LOWLAND TECHNOLOGY INTERNATIONAL Vol. 4, No.1, 21-36, June 2002
International Association of Lowland Technology (IALT), ISSN 1344-9656

BEHAVIOR OF THEORETICAL CURVE NUMBERS WITH RESPECT TO SOIL AND RAINFALL PROPERTIES

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ABSTRACT: A dimensional analysis fails to obtain a universal, dimensionless relationship between the maximum retention potential (S) of the Soil Conservation Service (SCS), runoff curve numbers (*CN*), and saturated hydraulic conductivity K_s, sorptivity, rainfall parameters, surface runoff, and soil moisture. Next, both S and the theoretical *CN* (CN_T) were calibrated based on the numerical solutions of Richard's equation applied to homogeneous soil columns. Results show that S and CN_T are directly related to soil but inversely related to rainfall properties. As a temporally-lumped model, *CN* generally estimated lower cumulative infiltration than that of Richard's equation; and in practice, the *CN* method may perform poorly if (1) the Antecedent Moisture Conditions (AMC) is low, (2) the initial rainfall is much higher than K_s which leads to Horton overland flow, or (3) the rain pulses after the initial abstraction is satisfied are small. Before applications, adjustments of CN_T with respect to the standard *CN* of fallow/idle land to reflect the effects of landuse, land treatment, and hydrologic conditions are recommended.

INTRODUCTION

The runoff curve number (*CN*) method of the former U.S. SCS is a simplified, empirical model that was developed to estimate the runoff depths resulting from large, single-storm events occurring over small agricultural areas or watersheds. Even though the SCS does not regard it an infiltration loss model, it tends to be interpreted as such by researchers (e.g., Ponce and Hawkins 1996). Using real data from a watershed, Morel-Seytoux et al. (1982), and recently Silveira et al. (2000) showed that it is an infiltration loss model. It computes the runoff depth without considering the spatial and temporal variability of the rainfall process, and so it is not expected to be perfectly predictive. However, it has been widely used in United States and other countries because it is easy to use, it requires minimum information, and is related to obvious watershed properties like soil types, landuse, surface and AMC (Ponce and Hawkins 1996). Given that complex, physics-based infiltration models cannot be easily applied because of mathematical difficulties and a lack of adequate data, the *CN* method provides a simple solution to a highly complex problem in watershed hydrology. Moreover, real world problems such as soil heterogeneity, soil swelling and shrinkage, cracks and fissures at times cause simple models to perform the same or better than complex based models.

The SCS method (McCuen 1998; SCS National Engineering Handbook 1963) relates a *CN* to four factors: the soil groups, the landcover complex, land treatment, and the AMC. The soil groups comprise of groups A, B, C, and D. The soil characteristics associated with each group are: Group A - deep sand, loess and silts; Group B - shallow loess, and sandy loam; Group C - clay and sandy loam, soils low in organic content but high in clay; Group D - swelling soils, heavy plastic clays, and some saline soils. The landcover complex includes a wide variety of land uses, land treatment and management, and hydrologic conditions. The

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Note: Discussion on this paper is open until December 25, 2002.

three AMC considered by the SCS method are dry (condition I), average (condition II), and wet (condition III). Other than the moisture condition, AMC is also implicitly used to account for other forms of variability.

In spite of its simple approach, *CN* attempts to incorporate some of the major physical principles of soil-water movement empirically, such as the effect of AMC and the decay nature of infiltration (Chen 1982). However, it has limitations compared to complex models such as Richard's Equation (Richards 1931). Besides, unless adequate observed data is available to estimate the *CN*, choosing a *CN* depends largely on the handbook estimates, and so it can be quite subjective, and prone to errors. Some researchers found the *CN* values obtained from the handbook are often different from the measured values, e.g., Smith and Eggert (1978). By expressing the SCS model to a 2-parameter relationship (i.e., by assuming the initial abstraction, I_a, not equal to 0.2S, where S is the maximum potential abstraction), Chen (1982) found that AMC plays an important role in determining *CN*. This is no surprise since S is also the volumetric capacity to store water in soils or on watershed surface. By assuming I_a not equal to 0.2S, Morel-Seytoux and Verdin (1980) also showed that I_a depended on rainfall properties.

Chong and Teng (1986) estimated the *CN* values of Molokai soil series from field observations of the maximum soil retention, saturated infiltration rate, and sorptivity under simulated rains. Pierson et al. (1995) used long-term observed rainfall data in combination with an infiltration model to produce the runoff data (runoff = rainfall - infiltration) for two test sites from which they derived the *CN*. They found that the *CN* they derived depend heavily on the infiltration model used, which were Green-Ampt, exponential and a constant infiltration models, and the storm characteristics. From the *CN* of pairs of rainfall-runoff amounts sorted in descending order, Hawkins (1993) found *CN* decreased asymptotically with increasing rainfall until a constant, rainfall-independent *CN* was reached, which shows that *CN* also depends on rainfall properties.

RESEARCH OBJECTIVES

Given that *CN* values are related to soil hydraulic properties, AMC, storm properties and landuse complexes, etc., objectives of this study are: (1) To find a possible general functional relationship between the maximum soil retention parameter (S) of *CN* with soil properties and hydrologic conditions using dimensional analysis, (2) To explore theoretical relationships of the *CN* with soil properties, hydrologic conditions and rainfall data, (3) to study effects of soil properties, types of surface runoff, and rainfall properties on the performance of *CN*, and (4) recommend adjustments to the theoretically derived *CN* in (2) to reflect the effects of landuse, land treatment, hydrologic condition and AMC. Since *CN* is well accepted and widely used, any information relating soil parameters and hydrologic conditions to *CN* should be useful to practitioners who use this approach.

RESEARCH METHODOLOGY

The basis for deriving the *CN* is to match the infiltration depth (F_{CN}) of the *CN* under single-storm events with the cumulative infiltration depth (F_R) determined by Richard's 1-D model for homogeneous soil columns with initially uniform soil moisture, è. Instead of laboratory or field data, numerical solutions of Richard's equation form the basis of this study. Richard's equation is by far the most physically based model for solving infiltration through homogeneous, saturated-unsaturated porous media. Albeit in real world situations physically based infiltration models may not necessarily be better than conceptual models (Hjelmfelt, 1991), in an ideal, homogeneous and bare soil profiles, Richard's equation should be better than these models. In addition, the results should provide some useful insight relating *CN* to soil properties, AMC and rainfall events, and the influence of such hydrologic and soil parameters on the performance of *CN*.

Soil Types and Rainfall Events

Four major soil classes selected for this study are sand, sandy loam, loam and silt loam. In a broad sense these soil classes represent the A, B, C, and D soil groups of the SCS method. Their hydraulic properties (listed in Table 1) are based on the work of Rawls et al. (1983) who analyzed approximately 5,000 soil horizons.

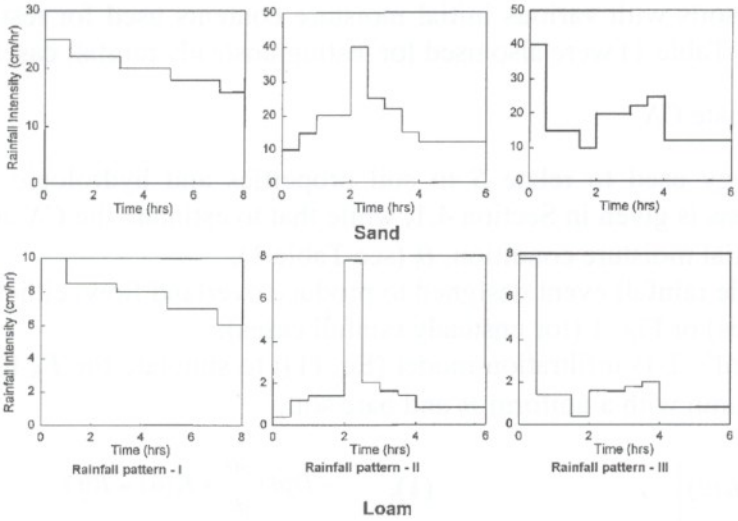

Fig. 1 Rainfall patterns I, II and III used in unsteady rainfall test cases for sand and loam

Table 1 The average hydraulic properties with ± one standard deviations about the mean (adopted from Rawls et al. 1983), initial soil moisture, and steady rainfall intensities of the four soil types tested in this study

Soil type	Saturated Hydraulic conductivity (cm/hr)	Pore Size Distribution Index	Bubbling Pressure (cm)	Residual Saturation (cm^3/cm^3)	Porosity (cm^3/cm^3)	Viscous correction factor	Initial moisture (cm^3/cm^3)	Rainfall* Intensity (cm/hr)
Sand	21.0	0.592 (.334-1.05)	7.26 (1.36-38.7)[#]	0.020 (.001-.039)	0.437 (.374-.500)	1.4	0.033, 0.091 0.280	21.5, 25.0
Sandy Loam	2.59	0.322 (.186-.558)	14.66 (3.45-62.2)	0.041 (-.024-.106)	0.453 (.351-.555)	1.4	0.095, 0.207	2.65, 5.18 10.36
Loam	1.32	0.220 (.137-.355)	11.15 (1.63-76.4)	0.027 (-.020-.074)	0.463 (.375-.551)	1.4	0.117, 0.270	1.40, 2.64 5.28
Silt Loam	0.68	0.211 (.136-.326)	20.76 (3.58-120.4)	0.015 (-.028-.058)	0.501 (.420-.582)	1.1	0.133, 0.330	0.70, 1.36 2.72

[#] Values inside the brackets are ± one standard deviations about the mean.
Rainfall intensities are that of steady rainfall test cases conducted for each soil type.

To explore the effects of the surface runoff mechanism on the *CN*'s performance, this study was conducted under a wide range of steady (Table 1) and unsteady rainfall events (Fig. 1) that were designed either to generate Hortonian (which means rainfall intensity exceeds infiltration capacity) or saturation overland flow as the mode of surface runoff. Since the primary purpose of the *CN* method is to compute runoff, the rainfall intensities are chosen to be higher than the K_s of each soil type to ensure that ponding occurs before the rainfall event ends. After ponding has occurred, the excess rainfall should contribute mostly to surface runoff. For each soil texture to create ponding situations within a reasonable time frame, rainfall intensities chosen were either slightly higher, or two to four times of the K_s. For sand, because of its high K_s of 21 cm/hour, it was necessary to use rainfall events of very high intensities even though such intensities (as shown in Fig. 1) are among the highest observed in the world (see Fig. 3-16, Linsley et al. 1982). In order to assess the effects of varying rain pulses, three unsteady rainfall patterns shown in Fig. 1, representative of certain natural rainfall events, were selected. The rainfall pattern I is taken from Morel-Seytoux et al. (1977), pattern II is a S-type distribution for 6-hour storms from the WMO (World Meteorological Organization), and rainfall pattern III is a modified version of rainfall pattern II. Pattern I is a series of rain pulses that represent a steady decline of rainfall intensities over time. Pattern II contains initially low rainfall intensities followed by a sudden burst of high rainfall intensity. The same sets of soils with various initial moisture contents used for testing steady rainfall cases (as shown in Table 1) were also used for testing unsteady rainfall cases.

Procedure to Estimate *CN*

The methodology used to relate S to soil properties and hydrologic conditions by the dimensional analysis is given in Section 4.1, while that to estimate the *CN* are outlined below:

1) Select an initial moisture condition, θ_i (see Table 1),

2) Select a single rainfall event designed to produce overland flow, either from Table 1 (for steady rainfall cases) or Fig. 1 (for unsteady rainfall cases);

3) Solve Richard's 1-D infiltration model (Eq. (1)) to simulate the F_R for a homogeneous, 1-m deep, soil column with a uniform θ_i and bare soil,

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial\theta}{\partial z} - K(\theta)\right] \quad (1), \qquad -D(\theta)\frac{\partial\theta}{\partial z} + K(\theta) = R(t) \quad (2)$$

To reduce the effects of hysteresis on $K(\theta)$ (Ungs et al. 1985; Jayaveerasingam 1990), Eq. (1) is solved as a function of θ by a time-centered, finite difference numerical scheme of Crank-Nicolson, and a general flux, upper boundary condition (Eq. (2)), where $R(t)$ is the rainfall at time t, $D(\theta)$ the diffusivity, and z is the vertical distance. At the lower boundary, e.g., z = 1 m, a Neumann type boundary condition (Eq. (3)), which means a constant water content gradient, is specified.

$$\frac{\partial\theta}{\partial z}(z = 1m, t) = 0,\ t \geq 0 \quad (3), \qquad K(\theta) = K_s\theta^{(2+3\lambda)/\lambda} \quad (4)$$

$K(\theta)$ and $D(\theta)$ are computed from Brooks and Corey's (1964) relationships (Eqs (4)-(5)), where λ is the pore size distribution index, ϕ the soil porosity and φ_b the bubbling pressure. For steady rain cases of various storm durations (4, 8, 11, 16 and 22 hours) and for unsteady rain cases, there are six sets of F_R per soil type, which form the calibration data used by Simplex mentioned in Step (11).

4) Estimate sorptivity, S_o. Given this study also involves unsteady rain, S_o was estimated using Parlange's (1972) approximate expression (Eq.(6)),

$$D(\theta) = \frac{-\varphi_b K_s}{\lambda(\phi - \theta_r)}\theta^{(2+1/\lambda)} \quad (5), \qquad S_o^2 = 2\int_{\theta_i}^{\theta_f}(\theta - \theta_i)D(\theta)d\theta \quad (6)$$

From Eqs. (5) and (6), and integration by parts, a close-form solution for S_o for each soil type under various θ_i is found to be

$$S_o^2 = \frac{-2\varphi_b K_s}{(1+3\lambda)}\left[\left(\frac{\theta_f - \theta_i}{\phi - \theta_r}\right)^{3+1/\lambda}(\theta_f - \theta_i) - \left(\frac{\theta_f - \theta_r}{\phi - \theta_r}\right)^{4+1/\lambda}\frac{(\phi - \theta_r)}{(4+1/\lambda)} + \left(\frac{\theta_i - \theta_r}{\phi - \theta_r}\right)^{4+1/\lambda}\frac{(\phi - \theta_r)}{(4+1/\lambda)}\right] \quad (7)$$

where θ_f and θ_r are the final and residual soil moisture respectively.

5) Compute the maximum soil water retention, S, of the SCS method by one of the three models (Eqs. 8 to 10) relating S to K_s, S_o, and rainfall intensity, R. Initially, model parameters P_1, P_2, P_3 and P_4 are assumed. These models are similar to that of Chong and Teng (1986) who used field measurements to determine their model parameters. For the unsteady rain cases, only Eqs. (8) and (9) were considered.

$$S = P_1 K_s^{P_2} \quad (8), \qquad S = P_1 K_s^{P_2} S_o^{P_3} \quad (9)$$

6) Upon finding S (in cm), CN is related to S by Eq. (11)

$$S = P_1 K_s^{P_2} S_o^{P_3} R^{P_4} \quad (10), \qquad CN = \frac{2540}{(S + 25.4)} \quad (11)$$

where theoretically $0 \le CN \le 100$, $\infty \ge S \ge 0$, and terms on the right-hand side of Eq. (11) are in cm (units of this study). Since S represents the watershed's capacity to retain storm moisture, it should have an upper limit, which means that $CN > 0$. This means that the soil profile has a finite storage depth. Even though this is different from a 1-m deep soil column with constant soil moisture gradient as the lower boundary, it should not affect the results because of the short durations considered (6 to 24 hours), and the CN method uses storm duration up to 24 hours to compute the runoff, Q.

7) Compute Q by the SCS method (Eq. (12))

$$Q = \frac{(P - I_a)^2}{(P - I_a) + S} \quad (12), \qquad \frac{F_{CN}}{S} = \frac{Q}{P - I_a} \quad (13)$$

where P is the total rainfall at time Δt and I_a the initial abstraction. If $P < I_a$, Q is assumed zero. Albeit experience shows that $0 \le I_a \le 0.3S$ (e.g., Hawkins 1984), the standard approach assumes I_a to be $0.2S$. Here, I_a is assumed to be the total amount of rainfall just before ponding.

8) Compute F_{CN} according to the rainfall-runoff relationship of the CN method (Eq. (13)). By this assumption, we can estimate F_{CN} and Q through only one parameter, the CN.

9) Repeat steps (7) and (8) for $\Delta t = 1, 2, 3, \ldots, n$ hours, and for every combination of θ_i and rainfall event.

10) With each set of P_1, P_2, P_3 and P_4 of Eqs. (8) to (10), which give rise to several sets of $F_{CN,\Delta t}$, where $\Delta t = 1, 2, 3, \ldots, n$, compute either the least square objective function (OF) for differences between un-transformed (OF_1), or log-transformed (OF_2), F_R and F_{CN} data. Both OF_1 and OF_2 assume the presence of Gaussian, independent homogeneous variance errors.

$$OF_1 = \sqrt{\frac{\sum_{\Delta t=1}^{n} (F_{R,\Delta t} - F_{CN,\Delta t})^2}{n}} \quad (14), \qquad OF_2 = \sqrt{\frac{\sum_{\Delta t=1}^{n} (\log F_{R,\Delta t} - \log F_{CN,\Delta t})^2}{n}} \quad (15)$$

11) A powerful, optimization algorithm of Nelder and Mead (1965), called the local Simplex, uses three operations (reflection, expansion, and contraction) to automatically search for optimum P_1, P_2, P_3 and P_4 in the parameter space. Ideally, the parameters found are optimum when either OF_1 or OF_2 is minimized. The search continues until either the OF, or the change of OF value from one iteration to the next drops below a pre-set criterion, or the number of iterations exceed the maximum allowable.

A flow chart summarizing the above procedure is shown in Fig. 2. By subjecting these four major soil classes to various rainfall events and AMC (Table 1), the parameters were calibrated. The optimum parameter values obtained through Simplex for test cases S2, S3, . . ., to US3Log, listed in Table 2 represent a combination of model type (2 represents 2-parameter and 3 represents 3-parameter models), rainfall type (S for steady and US for unsteady), and either non-transformed (such as S2 or US2) or log-transformed calibration data (such as S2Log or US2Log). Each set of the six steady rain test cases was repeated for storm duration of 4, 8, 11, 16, and 22 hours but Table 2 only shows the parameters obtained for 11 and 22 hours.

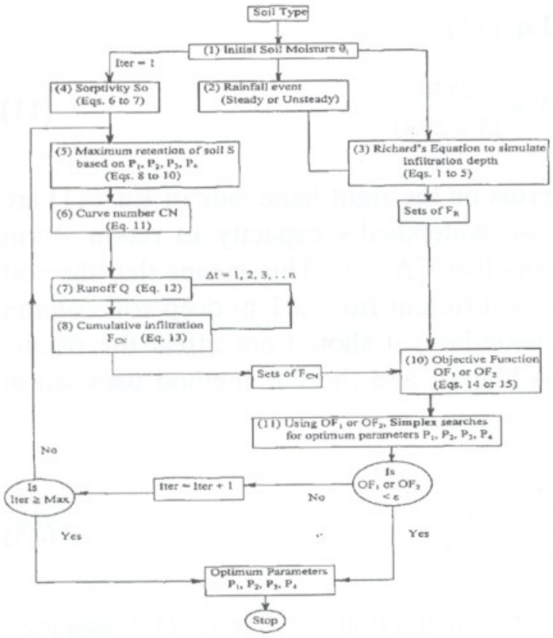

Fig. 2 The model parameters of Eqs. 8 to 10, rainfall and calibration data types used, and corresponding optimized parameter values derived (using the local Simplex algorithm) for 16 of the 36 test cases conducted in this study

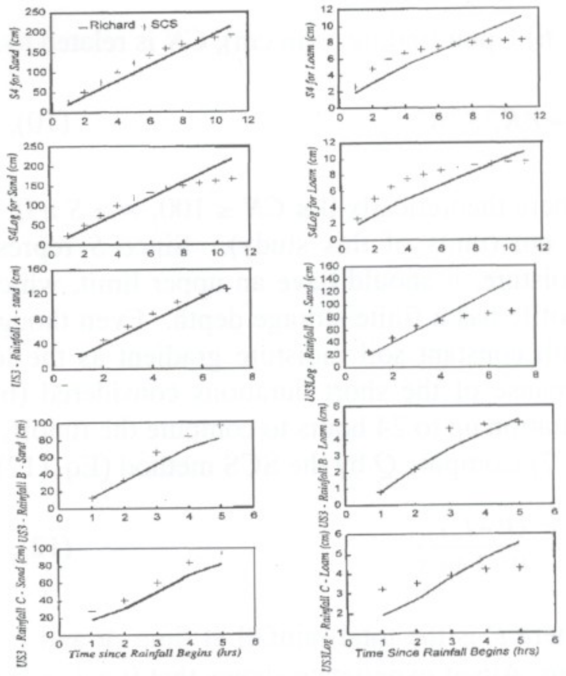

Fig. 3 Summary statistics of the calibration, validation and cross validation runs of 16 test cases based on Eqs (8)-(10), steady and unsteady rainfalls, and calibration data that are either transformed or untransformed

Figure 3 shows plots of cumulative infiltration versus time simulated by the *CN* method and Richard's model for sand and loam under different test cases. The calibration runs are based on the hourly *F* for four soil classes subjected to various degrees of AMC specified in terms of initial θ (see Table 1). Results shown in Table 3 are given in terms of the root-mean-square error (RMSE), the bias (BIAS), the coefficient of Efficiency (E_f) (Nash and Sutcliff 1970) and the coefficient of determination, R^2. To ensure a proper validation of the *CN* method, in the validation and cross-validation stages only *F* estimated by the *CN* method that were not used as calibration data are compared with that estimated by Richard's model. These are the 12^{th}-hour (for calibration involving 11-hour storm) and the 23rd-24th hour (for calibration involving 22-hour storm) *F* for the steady rainfall cases and the 8th-hour cumulative infiltration for the unsteady rainfall cases.

DISCUSSIONS OF RESULTS

Results of the dimensional analysis are first presented and then the theoretical *CN* obtained by the Simplex method.

Dimensional Analysis

Table 2 The model parameters of Eqs. 8 to 10, rainfall and calibration data types used, and corresponding optimized parameter values derived (using the local Simplex algorithm) for 16 of the 36 test cases conducted in this study

Storm Duration (T) used for calibration	Test Case	Equation used	Rainfall Type	Calibration Data	P_1	P_2	P_3	P_4
11	S2	8	Steady	nt#	16000$	0.933		
11	S3	9	Steady	nt	33200	1.01	-0.0007	
11	S4	10	Steady	nt	1120	1.28	0.028	-0.652
11	S2Log	8	Steady	Lt#	3000	0.795		
11	S3Log	9	Steady	Lt	15100	0.738	0.284	
11	S4Log	10	Steady	Lt	1030	1.04	0.142	-0.472
22	S2	8	Steady	nt	40700	0.960		
22	S3	9	Steady	nt	53100	0.985	0.0013	
22	S4	10	Steady	nt	24800	1.05	0.015	-0.156
22	S2Log	8	Steady	Lt	9460	0.842		
22	S3Log	9	Steady	Lt	26000	0.738	0.283	
22	S4Log	10	Steady	Lt	22300	0.899	0.553	-0.390
	US2	8	Unsteady	nt	38400	1.080		
	US3	9	Unsteady	nt	35600	1.080	-0.017	
	US2Log	8	Unsteady	Lt	1190	0.789		
	US3Log	9	Unsteady	Lt	6560	0.620	0.468	

nt = Nontransformed data, Lt = Log-transformed data

$ = All the parameters of the steady rain cases (S2 to S4Log) shown above were calibrated from storms either of durations equal to 11 or 22 hours.

A possible way to generalize the functional relationship among the eight variables, S, K_s, S_o, T (time), R, F, Q, and θ_i is to form dimensionless groups of π-terms through dimensional analysis. Because the variables include three independent dimensional units - mass, length, and time - the Buckingham π-theorem (Buckingham 1914) allows us to write a relationship in terms of five non-dimensional groups. One possible relationship is Eq. (16)

$$\frac{S}{TK_s} = f\left(\frac{S_o}{K_s T^{0.5}}, \frac{K_s}{R}, \theta_i, \frac{QT}{F}\right) \quad (16), \qquad \frac{S}{TK_s} \approx f\left(\frac{S_o}{K_s T^{0.5}}\right) \quad (17)$$

Among the four groups on the right-hand side of Eq. (16), the first group $S_o/K_sT^{0.5}$ has been found to be most relevant in this study. Since R is quite arbitrarily set in the experiments, which directly influence Q and F, the groups K_s/R and QT/F turn out to be not much related to the dependent group S/TK_s. Therefore Eq. (16) is approximated by Eq. (17)

Using the S values derived from Eq. (8) that does not involve S_o, Eq. (17) is plotted for steady and unsteady rainfall cases (S2 and US2) in Fig. 4. Apparently S/TK_s and $S_o/K_sT^{0.5}$ are somewhat inversely related but that relationship is not clear-cut. Such a weak inverse relationship is confirmed by the small negative values of -0.0007 and -0.017 obtained for the P_3 of S3 and US3 respectively (see Table 2). However, the inverse relationship of S/TK_s and $S_o/K_sT^{0.5}$ changes to a direct one when the S values are taken from log-transformed calibration data, e.g., US3Log. This direct relationship is again confirmed by the positive P_3 of 0.468 obtained for US3Log. Even though the opposite results obtained for original and log-transformed calibration data show that it is likely difficult to obtain a universal, dimensionless relationship among these variables, the results at least confirms the validity of the parameters (such as P_3) obtained in this study.

Estimate Theoretical Curve Numbers (*CN*) Using Simplex

On a whole, the calibration results are encouraging (Table 3), with R^2 and E_f mostly exceeding 90% and bias within 10%. The *CN* derived from the calibrated parameters of Eqs. (8) to (10) also mainly fall within the practical range of 40 to 98 recommended in the literature (e.g., Van Mullem 1989). Essentially, higher *CN* means smaller S and vice versa. This means the *CN* of sands should be much smaller than that of silt loam, while the *CN* of sandy loam and loam should be somewhere in between (see Fig. 6). Under various combinations of rainfall, AMC and calibration data types, the *CN* derived are between 7 and 37 for sand, between 30 and 76 for sandy loam, between 50 to 90 for loam, and 60 to 93 for silt loam. The *CN* values derived are mostly within the practical range of 40 to 98, except for sand that drops below the lower limit of 40 because we rarely encounter catchments having sand as the predominant soil type.

Table 3 Summary statistics of the calibration, validation and cross validation runs of 16 test cases based on Equations 8 to 10, steady and unsteady rainfalls, and calibration data that are either transformed or untransformed

Storm Duration	Test Case	Calibration				Validation				Cross Validation*		
(T) used for Calibration		BIAS	E_f	R^2	RMSE	BIAS	E_f	R^2	RMSE	BIAS	E_f	RMSE
11	S2	3.55	98.9	99.5	16.5	-20.2	94.6	99.8	30.4	-38.1	83.4	61.2
11	S3	1.95	98.8	99.5	16.8	-20.9	95.0	99.8	29.2	-38.2	83.3	61.3
11	S4	4.06	99.0	99.6	15.6	-17.7	94.9	99.7	29.4			
11	S2Log	-3.95	96.5	98.8	28.8	-32.9	81.5	99.7	56.3	-60.5	43.9	112.
11	S3Log	-0.04	97.9	99.0	22.5	-27.4	87.4	99.4	46.4	-60.2	43.5	113.
11	S4Log	-2.16	97.4	99.0	24.8	-29.2	84.6	99.5	51.3			
22	S2	3.77	98.8	99.5	17.0	-14.2	97.9	99.9	18.9			
22	S3	3.31	98.8	99.5	17.1	-14.3	98.0	99.9	18.3			
22	S4	3.79	98.9	99.5	16.6	-13.8	98.0	99.9	18.5			
22	S2Log	-3.68	96.8	98.9	28.4	-26.8	88.8	99.9	43.9			
22	S3Log	-3.81	96.4	98.7	30.0	-26.9	88.1	99.7	45.3			
22	S4Log	-1.76	96.9	98.6	27.9	-22.7	90.4	99.0	40.6			
	US2	10.1	96.9	99.2	27.8	-3.08	97.1	98.7	23.0	12.6	97.6	21.0
	US3	10.1	96.9	99.2	27.8	-3.07	97.1	98.7	23.0	15.5	97.4	22.1
	US2Log	-4.20	90.8	96.1	47.7	-27.9	81.1	96.3	59.2	13.7	97.8	20.4
	US3Log	-5.56	88.9	95.3	52.3	-29.2	78.3	95.3	63.4	15.6	97.4	21.9

*For T = 11-hour, S2, S3, S2Log, and S3Log, model parameters P_1, P_2, and P_3 for computing the maximum retention S are based on those derived from their unsteady rainfall counterparts, US2, US3, US2Log, and US3Log respectively. For computing the maximum retention S of US2, US3, US2Log and US3Log, the same strategy of using parameters derived from their steady rainfall counterparts was also adopted

Calibration Under Steady Rainfall

Given that the basis of calibration comes from the numerical solutions of Richard's equation for homogeneous soil columns, these *CN* values likely reflect a barren, un-crusted soil top as the landuse without any effect of land treatment and management. For steady rain cases, the average *CN* obtained for sand, sandy loam, loam and silt loam are 17, 55, 72 and 82 respectively (Fig. 5 and Table 4). The averaged *CN* recommended by SCS for noncultivated agricultural land, meadow, forestland (deciduous, brush and woods), under non-treated, well managed condition and AMC-II are 29, 56, 70 and 77 for soil groups A, B, C and D, respectively (McCuen 1998). If sand is approximately equated to soil group A, sandy loam to B, loam to C and silt loam to D, then the mean *CN* derived for sand is slightly low (17), but the *CN* for the other three soil types are quite comparable. However, when compared to the recommended *CN* for fallow or idle land (77, 86, 91 and 94), these *CN* are generally quite small (which means higher infiltration) partly because the steady rainfall data applied are generally higher than that of nature.

Interestingly, the average *CN* for the same four soil types under unsteady rainfall (US2 to US3Log) increase to 28, 75, 87 and 93, respectively, which, with the exception of sand, would be comparable to that of fallow or idle land. It is obvious that the theoretical *CN* (CN_T) depend not only on the soil types but also on the rainfall properties. CN_T tends to decrease with an increase in storm intensity or duration or both, and vice versa. Figure 6, which gives the steady rain *CN* in terms of maximum, minimum, mean (μ), and $\pm$ 1 standard deviation (σ) about μ, confirms that *CN* decreases with the storm duration consistently for all the soil types. This result agrees with that of Hawkins (1993) and it came as no surprise since storms of the same intensity but longer duration result in larger S, and, therefore, smaller *CN*.

Given CN_T are based on homogeneous soil columns with no vegetation cover, they should in theory more closely agree with that of fallow/idle land (such as the average *CN* of unsteady rainfall) than that of meadow or forestland (such as the average *CN* of steady rainfall). In this regard, it seems the CN_T obtained for the unsteady rainfall cases are more acceptable than that of steady rainfall because the latter are generally too heavy compared to actual storm events occur in nature. The former rainfall events are more realistic and so are their CN_T.

Calibration runs under steady rainfall (S4 and S4Log) in Fig. 3 show that initially *F* values of the *CN* method are higher than those of the Richard's, but as time goes on, as *F* approaches *S* of the *CN* method, the reverse occurs. Given that *CN* is a temporally lumped model, it is no surprise to find that the *CN* method cannot quite replicate the temporal infiltration rate of the Richard's model, or vice versa (as shown in Fig. 3). The problem of under-estimating *F* gets worse with a decrease in *K(è)*, such as from sand to loam. For loamy soil, this problem persists even when the AMC specified is very low, i.e., 0.033. Besides the soil type, the accuracy of *F* estimated also depends on whether the calibration data used is log-transformed or not. The effects of log-transforming the calibration data have been mixed, giving rise to larger *CN* than non-transformed data for sand but not necessarily so for the other three soil types (Fig. 5). An explanation to this phenomenon is given in Section 4.5. The above two factors, however, are relatively minor for the root problem lies in the assumption that the total retention, $F+I_a$, asymptotically approaches $S+I_a$, as the precipitation $P \rightarrow \infty$, by

$$F = \frac{(P-I_a)S}{(P-I_a+S)} \tag{18}$$

To increase *F* at later hours, an easy way out is to pick a smaller *CN* to increase *S*. The approach, nonetheless, is ad hoc and subjective, and ultimately *F* will still approach *S*, unless Eq. (16) is modified such that *F* becomes unbounded. An example of making *F* unbounded is Hawkins' (1992) proposed model, $Q = f(P - I_a)$, which means that $F = f(P - I_a - Q)$. This implies that when $(P - I_a) \rightarrow \infty$, *F* also $\rightarrow \infty$, a feature found in infiltration models like Green-Ampt and Philip. However, this feature also creates an infinite retention even though soil profiles have finite depth.

It is found that the difference in the *S* values estimated via the 2-, 3-, and 4-parameter models (Eqs. (8) to (10)) is only marginal, as reflected by the *CN* plotted in Fig. 5, and the *F* shown in Fig. 7. This implies that *S* depends mostly on K_s, *R* and *T*, while S_o does not contribute much to *S*. The lack of correlation between S_o and *S* indicates the difficulty in accounting for the effect of AMC on CN_T through Eqs. 8 to 10. Apparently the effect of AMC on the value of CN_T should be accounted for at a later stage (see Section 5). Since under-estimating *F* means over-estimating *Q*, for steady rain cases whose pulses are greater than K_s, the over-estimation of *Q* gets worse with time. It is probably because of this reason that the SCS method was developed for event-based applications.

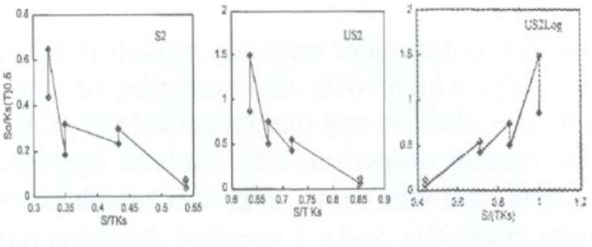

Fig. 4 Plots show the functional relationship between non-dimensional groups S/TK_s and $S_o/K_sT^{0.5}$ (Eq. 17).

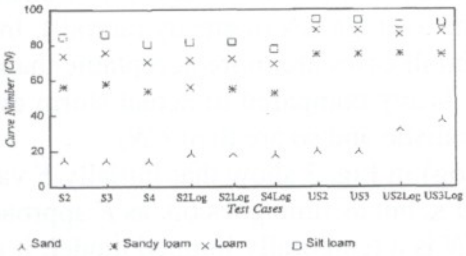

Fig. 5 Mean runoff curve numbers (*CN*) derived for the 4 soil types - sand, sandy loam, loam and silt loam - subjected to various combinations of rainfall events, soil retention models, and calibration data types (S2, S3, . . to US3Log)

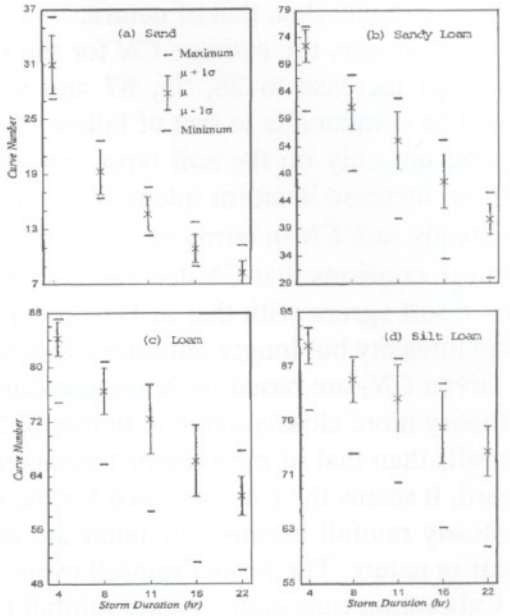

Fig. 6 Modified box-plots showing the curve numbers (*CN*) obtained for test cases S2, S3, . ., and S4Log of (a) sand, (b) sandy loam, (c) loam, and (d) silt loam, under steady rainfall events of intensities given in Table 1 and durations of 4, 8, 11, 16, and 22 hours respectively

Calibration Under Unsteady Rainfall

The effects of not accounting for the Hortonian mechanism become obvious under the test cases involving unsteady rainfall patterns II and III. The rain pulse of II at the third hour is much larger than the K_s of the soil (see Fig. 1), and so it should give rise to the Horton overland flow condition. Because the *CN* method does not account for the Hortonian or other overland flow sources, it estimates a value of F higher than that of Richard's model after that particular rain pulse (US3 and US3Log cases of Fig. 3). Because of the Hortonian mechanism, Richard's model generally takes a few more hours to "catch up" with the *CN* method in the amount of F estimated.

The rainfall pattern-I is similar to II except that the large rain pulse has been moved to the first hour. Therefore under C, the *CN* method tends to estimate a higher F right from the beginning because it ignores the Hortonian mechanism. Except for sand, despite with higher F values initially, the *CN*'s estimated F values quickly fall behind that of Richard's model because after satisfying I_a, it predicts that the subsequent rain pulses will always contribute to runoff, even when these pulses are very small. Since the subsequent rain pulses of C are fairly small, Richard's model estimated that most of the rain pulses contributed to infiltration and so its F values quickly surpassed that of the *CN*.

For test cases under rainfall pattern-I, the estimated F values of the *CN* are similar to the steady rainfall counterparts. With a rainfall pattern that decreases with time, the *CN* generally over-estimates F in the first few hours and then does the reverse in the next few hours when F approaches S. Again, the problem of under-estimating F gets worse as K_s decreases (such as from sand to loam) because S is closely linked to K_s.

Validation (V) Experience

Given that during calibration the *CN* method generally under-estimated *F* at later hours, it is expected to under-estimate *F* at the validation stage that involves *F* values at the latest hours of each rainfall event (8^{th} hour for unsteady rain, 12^{th}-15^{th} or 23^{th}-24^{th} hour for steady rain). Again, this is mainly because of *CN*'s asymptotic assumption of *F*. At the validation stage, for steady rain cases there is little difference between the 12^{th}-hour *F* values estimated under S2, S3 and S4 (Fig. 7), which again confirms that S primarily depends on the K_s of the soil, while S_o has little influence on S. Chong and Teng (1986) also found that K_s was much more important than S_o in estimating S even though they only used the Molokai soil in their study.

For steady rainfall cases subjected to log-transformed calibration data (S2Log, S3Log, S4Log), there are more obvious differences in the 12th-hour *F* values estimated even though the differences are still marginal. Because the 12th-hour *F* value of sand is about one order larger than that of silt loam or loam, using original data in S2, S3, and S4 leads to model parameters that reflect a better performance for sand at the expense of loam or silt loam (Fig. 3). The trend becomes opposite when transformed calibration data were used (S2Log, S3Log, and S4Log). In terms of summary statistics, results of S2, S3 and S4 are better than S2Log, S3Log, and S4Log because the former estimated large *F* values more accurately while the latter estimated small *F* values more accurately, and large *F* values exert a larger influence on the summary statistic. The same applies to the unsteady rainfall test cases US2, US3, US2Log, and US3Log.

Test cases involving 12 consecutive hours of steady rain pulses are somewhat idealized except under simulated rainfall situations or in some very wet spots in the tropics. In this sense, unsteady rainfall cases involving three patterns of 8-hour storms (I, II and III) are more realistic and interesting than the steady rainfall cases, albeit their results are not as good as the latter cases. Compared to Richard's model, most *F* values for the *CN* method under unsteady rain are still under-estimated except for several cases when the reverse happened (see Fig. 8), e.g., for sand or when the model parameters used to compute the *CN* for unsteady test cases were taken from the steady rainfall counterparts.

Cross-Validation (C-V) Experience

Results in the validation tests are quite predictable even though the *F* values compared at the validation (V) stage were not used in the calibration stage because both stages involve the same storm events. The cross-validation (C-V) experience is partly designed to study the influence of rainfall type and intensity on *CN*. Under C-V (Table 3), P_1, P_2, and P_3 used for estimating S (by Eqs. 8 and 9) for test cases S2, S3, S2Log, and S3Log were taken from their unsteady rainfall counterparts, US2, US3, US2Log and US3Log respectively, and vice versa. By using a storm event in the V stage different from the calibration stage, we expect the performance of the *CN* method to be less predictable. Furthermore, the C-V results should be more valid than V results (Section 4.5) because now the model parameters tested were derived from different rainfall patterns.

From Table 3 and Fig. 8, it is found that C-V results for S2, S3, S2Log, and S3Log of 11-storm duration are poorer than their corresponding V results, especially for S2Log and S3Log. For example, under validation the E_f for S2Log and S3Log were 81.5% and 87.4%, but under C-V the E_f were 43.8% and 43.5%, respectively. The corresponding bias values under C-V were about -60% but only about -30% under validation. Since the total rainfall depths of steady rains are more than that of unsteady rains, one would expect *S* estimated from P_1, P_2, and P_3 calibrated with the former to be larger than that calibrated with the latter. With larger *S* values, the *CN* method will estimat e larger *F*. Given that using P_1, P_2, and P_3 calibrated from unsteady rainfall lead to lower S values, which lead to lower F values, it makes sense to find the C-V results for the steady rainfall cases S2, S3, S2Log, and S3Log to be worse than their corresponding validation results.

For the same reason, the C-V results from US2, US3, US2Log, and US3Log turn out to be slightly better than their corresponding V results (see Fig. 8 and Table 3). For example, the bias (E_f) for US2Log and US3Log under C-V were about 14% (97%) whereas that under V was close to -30% (80%). In fact, C-V results for US2 and US3 show that for the first time the F values of CN exceed that of Richard's because these F values were based on the S values estimated from P_1, P_2, and P_3 of their steady rainfall counterparts. Albeit no strong relationship between S and R has been found, given that better results are obtained when the CN derived from heavier, steady rainfall events are applied to test cases using lighter, unsteady rainfall events, and vice versa, it is obvious that R and T has some influence on the CN values calibrated, as confirmed by Fig. 6.

In summary, the CN method has difficulty emulating the temporal infiltration rates of Richard's model or vice versa simply because it is meant to be a temporally-lumped model. Nonetheless, it seems that CN derived from K_s can still estimate reasonable values of F at the 12th, 23rd to 24th or 8th-hour period, particularly when applying CN derived from heavier rainstorms to cases involving lighter storms.

Table 4 Average theoretical CN derived from steady and unsteady rainfall cases, and the corresponding average CN recommended for meadow/forestland, and fallow/idle lands respectively

Major Soil Classifications of SCS	A	B	C	D
Mean CN of Meadow, Forestland, Brush and Wood (good hydrologic conditions)	29	55	70	77
CN of Fallow/ Idle land	77	86	91	94
Major soil Types Tested	Sand	Sandy Loam	Loam	Silt Loam
Average CN under steady rainfall (S2 to S4Log)	17	55	72	82
Average CN under unsteady rainfall (US2 to US3Log)	28	75	87	93

On a whole, it seems that more realistic CN_T are obtained from unsteady rainfall than the steady rainfall (Section 4.3 and Table 4) because the former tends to more resemble nature, while the latter chosen in this study tends to be heavy, especially that of sand. Therefore if we know the dominant soil type, derive the CN via the method described in Section 3 under moderate unsteady rainfall. Even though the results are numerically simulated, it seems that in practice the runoff CN method will perform poorly (even when the CN chosen is realistic) if (1) the AMC is low, (2) the initial R is much higher than the dominant K_s which leads to Horton overland flow not accounted for by the CN method, or (3) the rain pulses after the I_a is satisfied are small because the CN method still simulates surface runoff albeit under such circumstances there should only be infiltration.

RECOMMENDATIONS TO ADJUST THEORETICAL CN (CN_T)

Given CN_T are obtained from Richard's solution under homogeneous soil columns of barren soil top, they should be adjusted before they can be applied to real world problems. As a minimum, these CN_T can serve as an initial estimate to the actual CN to be used, which the user should confirm with field observation (if possible) and the literature, e.g., the SCS handbook. The CN_T obtained from using moderate, unsteady rainfall events should reflect the CN of the dominant soil type under a landuse condition similar to that of fallow or idle land with no vegetation (Section 4.2). To reflect the effects of landuse, land treatment and management, and AMC, some recommendations are given below to adjust CN_T so that it will be applicable to cultivated, non-cultivated agricultural lands, or forestlands.

From SCS's recommended *CN* for the above landuse types (e.g., McCuen 1998), the difference (reduction) in *CN* between these landuse types and that of fallow or idle land (77, 86, 91 94) under AMC-II are estimated. Cultivated agricultural lands have smaller *CN* than fallow (which means more infiltration losses) because land plowing and the presence of plant roots (which give rise to macropores) promote infiltration loss. The mean and standard deviations of these differences in *CN* categorized in terms of landuse, hydrologic conditions, and land treatments, are shown in Fig. 9. The standard deviations are relatively small, about 0.2 to 0.6 that of the mean.

Table 5 Difference in *CN* between cultivated agricultural lands and that of fallow/idle land (*CN* of 77, 86, 91 and 94) under various land treatment and hydrologic conditions

Landuse	Land Treatment	Hydrologic Condition	Soil			
			A	B	C	D
Row Crop	SR	P (poor)	5	5	3	3
	SR	G (good)	10	8	6	5
	CTi	P	6	6	4	4
	CTi	G	13	11	9	9
	C	P	7	7	7	6
	C	G	12	11	9	8
	CCTi	P	8	8	8	7
	CCTi	G	13	12	10	9
	CTr	P	11	12	11	12
	CTr	G	15	15	13	13
	CTrCTi	P	12	13	12	13
	CTrCTi	G	16	16	14	14
Small Grain	SR	P	12	10	7	6
	SR	G	14	11	8	7
	C	P	14	12	9	9
	C	G	16	13	10	10
	CCTi	P	15	13	10	10
	CCTi	G	17	13	10	10
	CTr	P	16	14	12	12
	CTr	G	18	16	13	13
	CTrCTi	P	17	15	13	13
	CTrCTi	G	19	17	14	14
Closed Seed	SR	P	11	9	6	5
	SR	G	19	14	10	9
	C	P	13	11	8	9
	C	G	22	17	13	11
	CTr	P	14	13	11	11
	CTr	G	26	19	15	14

SR = straight row, C = contoured, CTi = conservation tillage, CCTi = contoured and conservation tillage, CTr = contoured and terraces, CTrCTi = contoured and terraces and conservation tillage.

For a given dominant soil type, first obtain CN_T as outlined in Section 3. Check CN_T with the values in Table 4 and its dominant soil property to determine which soil class of SCS (A, B, C, D) it belongs to. The *CN* adjustment should then be based on the soil class identified. If it is difficult to pinpoint which soil class it belongs to, it may be necessary to interpolate between the adjustments obtained from two soil classes. However, in view of the uncertainty involved, the additional refinement through interpolation may not be justifiable or unfruitful. If the landuse belong to cultivated agricultural lands such as row crops, small grain and closed seeds, CN_T should be reduced by an amount ranging from 3 to 26 (Table 5), but on the average the reduction should be between 9 (soil D) to 18 (soil A) (Figure 9). In general, as expected soil A has the largest reduction while soil D has the smallest reduction in *CN*.

The corresponding reduction in *CN* for non-cultivated, pasture and forestland are larger especially for soil A (5 to 71). The average reduction for these landuse ranges from about 10 (soil D) to 40 (soil A). Compared to pasture and range lands with grass, shrubs, and forestland, plant roots of cultivated crops are generally smaller. The former tends to have more diversified vegetation, and usually with bigger roots and so bigger macropores resulted from dead roots, tree trunks, cracks, etc., which encourage more infiltration loss than the latter.

As a result, compared to cultivated agricultural lands, the non-cultivated pasture and forestland generally have a larger reduction in *CN* (Figure 9).

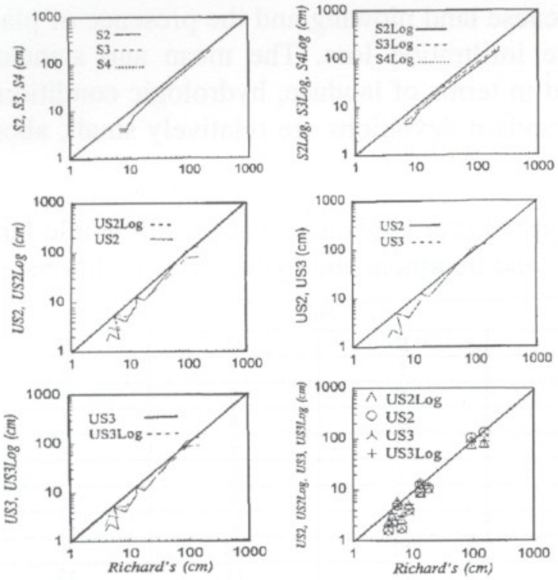

Fig. 7 Scatterplots of the cumulative infiltration (*F*) between the SCS method and the Richard's equation under the validation stage

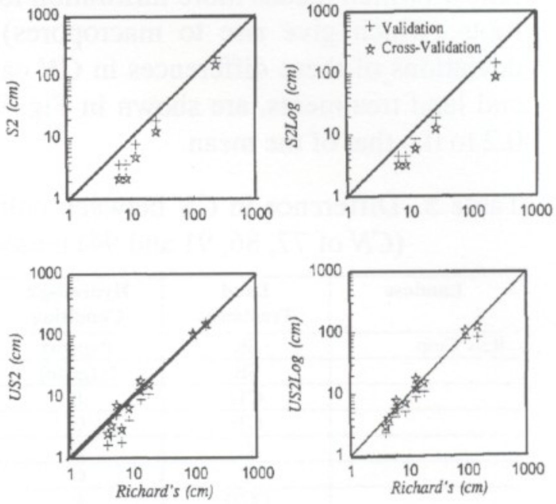

Fig. 8 Scatterplots of the cumulative infiltration (*F*) between the SCS method and the Richard's equation under validation and cross-validation runs

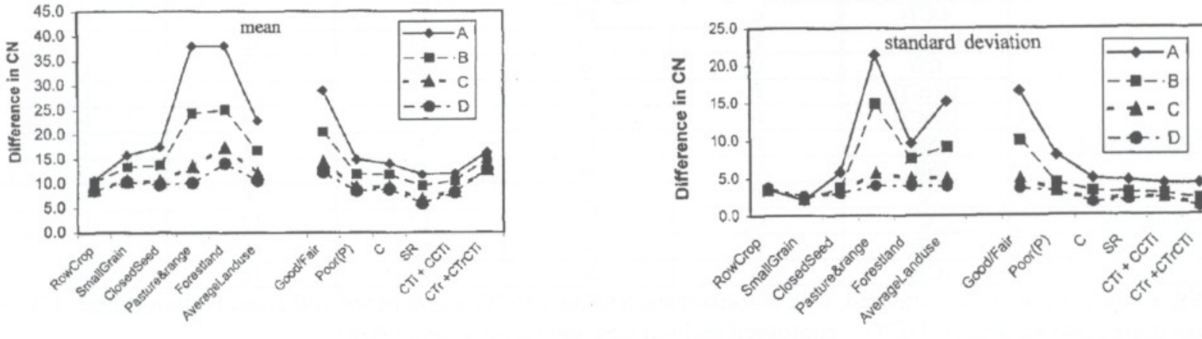

Fig. 9 The mean and standard deviation of the difference (reduction) in *CN* between various landuse (row crop, . . , forestland, landuse average), hydrologic conditions (good/fair and poor), and land treatments (Contour, . ., Contour tillage), with that of fallow/idle land under soil types A, B, C, D, and AMC-II

As expected, in terms of hydrologic conditions, lands classified as good/fair have larger reduction in *CN* (average 12 to 29) than lands of poor conditions (average 8 to 15) for we expect more infiltration loss in the former than the latter. In terms of land treatments, straight row generally incur the least reduction (average 6 to 12) while contour/terraced and contour/tillage incur the most reduction (average 13 to 17) in *CN*. Such land treatments involve plowing, turning and loosening of topsoils, which encourage more infiltration loss and hence lower *CN*.

Below is an example on how to adjust the CN_T obtained for say, loam (which more or less corresponds to soil type C), whose mean *CN* computed under unsteady rainfall is 87 (Section 4.2). For agricultural land under good condition, reduce the *CN* by about 6 to 15 (average about 10), but for cultivated agricultural land under poor condition, reduce the *CN* by about 3 to 12 (average about 7). If small grain is the landuse, straight row the land treatment and its hydrologic condition is poor, the reduction in *CN* should be about 10 (Table 5). To reflect data variability, we could further adjust the *CN* by say, ± 1 standard deviation for small grain (about ±2). Finally, the resultant *CN* (87-10 ± 2) can be further adjusted for AMC-I (or AMC-III) based on the guideline given in the SCS handbook.

SUMMARY AND CONCLUSIONS

A dimensional analysis shows that it is likely difficult to obtain a universal, dimensionless relationship between the maximum retention potential (*S*) of the *CN* method with K_s, S_o, T (time), R, F, Q, and θ_i. Next, the *CN* were derived through three models relating S to K_s, S_o, and R. The calibration of model parameters was based on the numerical solutions of Richard's 1-D model applied to columns of four homogeneous soil classes. A direct search, optimization algorithm called Simplex was used to calibrate the parameters. Results show that under ideal settings, S depends mainly on K_s, rainfall intensities and distributions, and partly on whether the calibration data used was un-transformed or log-transformed.

The average *CN* obtained under steady rainfall sand, sandy loam, loam and silt loam were 17, 55, 72 and 82. However, under unsteady rainfall, the average *CN* for these four soil types increase to 28, 75, 87 and 93 which, with the exception of sand, agree more closely with the recommended *CN* for fallow or idle land under soil types A, B, C and D. The results (and the cross-validation results) show that CN_T are directly related to soil properties and inversely related to storm intensities and durations, which also means that CN_T obtained from unsteady rainfall cases are more realistic than that of steady rainfall which are generally too heavy. Except for sand, the *CN* derived are generally within the practical range of 40 to 98. Given that *CN* is a temporally-lumped model, its F tends to be lower than that of the Richard's equation, unless the *CN* calibrated from heavier, steady rainfall events were applied to test cases using lighter, unsteady rainfall events (Section 4.6).

Even though this study is based on the numerical solutions of Richard's model, it probably shows that in practice the *CN* method (asymptotic approach of estimating F) may perform poorly when, (1) the AMC is low, or (2) the rain pulses after the I_a is satisfied are small because the *CN* method still simulates surface runoff albeit under such circumstances there should only be infiltration, or (3) the initial R is much higher than the dominant K_s which leads to Horton overland flow not accounted for by the *CN* method. Under the first two conditions, *CN* tend to under-estimate F, while under (3) it tends to over-estimate F. Lastly, since these *CN* were theoretically derived under ideal environment, some adjustments to the *CN* are recommended (Section 5) to reflect the effects of landuse, land treatment, hydrologic conditions and AMC.

Strictly speaking, because of soil heterogeneity, soil swelling and shrinkage, cracks, root growth and decay, insect activities, and other factors, all infiltration models (including the Richard's model) are inadequate in real world applications. The *CN* method is a practical approach that bypasses all complications. Future research should focus on: (1) Conduct more realistic simulations of the infiltration process, such as using 2-D, or 3-D Richard's model and soil columns whose properties either vary vertically, or horizontally, or both; (2) Instead of using sorptivity, try other provisions to incorporate AMC into *CN* explicitly; and (3) Attempt to achieve a more precise linkage between *CN* and rainfall intensities (R) or duration (T).

ACKNOWLEDGMENTS

The work was partially supported by a funding from the Natural Science and Engineering Research Council of Canada. Some of the numerical solutions of the Richard's equation were taken from the M. Eng. thesis of Mr. Roy Jayaveerasingam at the Asian Institute of Technology, Bangkok. The suggestions of J. K. Koelliker and T. J. Ward improve the quality of this paper.

REFERENCES

Brooks, R. H. and Corey, A. T. (1964). Hydraulic properties of porous media. Hydrology Paper no. 3. Colorado State University.

Buckingham, E. (1914). On physically similar systems; illustrations of the use of dimensional equations. Phys. Rev. IV. 4: 345.

Chen, C. L. (1982). An evaluation of the mathematics and physical significance of the Soil Conservation Service curve number procedure for estimating runoff volume. Proc. Int. Symp. on Rainfall-Runoff Modeling, Water Resources Publication, Littleton, Colo.: 387-418.

Chong, S. K. and Teng, T. M. (1986). Relationship between the runoff curve number and hydrologic soil properties. J. Hydrology. 84: 1-7.

Hawkins, R. H. (1984). A comparison of predicted and observed runoff curve numbers, Proc., Spec. Conf., Irrigation and Drainage Div., ASCE, New York: 702-709.

Hawkins, R. H. (1992). Variety, classification, and association in runoff response, Rep., School of Renewable Natural Resources, Univ. of Arizona, Tucson, Ariz.

Hawkins, R. H. (1993). Asymptotic determination of runoff curve numbers from data. J. Irrig. and Drain. Div., ASCE, 119: 334-345.

Hjelmfelt, A. T. Jr. (1991). Investigation of curve number procedure. J. of Hydraulic Div., ASCE, 117(6): 725-737.

Jayaveerasingam, M. R. (1990). Comparison of simplified infiltration models under idealized condition, M. Eng. thesis No. WA-90-13, Asian Institute of Technology, Bangkok, Thailand.

Linsley, R. K., Kohler, M. A. and Paulhus, J. L. H. (1982). Hydrology for Engineers. 3rd Edition, McGraw-Hill.

McCuen, R.H. (1998). Hydrologic analysis and design. 2nd Edition, Prentice Hall, New Jersey: 157-159.

Morel-Seytoux, H. J., Thomas, A. P. and Jonch-Clausen, T. (1977). Computation of infiltration for unsteady uninterrupted high rainfall. J. of Hydrology. 35: 221-234.

Nash, J. E. and Sutcliffe, J. V. (1970). River flow forecasting through conceptual models; Part 1 - A discussion of principles. J. of Hydrology. 10(3): 282-290.

Nelder J. A. and Mead R. (1965). A simplex method for functional minimization. Computer Journal. 9: 308 -313.

Parlange, J. Y. (1972). Analytical theory of water movement in soils. in Joint Symposium on Fundamentals of Transport Phenomena in Porous Media, Univ. Guelph, Guelph, Ontario, Canada.

Pierson, F. B., Hawkins, R. H., Cooley, K. R. and Vactor, S. S. Van (1995). Experiences in estimating curve numbers from rainfall simulation data. Proceedings of the Symposium on watershed management - planning for the 21st Century, ASCE, San Antonio, Texas.

Ponce, V. M. and Hawkins, R. H. (1996). Runoff curve number: Has it reached maturity? J. Hydrologic Engr., ASCE. 1(1): 11-19.

Rawls, W. J., Brakensiek, D. L. and Miller, N. (1983). Green-Ampt infiltration parameters from soils data. J. Hydraulic Div., ASCE. 109(1): 62-70.

Richards, L. A. (1931). Capillary conduction through porous mediums. Physics. 1: 313-318.

SCS (1963). National Engineering Handbook, " Section 4: Hydrology.", Soil Conservation Service, USDA, Washington, D.C.

Smith, R. E. and Eggert, K. G. (1978). Discussion of infiltration formula on SCS curve number. J. Irrigation and Drainage Div., ASCE. 104: 462 - 464.

Ungs, M. J., Boersma, L. and Akratanakul, S. (1985). Or-nature: The numerical analysis of transport of water and solutes through soil and plants, Vol. 1. Theoretical Basis, Special Report 753, Agricultural Experiment Station, Oregon State University, Corvallis.

Van Mullem, J. A. (1989). Runoff and peak discharges using Green-Ampt infiltration model. J. Hydraulic Div., ASCE. 117(3): 354 - 370.