

## ANALYSIS OF SOFT GROUND REINFORCED BY NON-HOMOGENEOUS GRANULAR PILE-MAT SYSTEM

Jagdish T. Shahu<sup>1</sup>, Shigenori Hayashi<sup>2</sup> and Madhira R. Madhav<sup>3</sup>

**ABSTRACT:** Due to differences in compaction characteristics and overburden stresses, modulus of the deformation of granular piles shows variation with depth and the granular piles may be considered as non-homogeneous. A simple theoretical approach to analyze soft ground reinforced by non-homogeneous granular piles with granular mat on top is presented. The proposed model is validated by comparison with other numerical models and field test results.

A non-homogeneous granular pile is found to be more beneficial than the homogeneous one. For the case of non-homogeneous granular piles, due to increase in granular pile modulus with depth, shear stresses between the granular pile and the surrounding soft soil and the total settlement reduce by a significant amount. The variation of stress concentration factor with depth tends to become more uniform as the rate of variation of granular pile stiffness with depth increases. The reduction in shear stresses due to non-homogeneity of granular pile is found to be very high at low values of area ratio. This is highly desirable for stabilization of soft soils having low shear strength.

### INTRODUCTION

Granular piles or stone columns are frequently used for stabilization of soft soil and construction of infrastructure facilities in coastal and lowland areas. For low-rise buildings and structures such as liquid storage tanks, abutments, embankments, etc., that can tolerate some settlement, granular piles provide the most popular method of support owing to their cost-effectiveness and easy installation. Granular piles increase stiffness of the ground, reduce settlement, facilitate consolidation of the soft ground, and minimize the likelihood of liquefaction due to earthquake. Bergado et al. (1996) present a state of art review of granular pile analysis, design and installation.

Several theories for the analysis of granular pile reinforced ground have been proposed by different researchers considering different failure modes, load sharing mechanisms, and so forth (e.g., Greenwood 1970; Madhav and Vitkar 1978; Balaam and Booker 1981; Schweiger and Pande 1986; Canetta and Nova 1989; Alamgir et al. 1993). However, most of these approaches are based on the evaluation of settlement behaviour of the granular pile and the surrounding soft soil, assuming homogeneous material properties for the granular pile. Since, in real practice, the material properties vary significantly with depth owing to differences in overburden stresses, compaction procedures at top and bottom portions, etc., these approaches may not be able to predict the actual behaviour of the granular pile reinforced ground accurately. Also it is common knowledge that bulging and subsequent failure of granular piles occur mainly due to high stress concentration near top of the granular pile (within few diameters). Since stresses in the granular

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1 Lecturer, Department of Civil Engineering, Visvesvaraya Regional College of Engineering, Nagpur 440 011, INDIA.

2 Professor, Institute of Lowland Technology, Saga University, 1 Honjo, Saga 840-8502, JAPAN.

3 Professor, Department of Civil Engineering, Indian Institute of Technology, Kanpur 208016, INDIA.

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pile and the surrounding soft soil are significantly influenced by the material properties of the granular pile, it is rather important to evaluate the influence of non-homogeneity of the granular pile on these stresses.

Alamgir et al. (1993) have proposed a simple approach to evaluate shear stresses at granular pile/soil interface based on the unit cell concept. In this paper, the one dimensional (mechanistic) model has been extended to analyze stresses and settlements of soft soil reinforced with non-homogeneous granular piles with granular mat on top. The model is validated by comparisons with other numerical models and measured field test results. The effect of non-homogeneity of the granular pile on treated ground responses is evaluated by comparing predicted results for the case of homogeneous and non-homogeneous granular piles. The comparison shows that stresses transferred to the granular pile and the soil, shear stresses at granular pile/soil interface and stress concentration factor for the homogeneous case are significantly different than those for the non-homogeneous case. The response of non-homogeneous granular pile treated ground is found to be more beneficial than that of homogeneous granular pile treated ground. Thus, more economical designs can be achieved by using actual non-homogeneous material properties of granular piles. A detailed parametric study is also carried out using this model to evaluate the relative influence of various parameters on the effect of non-homogeneity of granular pile on treated ground responses.

## FORMULATION AND SOLUTION

The reinforced ground and the unit cell representing the zone of influence of a granular pile are shown in Figs. 1(a) and (b). Given the total area of the reinforced ground ( $L \times B$ ) and the number of granular piles  $m$ , the equivalent diameter of the unit cell can be calculated as

$$d_e = \sqrt{\frac{4LB}{m\pi}} \quad (1)$$

The details of the unit cell are given in Fig. 2. The granular mat is assumed as a rigid smooth layer through which the uniform load is applied. Since granular mat in practice is likely to be flexible or semi-rigid, this assumption constitutes a shortcoming of the theory presented. The unit cell is discretized into  $n$  number of elements. For any given element ' $i$ ', the average stresses at the midheight in the granular pile and the soft soil are  $q_{gpi}$  and  $q_{si}$  respectively. Equilibrium of vertical forces at any depth inside the unit cell can be expressed as

$$q_0 = q_{gpi}A_r + q_{si}(1 - A_r) \quad (2)$$

where  $q_0$  is applied stress on unit cell, and  $A_r$  is area ratio equal to  $(d/d_e)^2$  with  $d$  and  $d_e$  being the radii of the granular pile and the unit cell, respectively.

It is assumed that the modulus of deformation for the granular pile varies linearly with depth. The modulus of deformation at the top of the pile is  $E_{gp}$  and it increases with depth at a constant rate,  $\alpha$ . Thus the modulus of deformation for any element of the granular pile,  $E_{gpi}$  can be expressed as

$$E_{gpi} = E_{gp} \left( 1 + \alpha \frac{z_i}{H} \right) \quad (3)$$



For convenience, the above parameters are normalized in the following way:

$$q_0^* = q_0 / \sigma'_{av}; \quad q_{si}^* = q_{si} / \sigma'_{av}; \quad q_{gpi}^* = q_{gpi} / \sigma'_{av};$$

$$\sigma_{i0}^* = \sigma_0 / \sigma'_{av}; \quad E_{gp}^* = E_{gp} / \sigma'_{av}; \quad \text{and} \quad z_i^* = z_i / H;$$

where  $\sigma'_{av}$  is average initial stress at the mid height of the soil layer and is equal to  $(\gamma_{sub} H/2)$ , and  $\gamma_{sub}$  is submerged unit weight of the soil. Using the above normalized parameters, Eqs. (2) and (6) may be rewritten as

$$q_0^* = q_{gpi}^* A_r + q_{si}^* (1 - A_r) \quad (7)$$

and

$$q_{gpi}^* = R_s (1 + \alpha z_i^*) \cdot \ln \left( 1 + \frac{q_{si}^*}{\sigma_{0i}^*} \right) \quad (8)$$

where

$$R_s = 0.434 \frac{C_c}{(1 + e_0)} \frac{E_{gp}}{\sigma'_{av}} \quad (9)$$

The effect of the granular pad or mat is taken into consideration by taking the value of  $\sigma_{0i}^*$  as given below:

$$\sigma_{0i}^* = \frac{\gamma_f h_f + \gamma_{sub} z_i}{(\gamma_{sub} H / 2)} \quad (10)$$

where  $\gamma_f$  is the unit weight of the material of granular mat, and  $h_f$  is the thickness of granular mat. Eq. (10) may be further simplified as follows:

$$\sigma_{0i}^* = 2z_i^* + f_s \quad (11)$$

where  $f_s$  is normalized surcharge due to the granular mat and is equal to  $(\gamma_f h_f / \sigma'_{av})$ .

For any element  $i$ , Eqs. (7) and (8) may be solved iteratively to evaluate the value of  $q_{si}^*$  and  $q_{gpi}^*$  for applied load  $q_0^*$ . The results are obtained in form of  $q'_{si}$  and  $q'_{gpi}$  where

$$q'_{si} = q_{si}^* / q_0^* = q_{si} / q_0$$

and

$$q'_{gpi} = q_{gpi}^* / q_0^* = q_{gpi} / q_0$$

The above procedure is repeated and  $q'_{si}$  and  $q'_{gpi}$  are obtained for all the elements from 1 to  $n$ .

The shear stress mobilized at the soil-granular pile interface,  $\tau'_i$  (Fig. 2) is obtained as:

$$\tau'_i = \frac{(d/H)}{4(\Delta h_i/H)} \{q'_{gpi} - q'_{gpi+1}\} \quad (12)$$

where  $q'_{gpi+1}$  is normalized granular pile stress at the center of the element  $(i+1)$ . If the unit cell is discretized into  $n$  number of equal elements, Eq. (12) reduces to

$$\tau'_i = \frac{n}{4(D_r)} \{q'_{gpi} - q'_{gpi+1}\} \quad (13)$$

where  $D_r$  is depth ratio and is equal to  $(H/d)$ . The shear stress for  $n$ th element is evaluated based on the assumption that the difference in the shear stress between element  $(n-2)$  and element  $(n-1)$  and that between element  $(n-1)$  and  $n$  remains the same.

$$\tau'_n = \tau'_{n-1} - (\tau'_{n-2} - \tau'_{n-1}) \quad (14)$$

As the number of element  $n$  increases and  $\Delta h_i$  decreases, it is expected that the error involved due to this assumption would be insignificant.

The displacement of  $i$ th element,  $\Delta S_i$  (same as  $\Delta S_{gpi}$  or  $\Delta S_{si}$ ) may be obtained from Eq. (5) which may be rewritten as:

$$\frac{\Delta S_i}{H} = \frac{1}{C_1} \frac{\Delta h_i}{H} \ln\left(1 + \frac{q_{si}}{\sigma_{oi}}\right) \quad (15)$$

with

$$C_1 = \frac{(1 + e_0)}{0.434C_c} \quad (16)$$

where  $C_1$  is soil stiffness factor. Total normalized displacement,  $\delta_i/H$  for element  $i$ , is then obtained by adding the displacements of all the elements up to that level as:

$$\frac{\delta_i}{H} = \sum_i^n \frac{\Delta S_i}{H} \quad (17)$$

Stress concentration factor for any given element ' $i$ ' is evaluated as:

$$(SCF)_i = \frac{q_{gpi}}{q_{si}} \quad (18)$$

The total settlement of the untreated ground,  $S_0$  is obtained by using the conventional  $e$ - $\log(p)$  relationship and dividing the ground into the same number ( $n$ ) of elements. Settlement reduction factor,  $\mu$  may then be evaluated as:

$$\mu = \frac{S_t}{S_0} \quad (19)$$

where  $S_t$  is the total settlement of the granular pile reinforced ground and also the same as the total normalized displacement for element 1 ( $\delta_1$ ).

## RESULTS AND DISCUSSION

The response of soft ground reinforced by non-homogeneous granular pile with granular mat on top is presented as a set of representative nominal values of different input parameters of the reinforced ground (Table 1). The response has been evaluated for four different variations of modulus of deformation of the granular pile with depth (i.e.,  $\alpha = 0, 0.5, 1.0$  and  $2.0$ ). Next, a detailed parametric study is carried out to evaluate the relative influence of each parameter on this response. The unit cell was divided into 20 elements because shear stresses converge only for number of elements,  $n$  becoming equal to or greater than 20.

Table 1 Nominal value of input parameters

Name of the Parameter	Nominal value
Normalized surcharge due to granular mat $f_s$	0.50
Relative stiffness $R_s$	20
Area ratio $A_r$	0.25
Normalized applied stress $q_0^*$	2.00
Number of elements $n$	20
Depth ratio $D_r$	10
Soil stiffness factor $C_1$	7.68

## EFFECT OF NON-HOMOGENEITY OF GRANULAR PILE ON TREATED GROUND RESPONSES

The effect of variation of granular pile modulus on treated ground responses is depicted in Figs. 3 to 7. As the rate of variation of the modulus of deformation of the granular pile with depth,  $\alpha$  increases, the pile section of any given element becomes stiffer. Hence, at any given depth, the granular pile stress increases and hence, the soil stress decreases with increasing  $\alpha$  (Figs. 3 and 4). Moreover, as the depth increases, the modulus of deformation of the granular pile,  $E_{gp}$  increases, and hence the granular pile becomes progressively stiffer and the percentage of the total load supported by the pile section increasing. Consequently, the distribution of granular pile stress and hence soil stress becomes uniform with depth (Figs. 3 and 4). The higher the  $\alpha$  value, the more uniform the stress distributions. As a result, the distribution of stress concentration factor with depth also becomes more uniform with increasing  $\alpha$  (Fig. 5). Fig. 5 shows that at  $z/H$  of 0.025, the stress concentration factor for  $\alpha$  equal to 2.0 is slightly higher (about 6%) than that for the homogeneous case. However, it may be noted that the maximum value of stress concentration factor at the top will remain the same in both the cases because the modulus of deformation of the granular pile at  $z/H$  equal to zero remains the same for both the cases.

As  $\alpha$  increases, the treated ground becomes stiffer and the displacement at any given depth

decreases significantly (Fig. 6). Normalized shear stress at soil-granular pile interface results due to non-uniform distribution of granular pile stress or soil stress with depth (Eq. 13). As  $\alpha$  increases, the distribution of granular pile or soil stress tends to become more uniform with depth. Hence, the normalized shear stress at any given depth decreases with increasing  $\alpha$  (Fig. 7). Thus, a non-homogeneous granular pile may be considered as more beneficial than the homogeneous one.

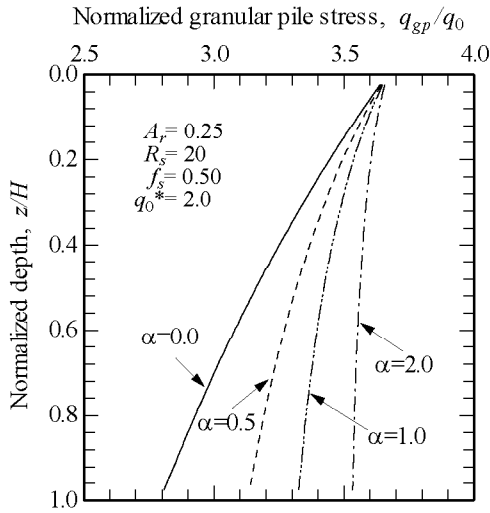


Fig. 3 Effect of non-homogeneity of granular pile on normalized granular pile stress

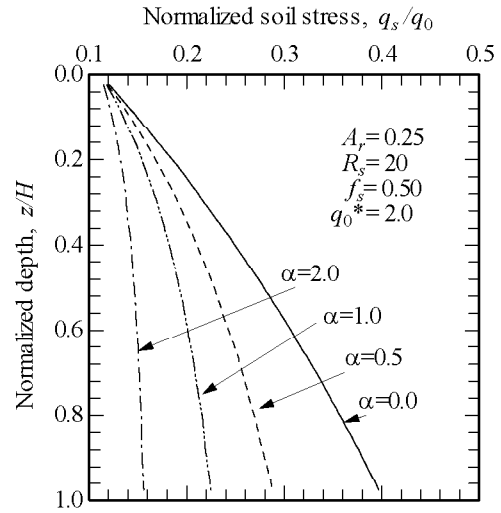


Fig. 4 Effect of non-homogeneity of granular pile on normalized soil stress

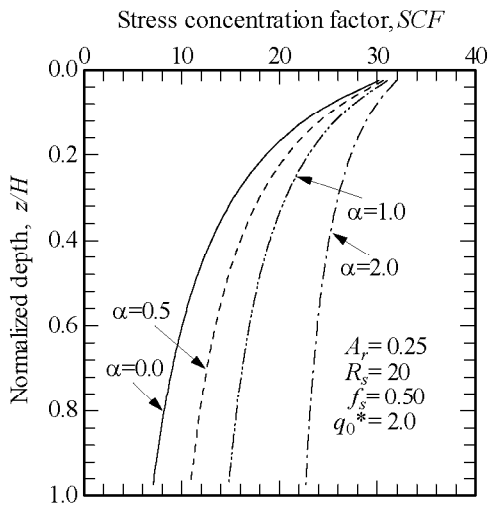


Fig. 5 Effect of non-homogeneity of granular pile on stress concentration factor

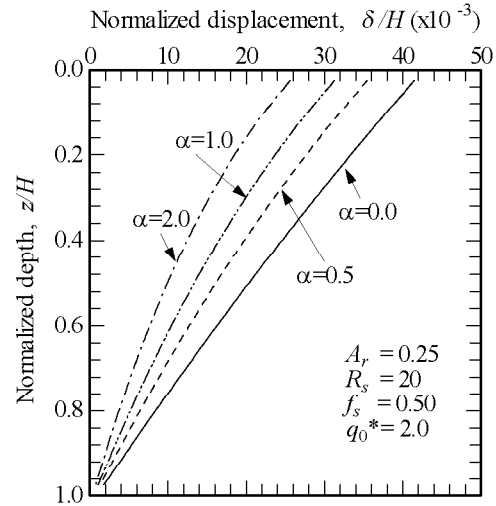


Fig. 6 Effect of non-homogeneity of granular pile on normalized displacement

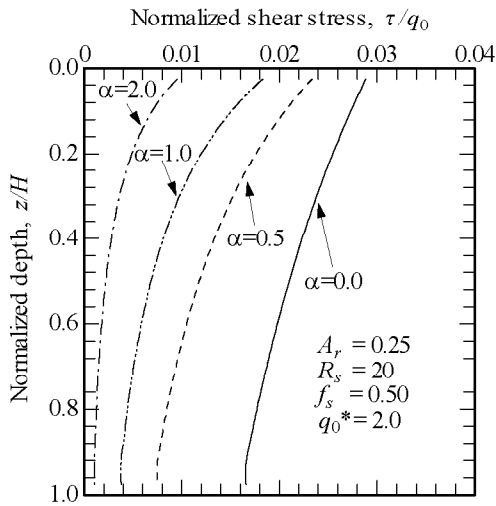


Fig. 7 Effect of non-homogeneity of granular pile on normalized shear stress

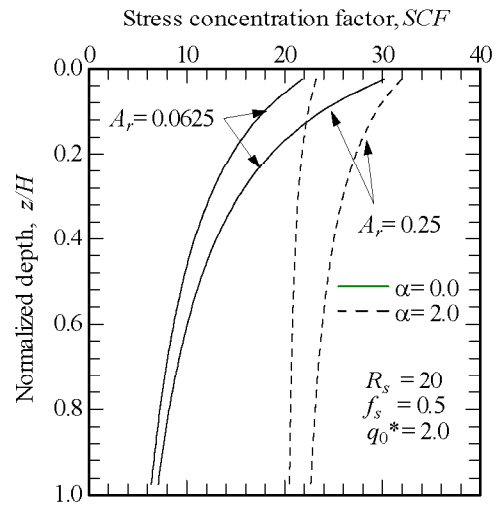


Fig. 8 Effect of area ratio on stress concentration factor for non-homogeneous granular pile

#### THE RELATIVE INFLUENCE OF VARIOUS PARAMETERS

The relative influence of various parameters on the response of soft ground treated with non-homogeneous granular pile is presented in Figs. 8 to 12. The effect of area ratio,  $A_r$  on stress concentration factor, shear stress at soil-granular interface, settlement and percentage load carried by granular pile at the top and the bottom is brought out in Figs. 8-11 respectively. As  $A_r$  increases, the area of the granular pile increases and the unit cell becomes stiffer. Hence, at any given depth, the granular pile stress increases and the soil stress decreases. This results in increase in the stress concentration factor (Eq. 18) at any given depth with increasing  $A_r$  (Fig. 8). The stress concentration factor increases with increasing  $\alpha$ . The higher the  $A_r$ , the higher the granular pile area and hence the higher the increase in the stress concentration factor with increasing  $\alpha$ .

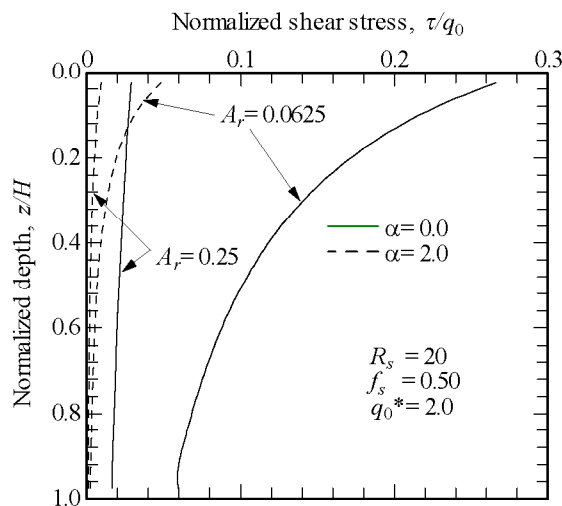


Fig. 9 Effect of area ratio on normalized shear stress for non-homogeneous granular pile



Normalized shear stress at soil-granular pile interface depends on the difference of the granular pile stresses (or soil stresses) at two consecutive elements (Eq. 13). A reduction in the normalized shear stress merely indicates a more uniform granular pile stress distribution with depth. The reduction in the normalized shear stress due to increasing  $\alpha$  is very high at low  $A_r$  as compared to that at high  $A_r$  (Fig. 9). At low  $A_r$ , a highly non-uniform granular pile stress distribution becomes uniform with increasing  $\alpha$  and this results in a significant reduction in the normalized shear stress. The low value of the normalized shear stress is highly desirable during stabilization of soft soils having low strength. At high  $A_r$ , even for the homogeneous case, the distribution of granular pile stress is uniform and the normalized shear stress is low. Hence, at high  $A_r$ , the effect of  $\alpha$  on the normalized shear stress is not significant.

$R_s$  is a measure of the relative stiffness of granular pile to soft soil (Eq. 9). As  $R_s$  or  $A_r$  increases, the unit cell becomes stiffer and the total settlement decreases. The percentile reduction in the total settlement with increasing  $\alpha$  increases with increasing  $A_r$  or  $R_s$  (Fig. 10). For example, the reduction in the settlement for the non-homogeneous case ( $\alpha = 2.0$ ) at  $A_r$  of 0.0625 is 35 % for  $R_s$  equal to 100 as compared to the corresponding value of 17 % for  $R_s$  equal to 20. On the other hand, the reduction in the settlement at  $A_r$  of 0.49 is 43 % for  $R_s$  equal to 100 as compared to the corresponding value of 40 % for  $R_s$  equal to 20. The higher the  $A_r$ , the higher the area of the granular pile and the higher the effect of  $\alpha$ . Also, the higher the  $R_s$ , the higher the stiffness of the granular pile and the higher the effect of  $\alpha$ .

As  $A_r$  increases, the granular pile becomes stiffer and hence percentage load carried by granular pile increases (Fig. 11). For the top element, this increase in the percentage load is not significant because the change in the granular pile modulus due to increasing  $\alpha$  is negligible for the top element. However, for the bottom element, the effect of  $\alpha$  is significant because the granular pile modulus at the bottom increases approximately three-fold for  $\alpha$  equal to 2.0.

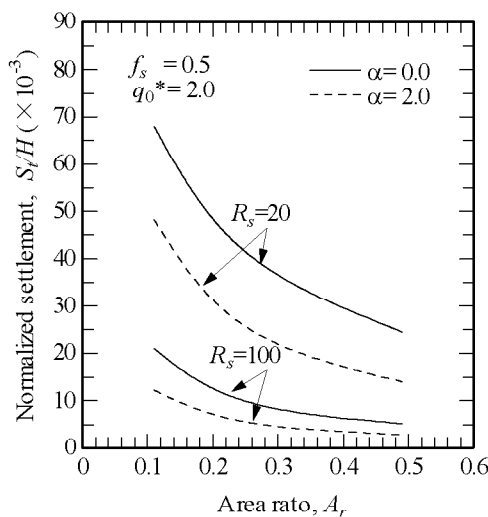


Fig. 10 Effect of area ratio and relative stiffness on settlement for non-homogeneous granular pile

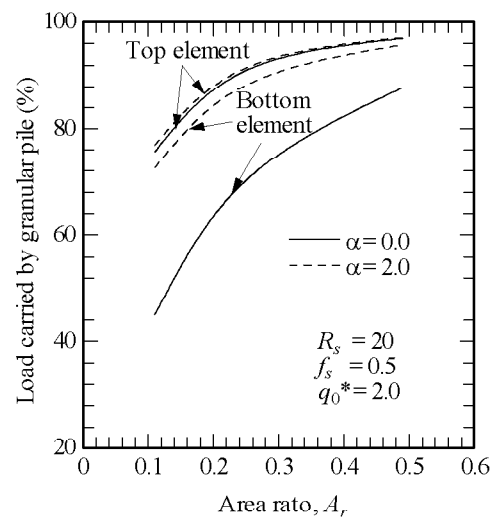


Fig. 11 Effect of area ratio on percentage load carried by non-homogeneous granular pile

As normalized surcharge due to the granular mat,  $f_s$  increases, the stress concentration factor at any given depth decreases (Fig. 12). This is because as  $f_s$  increases, overburden stress increases

and hence applied stress increases. The effect of the increase in applied stress is equivalent to increasing the stiffness of soil (owing to  $e$ - $\log p$  relationship, also evident from Eq. 5). Due to the increase in soil stiffness, the soil stress increases and the granular pile stress decreases at any given depth. Hence, the stress concentration factor, i.e. the ratio of granular pile stress to soil stress, decreases with increasing  $f_s$ . The other effect of increasing  $f_s$  is to make the distribution of stress concentration factor uniform with depth by reducing a very high stress concentration factor at the top by a significant amount (Fig. 12). As  $f_s$  increases, the percentile increase in stress concentration factor due to increasing  $\alpha$  shows a little decrease. In fact, this decrease is so small that it may not be evident in Fig. 12. However, on the basis of the actual values, it is observed that for increase in  $f_s$  from 0.05 to 0.5, the percentile increase in stress concentration factor for the non-homogeneous case ( $\alpha = 2.0$ ) for the top element decreases from 6.03 % to 5.87 %, whereas for the bottom element, the corresponding decrease is from 222.82 % to 220.65 %. The higher the  $f_s$ , the higher the soil stiffness and the lesser the effect of  $\alpha$  on stress concentration factor.

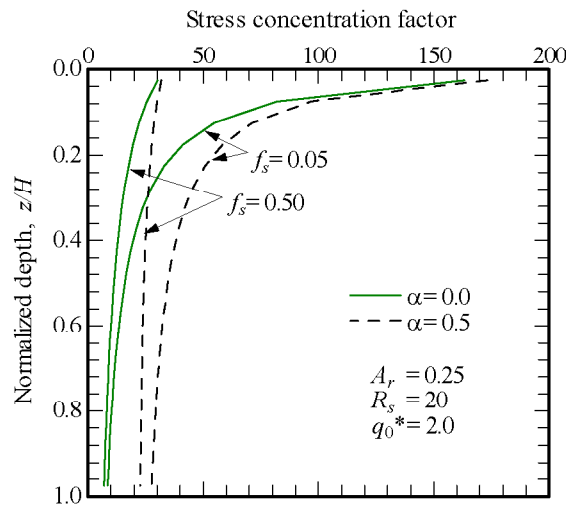


Fig. 12 Effect of granular mat on stress concentration factor for non-homogeneous granular pile

## MODEL VALIDATION

Figure 13 compares the predictions of the normalized granular pile stress at the midheight from the present analysis with published results available from other sources. Field test results by Bergado and Lam (1987) with  $E_{gp}/E_s$  of 16.67 compare well with the predictions by the present model with a relative stiffness ratio of 10. For high  $A_r$  values, the comparison is better with the predicted results for the case of homogeneous granular pile ( $\alpha = 0$ ). However, for low  $A_r$  values, the predicted results show a better comparison with the case of non-homogeneous granular pile ( $\alpha = 0.5$ ). Also shown in this figure are the results by Alamgir et al. (1994) for  $E_{gp}/E_s$  of 10. The present model appears to predict granular pile stresses better than those given by Alamgir et al. (1994).

Figure 14 compares the predicted relationship between settlement reduction factor and area ratio with some published field test results and Alamgir et al. (1994) model. The present model with  $R_s$  of 25 predicts the results comparable to Alamgir et al. (1994) model with  $E_{gp}/E_s$  of 15. Considering variations in relative stiffness and  $\alpha$  for different field test results shown in Fig. 14,

the range of values predicted by  $\alpha$  equal to zero and  $\alpha$  equal to 1.0 seems to cover most of the field test results quite well (e.g., Watt et al. 1967; Castelli et al. 1983; Baumann and Bauer 1974).. Notable exceptions are Venmans (1990) and Greenwood (1970) who record a higher value of settlement reduction factor.

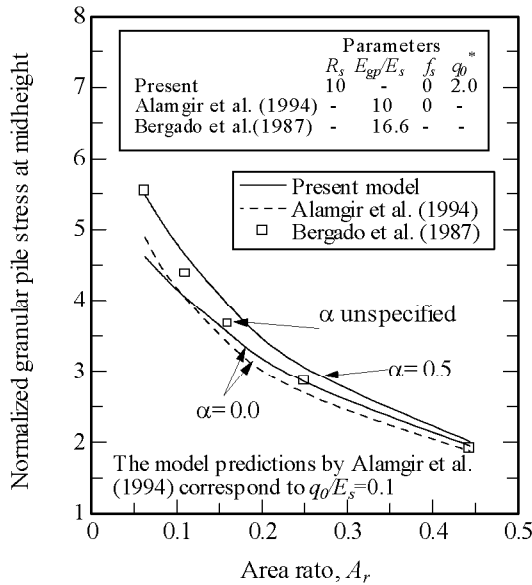


Fig. 13 Comparison of granular pile stresses at midheight with other published model and field test results

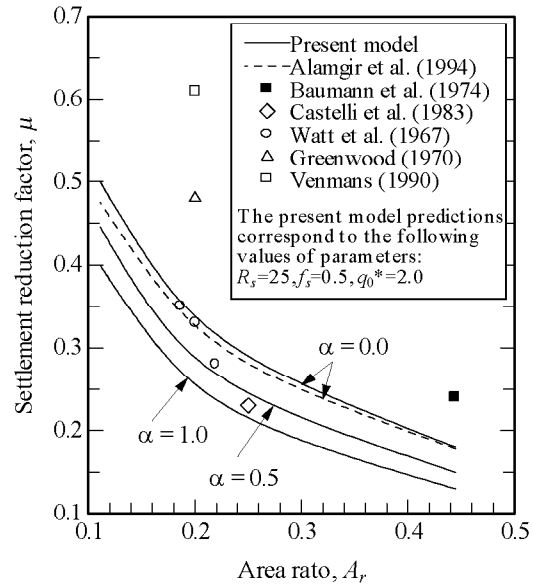


Fig. 14 Comparison of settlement reduction factor with other published model and field test results

SUMMARY AND CONCLUSIONS

A simple analysis to predict the deformation behaviour of uniformly loaded soft ground reinforced by non-homogeneous granular pile with granular mat on top is presented. The response of the treated ground is evaluated for four different variations of modulus of the deformation of granular pile with depth. A parametric study is also carried out to evaluate the relative influence of various parameters on this response. The following conclusions are drawn from this study:

- 1) As the rate of increase of granular pile modulus with depth ( $\alpha$ ) increases, the shear stress between the granular pile and the surrounding soft soil and the total settlement reduce by a significant amount. The variation of stress concentration factor with depth also tends to become uniform with increasing  $\alpha$ . Thus, a non-homogeneous granular pile may be considered to be more beneficial than the homogeneous one.
- 2) The higher the area ratio ( $A_r$ ), the higher the percentile reduction in the total settlement with increasing  $\alpha$ . The stress concentration factor at any given depth increases with increasing  $\alpha$ . The higher the area ratio ( $A_r$ ), the higher the increase in the stress concentration factor. The reduction in the shear stress between the granular pile and the soft soil due to increasing  $\alpha$  is very high at low  $A_r$  values as compared to that at high  $A_r$  values. This is highly desirable for stabilization of soft soils having low shear strength.
- 3) The higher the relative stiffness of the granular pile to the soft soil ( $R_s$ ), the higher the percentile reduction in the total settlement due to increasing  $\alpha$ .

4) The percentile increase in the stress concentration factor due to increasing  $\alpha$  shows a slight decrease with increasing  $f_s$  values.

The model presented herein predicts reinforced ground responses very similar to broad trends observed in the field and predictions by other numerical methods.

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