

UNCONSTRAINED CITY DEVELOPMENT USING THE EXTENSION OF STOCHASTIC EDEN SIMULATION

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ABSTRACT: In the quest for a special tool for lowland urban growth model, we have developed a model of a city based on the analogy of biological cell growth using the extension of the stochastic growth Eden simulation. In this paper, we described the theoretical observable fact on how a city grows if the land is flat and there is no natural barrier for the development. Similar researches on cellular urban growth are reviewed and we showed how they are different from our model. Urban growth model development using the extension of the Eden model as analogues to biological cell growth is explained. The model is applied to simulate a hypothetical unconstrained city development. The simulation results shows that for a mono-centric city without any development constraint, the expansion rate of the city area is equivalent to the ratio of the perimeter per area of the city. This result may be useful to predict the approximate growth rate of any city without any prior knowledge of the historical data. Furthermore, it was found that the growth of an unconstrained city is almost circular with a decreasing rate of the mean radius growth.

Keywords: Eden simulation, urban growth, cellular automata.

INTRODUCTION

Evolution of the physical growth of cities has been shaped by geography, environment, and economic, social and the political power of society. The shape and growth of a city has always been an important influence on the way cities are planned and conserved. Special case of urban growth may occur in the place where the cities are located in a flat ground that is affected by fluctuating water level, also called lowland. Flood and storm waters are commonly regarded as the most frequent and widespread natural hazard for such places. In connection with urban development, the improvement regulation of zone management is one of the most comprehensive and long-term solution for hazard mitigation. The long-term goal is to reduce the risks involved in the present occupation of flood-prone land and to deter further invasion of such areas [Smith and Ward (1998)]. To develop a strategic and comprehensive city plan for such lowland cities, an analysis tool such as an urban development model is needed.

Despite its importance to have a special urban development model for lowland city, there are only very few researches on urban model of lowland city (for example archeological research for Mayan lowland city in Ford (1997)). In fact, the range of application of urban development model has not been covered for the specific field of lowland city.

Lowland urban development model, however, may play an important role in many coastal belts, notably in Louisiana and Florida in the US, Venezuela, Netherlands, Italy, China, Japan, Korea, Indonesia, Thailand, India, Bangladesh, Sri Lanka, etc. Development of distinctive urban development models for lowland city could benefit planners and developers in such areas to explore potential growth scenarios of the city. Torrens and O'Sullivan (2001) stated that urban simulation models are abstract, simplified version of real world objects and phenomena that may be used as laboratories for exploring ideas about how cities work and change over time.

In the quest for a particular tool for Lowland urban planning, we have developed a model of city based on the analogy of biological cell growth using an extension of stochastic growth Eden simulation. Since Eden's stochastic growth model is a subset of what is called the Cellular Automata (CA) model, discussion about urban growth model using Eden model may not be completely isolated from the CA for urban model. In fact, many researches have been done to apply cellular automata for urban phenomena and urban growth model.

Among many researches about cellular urban growth modeling, two approaches divide the models. The first approach utilizes heuristic rules inside the CA to represent the city growth. Examples of this approach can be found in White and Engelen (1993) and Clarke,

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Hoppen and Gaydos (1997). The second approach is using CA with physical or biological analogy. Example of this approach are researches led by Batty (1991) which use Diffusion Limited Aggregation (DLA) as the aggregation model.

The lowland urban development model described in this paper is one of the applications of the second approach. Among many models of city growth and formation, analogy of a city with living cell organism has been proposed by several researchers (e.g. BBC City Cell and Keith Roberts Cell city). Eisner (1993, p. 253) stated that city plan must be adjustable to changing conditions that a city grow like “living cells”.

Among the analogical model of physical and biological growth, our particular interest on the stochastic Eden growth model came from the basis that the Eden stochastic growth model has been developed originally from biological studies of bacterial colonies and has been applied in many fields such as crystallography, solid state and image processing [Eden (1997)]. Benguigui (1986) first proposed an extension of the Eden model for a city, which he later called as p-model (Benguigui 1998). The application of Eden model for urban development, however, has not been fully uncovered, especially for lowland city.

CELLULAR GROWTH MODEL

To distinguish the original Eden model, Benguigui p-model and our extension of Eden model, in this section, we will give a comparison among the three models.

Murray Eden (1961) proposed a very simple stochastic growth model, which later is called after his name. It was first proposed as a biological growth model to describe the formation of a cell colony such as

bacteria or germ cells. The process begins with a single cell (called seed) placed into a node of a lattice and at every time step another site is selected randomly from the set of sites which have at least one nearest neighbor with previously occupied cells. The cells are added one after another to a growing cluster with equal probability. Eden was interested in the structural properties of the resulting colony of cells. Cluster grown by these rules is compact and non-fractal, only the surface is nontrivial and rough.

This recursive process allows every unoccupied neighbor equal chance to become occupied and every site of the lattice will finally become occupied. Therefore, the cells only grow at the periphery of the cluster and that colony is always connected. Wolfram (2002, p. 331) stated that Eden model is a simple aggregation model in which a new black cell is added at each step at a randomly chosen position adjacent to the existing cluster of black cells. The shape obtained is ultimately an almost perfect circle.

Benguigui (1986) extended the Eden model for city growth, which he called as p-model. The p-model has a binary value of developed or undeveloped cell, and the way cell grows is as follow: at the beginning, there is a single seed and the four neighborhoods are the potential sites. One of these potential sites are chosen at random and marked as visited. The neighboring sites of the visited site are also added to the list of potential site such that its probability of occurrence is increased by the number of times the same cell is visited. Only if the chosen site has been visited p times, then it is marked as developed cell.

To employ the Eden model as an analogy of biological cell growth to the urban growth model, there is a need for some modification and generalization of the model. The extension of the Eden model is necessary to incorporate the underlying behavior of the model as a city growth rather than as a biological cell. Our

Table 1 Comparison of original Eden model, Benguigui p-model and our extension

Original Eden Model	Benguigui p-Model	Our Extension for Urban Model
Seed start from a single cell	Seed start from a single cell	Seed can start from any number of cells
Seed only one time at the beginning	At least one seed at the beginning, any number of seed can be added at any time	At least one seed at the beginning, any number of seed can be added at any time
Cell represents biological cell (i.e. bacteria), cluster represent bacteria colony	Cell represents urban development, cluster represents city or part of the city	Cell represents urban development, cluster represents city or part of the city
Equal neighborhood probability	Equal neighborhood probability	Not necessarily equal neighborhood probability
Cell has only binary value (unoccupied and occupied)	Cell has only binary value (developed and undeveloped)	Cell has the value of development index
New cell is added only at the periphery of the cluster	New cell can be added at any place within the cluster or the periphery of the cluster.	New development index can be added at any place within the cluster or the periphery of the cluster.

modification has come to some extent that it may generalize the p-model. Moreover, equivalent behavior of Benguigui p-model could be obtained by setting a *minimum* development index as p from our model. Table 1 shows the comparison between the Eden model, p-model and our extension toward urban modeling.

Both Eden model, p-model of Benguigui and our extension of the Eden model have the similarity of using a regular square lattice or matrix as the base site with a discrete time step simulation. The location of each cell is specified by a node in a regular lattice. In the Eden model and p-model, the seed starts from a single cell and only place at the beginning of the simulation. This is to represent the growth of the organism, which begins with a single cell (fusion of two parent cells). In case of our lowland city growth model, however, the seed of development may start with several cells and the cells are added at any time.

The Eden and p-model assume that each cell is connected with at least one immediate neighboring cell. Benguigui model interprets the p value as the time between the first visits of a site until it is occupied which can be seen as the time between a decision to build and the effective building. Analogues to the biological cell model, our urban growth model, the connectivity may represent urban phenomena that development tends to be near or within the neighborhood of other existing developments. In many naturally developed cities, a new development usually tends to be built near the developed area to share the common cost. Placing a development in a distant place from a developed area has the consequence of high cost of utility and infrastructure that is likely to be avoided.

The cells in the Eden model and the p-model have only binary value of occupied and unoccupied, and the Eden model even always add new cell only on the periphery of the cluster. Our extension of the Eden model for city growth, however, assumes that the cell has the value of development index, which represent a relative scale of development value on that block.

THE MODEL DEVELOPMENT

The urban simulation model that we developed is a simplified form of a city in which the focus is on the physical development of city blocks. A general urban model consists of space, time and values. The space is the location of developments, time corresponds to the development stages and the value indicates the constraint, opportunity of development and the value of the space over time.

Conceptually, we model the city development as a local spatial interaction process. A city can be represented as many blocks of a development place, that

we call *cells*. Each cell represents a single development place at a time. Say for example, a cell is a 500-meter by 400-meter block or it can be a 20-meter-by-20-meter block. The precise size of a cell is not very important compared to the formation of the cell. Our scope is to answer the question of how the city is growing, that way, rather than finding the finer detail on how big is the city. Only the spatial expansion of an isolated city is modeled, while functional interaction within the cell (i.e. land use type) or interaction with other cities will be accomplished as our further study.

Each cell or development block, contains a value, called *development index*. The value of the development index represents a relative potential scale of development value, its interpretation may depend on the application of the model. For example, the development index may be interpreted as relative average land values on that block. In this case, a higher development index may indicate a higher land value. For other types of application, the development index may be read as the relative average number of floors of the buildings inside the cell. In this case, high development index may imply taller buildings in that block. Each time a development is implemented in a cell, the development index of the cell is increased. For simplicity, the development index is set as zero for undeveloped cells.

Each discrete time step of the simulation model represents a development stage rather than the continuous real time. Using discrete time step, development phase at one time step may represent one week, while the other time step may be associated with 2 months. Thus, the discrete time step indicates an interval of time. In each time step, one or several cells can be developed at the same time. However, the beginning and the end of the time step is discrete, the real development time may not precisely be at the beginning of the time step or stop at the end of the time step. The discrete time step is similar to say, "In these one and a half years, there are three development projects in this city." May be other developments had started this year that we do not count among the three because we count this development as the next development year.

We are interested on the whole city development structure. How some policies may influence the structure of urban pattern is more appealing than the detail of the development. With this kind of interest, a rough model of discrete time and space is still suitable.

The mechanism of city growth may depend on many factors. For example, when the economy of a city grows, some developers may consider raising development plans. After considering all the constraints, the plan may be realized into physical buildings. In a similar way, the model structure of each development consists of two phases:

1. Planning phase consist of 3 steps
 - a. Neighborhood Selection
 - b. Location Choice Model
 - c. Development Plan Evaluation
2. Implementation phase

The heart of the simulation model is in the planning phase. The planning phase consists of the selection of neighborhood location, choosing the exact location of development among the neighborhood and evaluating the constraint, rules and regulations available for that location. Only the development plan, which passes all the constraints and restriction, will be implemented. The implementation phase is the real construction of the building, detail plan on the scale of development or deterioration of the existing development over time.

The city is represented by an array or lattice of regular spaces called cells. At each time, a particular cell is in one of a finite number of allowed states represented by a development index. The state of a cell will change according to the states of neighboring cells in the array according to a set of transition rules.

A number of developments can be generated by chance at each time based on the economic growth of the city. For each development at a time step, the following transition rules are applied. Since we have three steps for the planning phase and one implementation phase, we divide the transition rules for each development into four steps:

1. Neighborhood Selection: among the developed cells, randomly select a cell location (x, y) of the center of a neighborhood.
2. Location Choice Model: inside the neighborhood, randomly select direction to point to the specific location of development plan.
3. Development Plan Evaluation: examine all the constraints of the specific location of development plan. If the specific location passes all the constraints, go to implementation stage, otherwise reconsider stage 1 or 2.
4. Implementation of Development Plan: increase or decrease (by nature as developers) the development index value of the cell.

Similarities of our model structure with the basic Eden model lead us to select the extension of Eden model as our base model.

The first two steps of neighborhood selection and location choice model is explained as follows. Suppose we have an m by n city matrix \mathbf{S} and $s(x, y)$ denotes the cell element of matrix at row = x and column = y . If $s(x, y) = 0$, we say that that cell is undeveloped. When the value of $s(x, y) > 0$, the cell is developed. The value of $s(x, y)$ represents the development index of that cell element.

A 3 by 3 square matrix \mathbf{N} is a sub matrix of \mathbf{S} , called a neighborhood matrix Two kinds of connectivity rules exist: the 4-neighbor rule (N, S, W, E) or called Von Neumann neighborhood and the 8-neighbor (N, S, W, E, NW, NE, SW, SE) rule or called Moore neighborhood. Although the Eden model uses the 4-neighbor rule, for urban modeling, the 8-neighbor rule appears more natural for the urban model than the 4-

neighbor rule. Which neighborhood rule to apply, is closely related to the measure of distance from the center of the neighborhood. In the four neighbor rule, the distance between center to vertical and horizontal adjacent cells (N, S, W, E) are one while the distance between center to the diagonal immediate neighbors

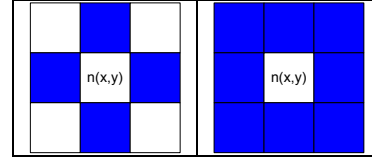


Fig.1 Neighbors rule for first layer neighbors. Left: 4 neighbors rule. Right: 8 neighbors rule.

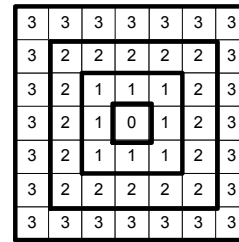


Fig. 2 Distance measurement from the center of a neighborhood matrix depends on the odd layer measured from center.

(NW, NE, SW, SE) is 2 in city block distance or $\sqrt{2}$ in Pythagorean distance. Using the 8-neighbor rule, the distance between the center and the first layer neighbors is always one; the distance to the second layer neighbors is always 2, regardless of using a city block or Pythagorean distance.

There are 8 neighbors that have a distance 1 from the center and 16 second-layer neighbors that have distance 2 from the center. In general, there are $8\xi = 4(\nu - 1)$ cells in ξ -layer which have distance ξ from the center, and the total number of neighbors is $4\xi(\xi + 1)$.

A path between two cells n_A and n_B is a sequence of developed cells, $n_A, \dots, n_j, n_{j+1}, \dots, n_B$ where each pair of consecutive cell n_j, n_{j+1} is such that n_j is the

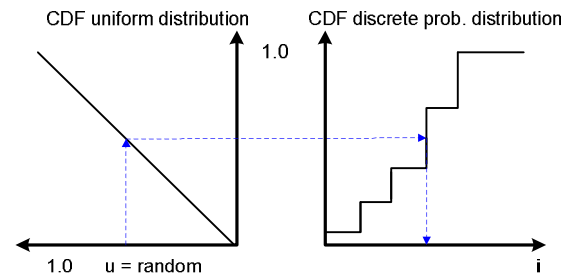


Fig. 3 Discrete Monte Carlo Model. Uniform random distribution is transformed by the discrete probability distribution to represent the later distribution.

adjacent cell (immediate neighbor) of n_{j+1} . We say that two developed cells are connected if there is a path between the two cells. A *development cluster* is a set of developed cells such that any two points in the set can be linked by a path, which lies entirely in that set.

If the neighborhood \mathbf{N} has a size of 3 by 3 then we can define $p_i^1, (i=1..9)$ as the probability distribution to select a location among cells in the neighborhood. The location choice model is a discrete Monte Carlo model to select a specific candidate location $i(x, y) \in \mathbf{N}$ based on the probability distribution p_i^1 . The model, as shown in the figure 3, is to generate the location i from the inverse discrete cumulative distribution function based on random uniform distribution.

The third step of development plan evaluation is established to consider the topographical and other constraint conditions that make the search more sensible. The probability of taking a cell with 'bad' condition is lowered by the constraint and the probability of choosing a good cell would be higher.

The fourth step, the implementation steps can be expressed such that if the candidate location $i(x, y)$ can satisfy all the constraint conditions η_j , then the corresponding cell in the lattice $s(x, y, t)$ may increase its value of development index. The additional development index, λ , is added for every time step. Thus, the transition rule can be written in short as

$$\begin{aligned} i(x, y) \in \mathbf{N} \leq \eta_j, \forall j &\Rightarrow \\ s(x, y, t) = s(x, y, t-1) \pm \lambda &\end{aligned} \quad (1)$$

The value of λ is set to be positive for growth and negative for decline. The four steps above are called the *basic model* of urban simulation. Additional factors and constraints will increase the complexity of the basic model.

SIMULATION RESULTS

Overall Shape

Though only local spatial interaction is defined in the model, emergent behavior of the city is produced as a self-organization phenomena of the model. This emergent behavior is mainly due to the set of neighborhood probability, number of seeds, location and the timing to input the seed. The neighborhood probability is a set of parameters $p_i^n, (i=1..9)$ that will influence the growth direction of the city. Setting equal probability (4 directions or 8 directions) will produce a circular shape, while two direction symmetrical probabilities create an elliptical shape. One direction of symmetry produce triangular or semi circle shape while linear irregular shape is produced using an irregular neighborhood probability. A linear city may be produced by setting strong probability in two main opposite direction and keep very small probability into other directions. Figure 4 shows several neighborhood probabilities setting.

Locating several seeds in undeveloped cells and letting them grow is the most convenient way to calibrate the shape of the city visually. This, however, is

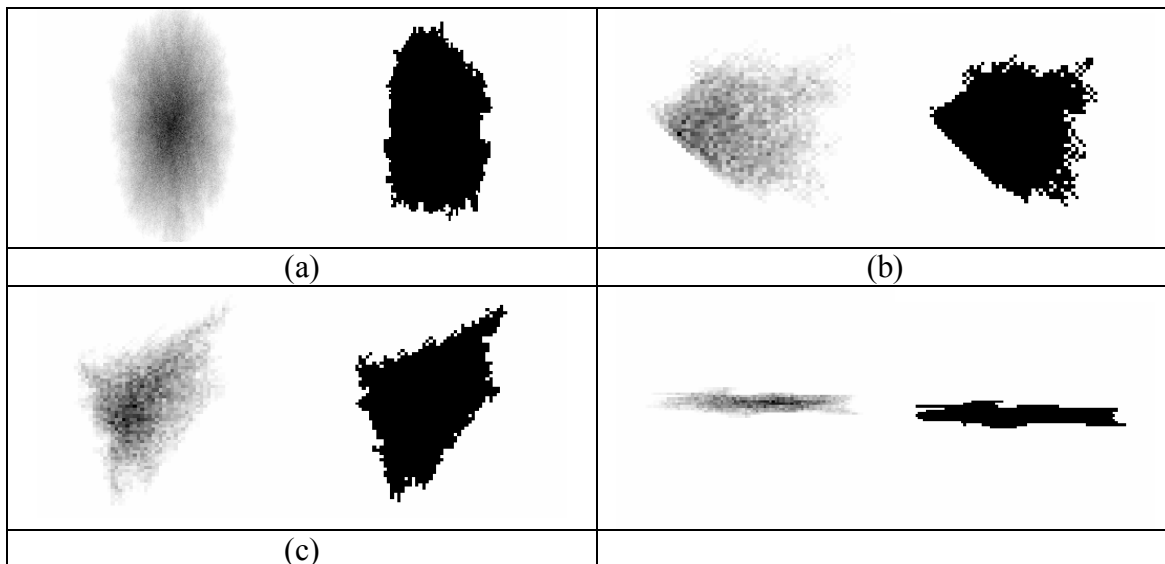


Fig. 4 Changing neighborhood probability will affect the direction of growth. This figure is produced using default Eden parameters and alter the neighborhood probability to be (a) two directions symmetrical (0, 40, 0; 10, 0, 10; 0, 40, 0) produce ellipse, (b) one direction symmetrical (0, 0, 40; 0, 0, 20; 0, 0, 40) produce triangular shape, (c) irregular neighborhood probability produce irregular shape (10, 0, 40; 0, 10, 0; 0, 40, 0), (d) Linear shape is come from high probabilities in two opposite main direction (0, 0, 1; 63, 0, 35; 1, 0, 0).



Fig. 5 City growth near the river and sea. Left: Constraint and seeds. Middle: Growth after 10,000 time steps. Right: city grows after 1 million time steps.

not the only way to calibrate the city shape. As the number of seeds, locations and timing of when to enter the seed is very important for the overall city shape, each of the seed will grow as a development cluster. The growth direction of the development cluster is directly controlled by the neighborhood probability parameters. All the development clusters, however, is using the same set of parameters. Thus, adjusting the growth direction of one seed will affect the other seed. The center of development is always near the location of the seed. Figure 5 shows an example of city growth in a lowland coastal site where the seeds were placed near the river.

Unconstrained City Growth

In the rest of this section, we discuss the result of applying the above-mentioned extension of Eden stochastic model for the urban growth. Application of the model on a theoretical observable fact on how a city grows if the land is flat and there is no natural barrier for the development. Particular problem for the application of the model is a simple theoretical question on how does development of a city look like if a city is growing without any constraint.

The simulation was done using default Eden parameters of neighborhood probability with 4 equal

neighbors without any development constraint. The extension of Eden model is equivalent to the Eden model with constant addition of development index. For simplicity the additional value of development index, λ , is assumed to be discrete and constant over time. At each time step, only a single development index can be added, ($\lambda = 1$). Since the discrete time step of the simulation model indicates time interval of development stage, a fixed development index may indicate that the growth rate of the city development is constant. The city is assumed mono-centric with a single seed in the middle as the starting point.

Figure 6 shows the typical city shape produced by the Eden model. The development cluster shape is circular with a rough perimeter and fully developed inner cells. The city center is developed near where the first seed was placed; it always has a higher development index than the outskirts of the city. The figure shows the shape in two ways, gray scale image with darker pixel indicates higher development index and black and white image. The gray scale image show clearly the location of the city center (near the original seed) where the development is the highest and lower developed cells near the edge of the city. The black and white image can show clearly the rough surface of the edge. The city boundary is seen clearly using the black and white image.

The time series of area and perimeter is shown in

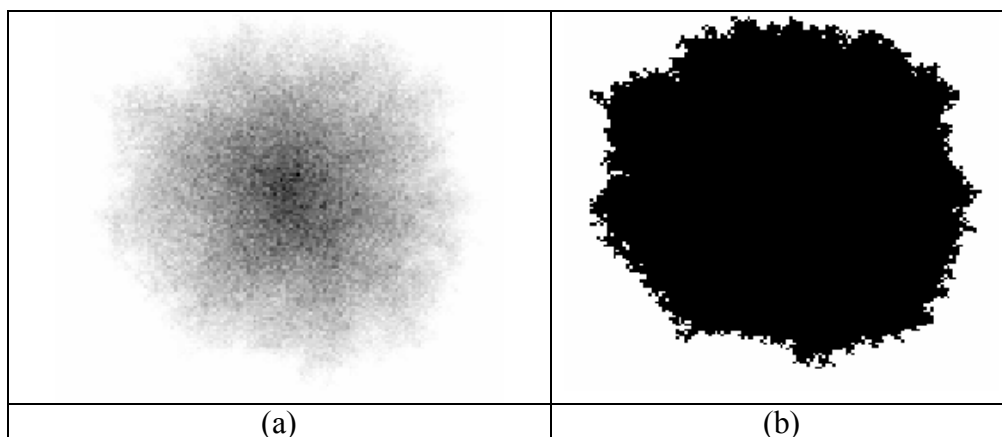
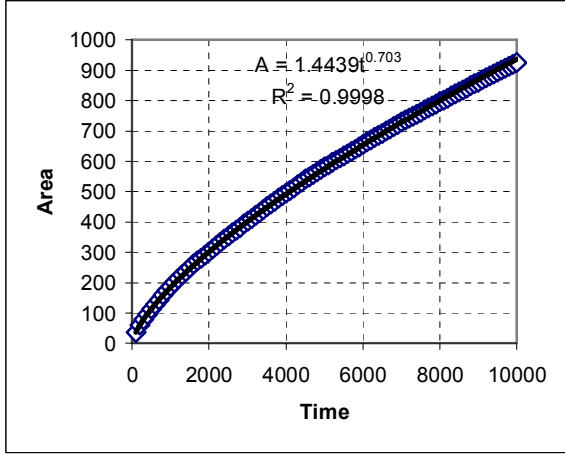
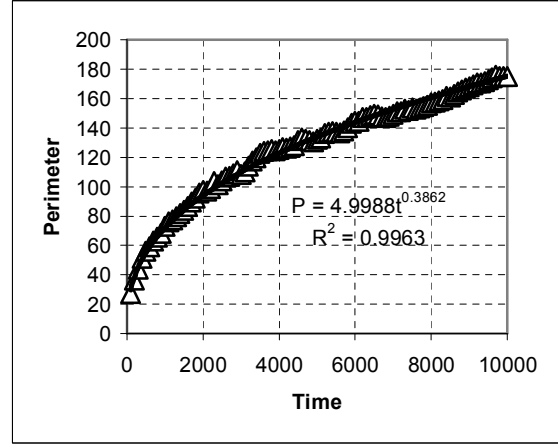


Fig. 6 The Eden model produces by the default parameters. (a) Gray value represents development index value, darker represents higher development index. (b) Black and white version shows that only the periphery has fractal surface, while the inner cells are compact.



(a)



(b)

Fig. 7 Growth trajectory of the city without any constraint (a) Area time series (b) Perimeter time series

Figure 7. Giving notation P as perimeter and A as area, the time series of area and perimeter using 10,000 simulation steps give the growth trajectory $A = 1.4439t^{0.703}$ and $P = 4.9988t^{0.3862}$ as shown in Figure 7. Continuing these simulation steps up to one millions produce

$$A = 2.2275t^{0.678} \text{ and } P = 9.2722t^{0.344} \quad (2)$$

The growth trajectory fit a power relationship. When the

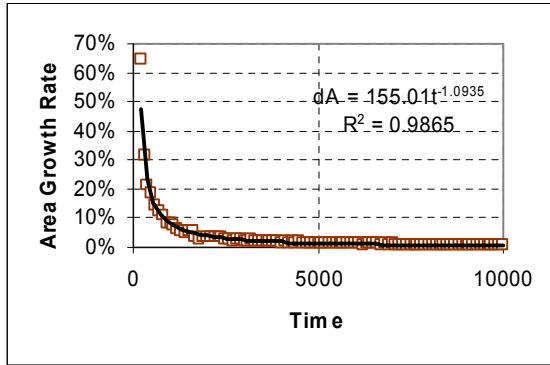


Fig. 8 Output of simulation: Area growth rate for small city is very high and medium and large city has very small expansion rate

city is small, the growth rate is high and as the city is getting bigger, the growth rate is getting much smaller as shown in Figure 8. These results may be interpreted that unconstrained city growth has a certain tendency when it is a small town; the growth of development is extensive to spread the area. For small towns where the area for expansion is available, the development of the cities tends to expand the city area. As the city is becoming larger, the growth of development is intensified in the existing inner part of the city with taller buildings rather than expanding the city area. When the time step is small, the city area is relatively small but the area growth rate is very high. As the city is rising, the area growth rate

decreases exponentially. Higher time step is correlated with bigger city area, as seen in Figure 7, but the area expansion rate is much smaller.

It is interesting to evaluate these phenomena by some quantitative values of area and perimeter. One of the simplest shape index relating area and perimeter of a general shape is the compaction ratio, $C_p = P/A$. The time series of compaction ratio produces

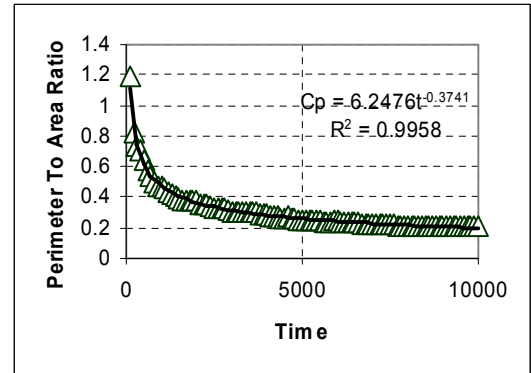


Fig. 9 The ratio of perimeter over area is declining over time due to smaller probability assigned to the edge.

$$C_p = 6.2476t^{-0.3741} \quad (3)$$

Deriving the area-growth equation produces $dA/dt = 1.015t^{-0.297}$, this has very similar scaling graph to the compaction ratio. In fact, after minimizing the sum square of error of the difference between the two variables, we can get the relationship

$$dA/dt = 0.3395C_p \quad (4)$$

Equation (4) has a very nice implication that without any development constraint, if the growth rate of a city development is assumed constant then the expansion rate

of the city area is equivalent to the ratio of perimeter to area of the city. Figure 9 shows the time series of compaction ratio (perimeter per area). This result is may be useful to predict the approximate growth rate of any city without any prior knowledge of the historical data. Without any information regarding historical data, based on above relationship, we can estimate the current city growth rate only by measuring the area and the perimeter of the city.

The reason for the above relationship lies on the Neighborhood Searching step. To expand the city perimeter or area, the cell selected must be in the edge (measured by perimeter) of the city, any inner cells selected will not make the city to expand. When the city grows bigger, the number of perimeter cells will become much smaller compared to the area. Because any developed cell has equal probability to be selected, then the probability of selecting the perimeter compared to the probability to select the inner cells are smaller as the city is getting bigger.

Generalizing equation (4) gives the relation that the rate dA/dt is proportional to the ratio P/A

$$dA/dt \propto P/A \quad (5)$$

and equation (2) can be generalized as

$$A \propto t^a \text{ and } P \propto t^b \quad (6)$$

It is possible to show that the equation (5) implies a relation between a and b . Since $dA/dt \propto a.t^{a-1}$, by inserting (6) in (5), one has $2a = b + 1$.

Suppose that the growth is exactly circular, it means that $A \propto P^2$ or that $a = 2b$. Associated with the relation $2a = b + 1$, we find that $a = 2/3$ and $b = 1/3$. These theoretical values are very near the found values from the simulation, in particular in the one million growths (see equation 2).

We can define a mean radius as the mean distance of the points at the periphery of the aggregate relative to the seed with general equation $dA \propto P dR$. Inserting in (5) gives the rate dR/dt ,

$$dR/dt = 1/A$$

or

$$dR/dt = t^{-a} \quad (7)$$

The rate is a decreasing function of t , with an exponent order of $2/3$. Thus, the growth in the proposed model is almost circular with a decreasing rate of the mean radius growth.

CONCLUSIONS

Eden model is one of the simplest stochastic growth models and the research reported in this paper has extended the stochastic urban growth Eden model for

lowland city. The model was applied to observe how a city grows if the land is flat and there is no natural barrier for the development. We have found that unconstrained city growth seems to have a tendency to extensively grow and to spread in area when it is still a small town and as the city becomes larger, it tends to intensify the development in the inner part of the city. It was also found out from the simulation results that for a mono-centric city without any development constraint, the expansion rate of the city area is equivalent to the ratio of perimeter per area of the city. This result may be useful to predict the approximate growth rate of any city without any prior knowledge of the historical data. Furthermore, it was found that the growth of an unconstrained city is almost circular with a decreasing rate of the mean radius growth with a scaling exponent order of $2/3$.

Further study may investigate the effect of urban decay by employing a negative value of development index. Calibration and validation of the model to the real lowland city may also be pursued.

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