the center be d_m, then the probability at d_m +1 must be zero. Thus, we take a straight line between (1, P_m) and (d_m +1,0) crossing a vertical line at the center we get (0, P_c). Call this line as an extension line.
3. From all other immediate neighbors, draw an extension line from (0, P_c) to this neighbor probability (example line (0, P_c)- (0, P_b)).
4. For all non immediate neighbors cell, intersecting a vertical line from the distance to the extension lines to get new points (in example above: (2, P_1), (2, P_2), (3, P_3)). Note that only positive points are taken.
5. Normalize the value of all neighboring points except the center (in example above: $T = P_1 + P_b + P_m + P_2 + P_3$, $P_1^{'} = P_1 / T$, $P_b^{'} = P_b / T$, $P_m^{'} = P_m / T$, $P_2^{'} = P_2 / T$, $P_3^{'} = P_3 / T$)

Aside from setting the maximum distance as the main parameter for *dispersion*, an unconnected dispersion shape can also be produced by setting a minimum value of development index. (Benguigui, 1995) proposed what he called as p-model extension of Eden for town growth. In the Benguigui model, the cell is in 3 states: vacant, visited and developed. After a cell is visited p number of times, it will become a developed cell. Generalizing the 3 states into a development index, we found that the p-value is equivalent to minimum development index that can be set as constraint in our Development Plan Evaluation step. If the development index of a cell is below the Benguigui p- value, it will not be considered as a developed cell and will not be considered in the output indices.

Another input parameter, the maximum development index, is a constraint development rather than affecting the overall shape. If the cell is considered but has a development index more than the specified value, the implementation step will not take place. If the Benguigui p-value (i.e. minimum constraint) is specified, the maximum development index must be specified at least the same as the minimum. If the maximum development index is higher than the Benguigui value, then the Benguigui value produces no meaningful result because it is always bounded by the maximum value automatically.

Simulating the model using the default values for an increase of dispersion parameter produces many outputs. One of the outputs is the area of the city in terms of number of cells. Relating the output with the input parameters through least square method yields some relationship. Important relationship between area, time and dispersion parameter is shown in equation (1). The best fit model is a power model and the parameter of the model shows the substantial weight of each factor. The number below each factor is the t statistic. Figure 5 shows the plot of the relationship. The left figure represents the simulation results while the right figure plots both model and simulation results. It seems that the dispersion affects very little adjustment while time affects the area exponentially. Due to this minimal effect, dispersion parameter (in terms of maximum distance layers of the neighborhood, d_m) will not be searched as a special case in our calibration strategy. A similar reasoning is adapted to other parameters of additional development index λ and maximum number of developments per year, e_m.

$$Area = \underset{(312.8)}{107.315} \cdot \underset{(210.2)}{Time^{0.748}} \cdot \underset{(15.9)}{Dispersion^{0.069}}$$

with $adj.R^2 = 0.978$ (1)

Fig. 6 Base aerial picture of Saga city, Japan in 1974

CALIBRATION STRATEGY

As mentioned in the previous section, the model has four parameters which consist of one set of neighborhood probabilities $\mathbf{N} = p_i, i = 1..9$, maximum distance (layers) of the neighborhood, d_m, additional development index λ and maximum number of developments per year, e_m. The last three parameters are simply three numbers that can be easily calibrated using Monte Carlo search with three tuples to minimize the difference between the real world developed cells and the simulation. The first set of parameter, that is neighborhood probabilities, however, is the most important parameter that is not easily detected using the search algorithm. The calibration of the neighborhood probability will influence the direction of city growth over time. A calibration strategy is needed to obtain the neighborhood probabilities. This section describes the general calibration strategy to obtain the neighborhood probabilities which will include segmentation of aerial image into real world developed cells. These developed cells serve as the basis of the search algorithm to obtain the three other parameters.

The calibration of neighborhood probabilities is based on the historical aerial pictures of the city. First, all of the aerial photographs are transformed into a single set of coordinate system through perspective transformation. At the end of this transformation, all the pictures have the same scale, and the same coordinate system with a single origin point. This transformation is necessary to ensure the consistency of the pictures. Second, the transformed aerial photographs are then converted into two categories, developed cells and undeveloped cells. Boundary of the developed cells is drawn as the city perimeter. Third, city center are positioned and the difference area from the city center to the 8 directions of the city perimeter are measured for each aerial image. These area differences are then normalized as the neighborhood probability distribution.

The calibration of the development index requires the interpretation of the development index. The development index could represent any spatially related developed value of the city, such as population density, and land value, number of floor area and so on. The calibration strategy is to select any combination of those spatially related developed data into a normalized development index. The trend of the development index is then compared to the simulation result. Adjustment of parameters such as range of additional development index, and range of number of developments per year will change the simulation results in terms of development index without affecting the overall shape. Overall fit of the development index of the real city is then compared with the simulation result through searching the parameters that produces those fit.

Image Calibration

Before the historical aerial photographs can be used for calibration of the urban model, they must be scaled into the same size and each pixel must indicate the same actual field location. Since the aerial photographs are taken from different years, they do not have the same size and due to inaccuracy of the picture taken, some skew angle may happen. To use them with sound reliability, image calibration is a necessary step to ensure all aerial photographs have the same scale, the same projection plane and the same coordinate system. To do that, we need to utilize one of the pictures as the basis coordinate. The choice of the basis picture is arbitrary as long as it is consistent. For example, the basis coordinates for the scaled Saga city is the aerial image of the year 1974.

Several pivot points are marked in all historical aerial photographs and the coordinate of the pivot points are gathered using the basis coordinate. For example, these pivot points may include fixed location of the road intersections, river intersections, or certain buildings that do not change over time. Since marking of the pivot points need visual human inspection, it is done manually. Those pivot coordinates indicates the same points in the real field that do not change over time. Assuming those pivot points had been distorted by scaling, translation and skew rotation, we want to recover the images from distortion and put them in the same size and the same coordinate system. Translation recovery can be done easily by matching pivot points after scaling and skew rotation has been restored. To recover the scaling and skew rotation, we use the affine regressions toward all the pivot points of two images. One of the image must be the basis image or images that has been fully restored (i.e. has the same basis coordinate) from all distortion. The affine regression formula is as follow:

$$\mathbf{x}_b = s_x(\mathbf{x}_i \cos\beta_x + \mathbf{y}_i \sin\beta_x)$$
$$\mathbf{y}_b = s_y(\mathbf{x}_i \cos\beta_y + \mathbf{y}_i \sin\beta_y) \quad (2)$$

Notation $(\mathbf{x}_b, \mathbf{y}_b)$ is the coordinate of basis year, while $(\mathbf{x}_i, \mathbf{y}_i)$ is the coordinate of other years. Both coordinates are represented in vector terms because they consist of many pivot points. Equation (2) can be put into multiple linear regressions as

$$\mathbf{x}_b = a + b\mathbf{x}_i + c\mathbf{y}_i$$
$$\mathbf{y}_b = d + e\mathbf{x}_i + f\mathbf{y}_i \quad (3)$$

The skew rotation horizontal and vertical angles are respectively $\beta_x = \arctan(c/b)$ and $\beta_y = \arctan(e/f)$. The scaling in widths and heights are respectively $s_x = b/\cos(\beta_x)$ and $s_y = e/\sin(\beta_y)$.

After all aerial photographs have been restored from the distortion; they are cropped into the same size. With this step, we have the same size of historical aerial photographs that has the same coordinate system.

Segmentation of Developed Area

The next step in the calibration strategy is to segment the aerial photographs into two categories, developed cells and undeveloped cells. This segmentation is important to serve two purposes. First it can be used to generate a constraint matrix based on the city planning and physical constraint of undeveloped cells such as agricultural field, river, cliff, sea, mountain, or forest etc. Second, the developed cells are utilized as the basis to compute neighborhood probabilities.

We have standardized the step to segment approximation of the developed area from the aerial image through a series of operation of edge detection filter, threshold the gray value images into binary using threshold value $T_h = \mu - k\sigma$, and dust and scratches noise filter and finally stamp filter to color the developed cell. For Saga city historical aerial pictures, we use $k = 1$ with μ and σ as mean and standard deviation of the pixel value from histogram, noise radius of 10 with threshold of 100, both light/dark balance parameter and smoothness parameters of one. A few manual noise-cleaning was done to ensure the results without affecting the overall segmentation. The segmentation pictures are shown in Fig. 7. The results of segmentation were highly correlated with the developed cells.

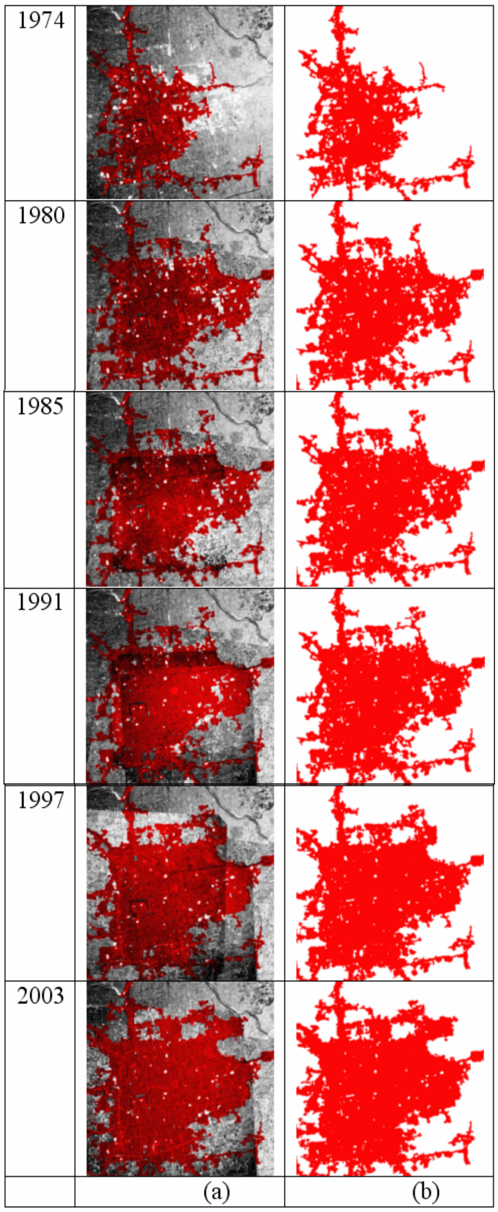

Fig. 7 (a) the historical aerial photographs overlay with the segmented developed cells (b) the segmented development based on the aerial photographs

Neighborhood Probability Distribution

Once the historical aerial images are segmented, the next step in the calibration is to compute the spatial distribution neighborhood probabilities. For each two consecutive historical aerial images, the difference area of the developed cells is determined. The center of

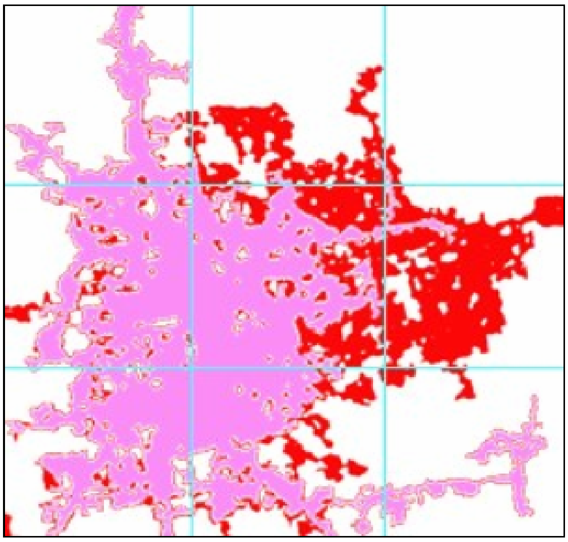

Fig. 8 Area differences of developed cells based on two historical aerial images (1974 and 1980. The 1974 part is shown only for clarity of description)

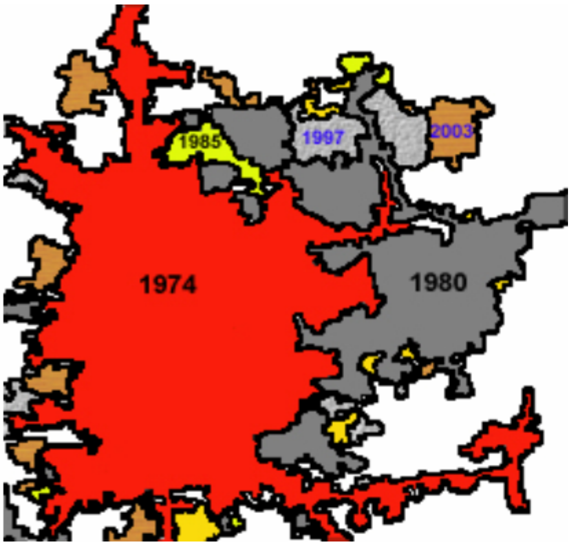

Fig. 9 The spread of Saga city from 1974 until 2003

gravity of the developed areas in the two input images are computed and averaged to provide the basis of division of the difference image. The difference image is then shifted so that the average centers of gravity as the center of the image without any rotation. This is to provide a common coordinate system for division of the image difference. Blank space replaces the uncovered area. Then, the regions of image difference are divided into 9 equal rectangles (see Fig. 8). The region of image difference in each rectangle is measured and normalized by division of the summation to give the spatial distribution of neighborhood probabilities.

$$\mathbf{N} = p_i \cong \frac{A_i}{\sum_i A_i}, i = 1..9 \quad (4)$$

Search Algorithm

Monte Carlo search (which is included in the family of Genetic algorithm and Simulated Annealing) is well known to be able to find the global optimum. We employ direct random search method as suggested by (Chapra and Canale, 1998) to find three parameters $\langle d_m, \lambda, e_m \rangle$, by minimizing the difference between the segmented developed area and the simulation results. The neighborhood probabilities are set between two bounding years. Since the changes of the neighborhood probability distribution over time are relatively larger than 5% threshold, we cannot use the average of the neighborhood probabilities over all the 30-year period to extrapolate the growth. Only interpolation of urban growth within those years is allowed.

Table 1 shows the shape descriptors of Saga city based on the aerial photographs. The corresponding growth is shown in Fig. 9. This growth is based on the 1974 map and the corresponding additional 5 year growths are shown in different shade patterns. The descriptors consist of Area of developed cell in square pixels and its centroid (x, y), perimeter length that bound the developed area, and ellipse representation of the urban growth in terms of major and minor axis and angle between the major axis with the horizontal line bounding the image. Thinness ratio is derived from area and perimeter $(Thinness\ ratio = \frac{4\pi A}{p^2})$. A value of 1.0 indicates a perfect circle. As the value approaches 0.0, it indicates an increasingly elongated polygon. More elaborate shape descriptors used in this research were explained in (Teknomo et al, 2004).

Table 1 Shape descriptors of Saga city based on Aerial Photograph

Year	Area	X	Y	Perimeter	Major	Minor	Angle	Thinness ratio
1974	18791	90.073	153.652	2793.666	177.553	134.751	117.556	0.03
1980	28230	110.81	145.989	3175.999	196.638	182.791	13.534	0.035
1985	29261	111.134	145.825	3191.698	199.767	186.499	22.162	0.036
1991	30186	111.66	146.896	3184.686	202.057	190.214	28.461	0.037
1997	33230	112.253	145.356	2918.353	214.372	197.366	37.01	0.049
2003	36100	111.225	141.955	3154.804	224.534	204.708	41.731	0.046

DISCUSSION

Having the simulation calibrated, now we discuss some applications and the theoretical implication of the simulation.

One of potential application of the simulation model is for the local city government planning agency to perform an assessment on the impact policy analysis such as putting additional constraint on the development. For example, zone management policies can be set as a constraint matrix where the flood prone areas are set prohibit additional development in these zones. The long term effect of the urban growth will eventually follow different paths and direction that can be interpolated using the simulation model. The detail of this application, however, shall be pursued in the future direction of this study. Knowing the constraint of development, three possible findings can be applied:

1. To find the expected level of development at a certain place (from the center of development) at certain time. Given $\mathbf{x} = (x, y)^T$, t , find $\lambda(\mathbf{x},t)$
2. To find the places where the development is expected at a certain level at a certain time. The answer is not a unique location. Given $\lambda(\mathbf{x},t)$, *t*, find $\mathbf{x} = (x, y)^T$
3. To find out when the development level at a certain location will be expected to reach a certain level. Given $\lambda(\mathbf{x},t)$, $\mathbf{x} = (x, y)^T$, find *t*.

Furthermore, the simulation results can be approximated theoretically as an exponential growth with radial diffusion. Taken the analogy of temporal-spatial diffusion to urban growth, total diffusing substance, M is analogue to total number of developments by the developer agents. Similar assumption to the simulation was taken that diffusion occurs in the direction of a decreasing gradient. The differential equation of such growth is discussed in (Banks, 1994) as shown in equation (5).

$$\frac{\partial N}{dt} = \delta\left(\frac{\partial^2 N}{\partial r^2} + \frac{\partial N}{r\partial r}\right) + \alpha N \qquad (5)$$

The term in the bracket is a basic diffusion equation while the last term in the right hand side represents the exponential growth. Notation N is the magnitude of growing quantity (i.e. Area of developed cells), δ is a dispersion coefficient (note that this is not the same value as the dispersion parameter of the simulation), r is the radius and α is the net growth rate. The solution of the equation is given by

$$N = \frac{M}{4\pi\delta t}\exp\left(\alpha t - \frac{r^2}{4\delta t}\right) \qquad (6)$$

Taking the development as radial, the total amount of new developments outside the boundary $r = R(t)$ at time *t* is denoted by *P*.

$$P = \int_R^\infty N(r,t)2\pi r dr = M\exp\left(\alpha t - \frac{R^2}{4\delta t}\right) \qquad (7)$$

Notation *R* stands for the radius of the expanding development which is defined as a circular area of new development. If the diffusion front or wave of development is defined by setting the exponent of the exponential equal to zero we have $\alpha t - \frac{R^2}{4\delta t} = 0$ or

$$R = 2t\sqrt{\alpha\delta} \qquad (8)$$

The velocity of new development is given by

$$u = \frac{dR}{dt} = 2\sqrt{\alpha\delta} \qquad (9)$$

The simulation without constraint will produce a radial growth as reported in (Teknomo et al, 2005). However, for radial growth, the area is $A = \pi R^2$. Thus, we have

$$t = \frac{\sqrt{A}}{4\pi\alpha\delta} \qquad (10)$$

For Saga city, plotting as in Fig. 10 we have the relationship of square root of area with time to be

$$\sqrt{A} = 1.53t - 2879.2 \qquad (11)$$

Where, area is measured in square pixels and time in year. Inputting equation 11 into equation 10 we have $\alpha\delta = 0.122 - 229.12/t$ and the velocity of new development as $u = 2\sqrt{0.122 - 229.12/t}$.

CONCLUSIONS

Motivated by the need to comprehend zone management policies in lowland cities, we have built a stochastic urban cellular model. We have described the underlying phenomena, model development and cellular transformation rule and the parameters in urban growth terminologies. Simulating the model, we have understood that the model characteristics have radial growth with higher development near the initial seeds.

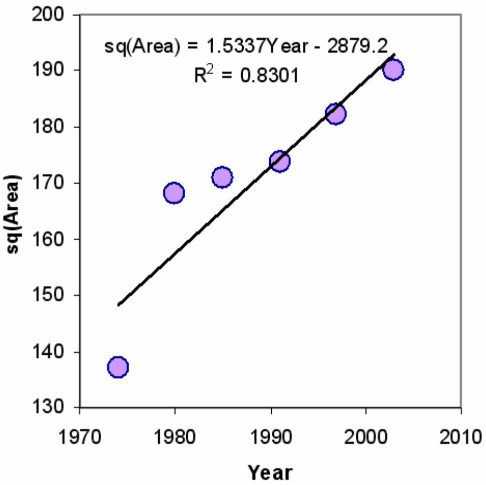

Fig.10 Total square root of developed area in Saga city measured in Pixels

Instead of representing growth as vacant and occupied states, we have generalized it into an index of development that can be used for different purposes of modeling such as land value, constraint of development, or simply population and so on. Thus, the simulation model is suitable for understanding the city growth phenomena that has similar characteristics to the model.

Emphasis on the calibration strategy of the simulation model using historical aerial photographs was elaborated toward image calibration, segmentation of developed cells, finding the neighborhood probabilities and finding the other parameters of the model. A case study of Saga lowland city in Japan was demonstrated. The link of the model with the mathematical model was also discussed. The analysis we have performed suggested that prediction of the growth is unreliable but interpolation of the growth between the knowing years can be done with great accuracy.

The model itself is quite generic to be transferable to any other concentric cities that are suitable with the assumption of the model.

Statistical goodness of fit for this model should be considered for further study. Further research may also follow the direction on establishing the linkage of land use and transportation, as well as more application of zone management and developer's net gain from the urban growth.

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