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ANALYSIS OF CULVERT APPROACHES WITH PILES OF VARYING LENGTH

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ABSTRACT: For the construction of culverts and their approaches on soft and highly compressible soils, an innovative approach is to provide piles with length decreasing with distance from the culvert. The equivalent stiffness of the piled strip as a function of the relative length of the piles estimated from Brown and Wiesner (1976), is bounded by linear and exponential variations with distance. An extended Pasternak type model is proposed for the culvert approaches with piles of varying length. The response of the system is shown to be governed by the relative stiffnesses of the granular bed, the culvert foundation, the approaches at the near and far ends and the relative pile length to diameter ratio. The settlement profiles are presented for the typical values of the above parameters. The relative stiffness of the granular pad has a significant effect on settlements and on the loads transferred to the culvert foundations.

INTRODUCTION

Construction on soft soils and reclaimed ground offers one of the most challenging jobs for the geotechnical engineers. Highway and embankments across water channels pose some very unique challenges. While the structure for the cross drainage has to be built so as not to suffer any significant settlement, the approaches have to be designed to provide a transition from the response of normal untreated ground to that of the structure which hardly settles. The foundation has to be provided so that it settles according to a desired profile such that vehicles using the highway do not encounter sharp bumps. That is, certain differential settlement is permissible provided its variation with the distance is commensurate with the function of the pavement.

The Column Approach, a recent innovation to obviate this problem, is to provide the culvert approaches with piles of gradually varying length in Fig. 1. The lengths of the piles decrease with the distance from the culvert. Poorooshasb (1997) presents preliminary results based on an extension of the unit cell concept (Poorooshasb et al. 1997) in which the depth of the unit cell is varied according to the depth of the piles along the culvert approach. The settlements varied from about 35 mm close to the culvert with piles of 12 m length to about 300 mm at a distance of 19 m from the culvert where the piles were only 1 m long. The concept of homogenisation of piled rafts has been investigated by Randolph (1994). In this method, the pile group is replaced by an equivalent pier of diameter, d_{eq}, and equivalent deformation modulus, E_{eq}, as:

$$d_{eq} = 1.13\sqrt{A_g} \tag{1}$$

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and $$E_{eq} = E_s + (E_p - E_s)(A_p / A_g) \qquad (2)$$

where A_g and A_p are the total area of the pile group and of the total cross sectional area of the piles, and E_s and E_p are the moduli of deformation of the soil and of the piles respectively.

In this paper, the approaches with the road base, the sub-base and piles of varying length are modelled as an extended Pasternak model with variable stiffness for the springs. The base and the sub-base of the pavement are assumed to be incompressible but to deform in shear. The approach is divided into a finite number of strips each strip being supported by piles of different length. The response of the piled strips is evaluated by the continuum approach and their settlement responses obtained. Corresponding to strips with piles of decreasing length with distance from the culvert, the spring stiffness decreases with the distance from the culvert supported on end bearing piles. Linear and exponential decreases of the equivalent spring stiffnesses with the distance, are considered.

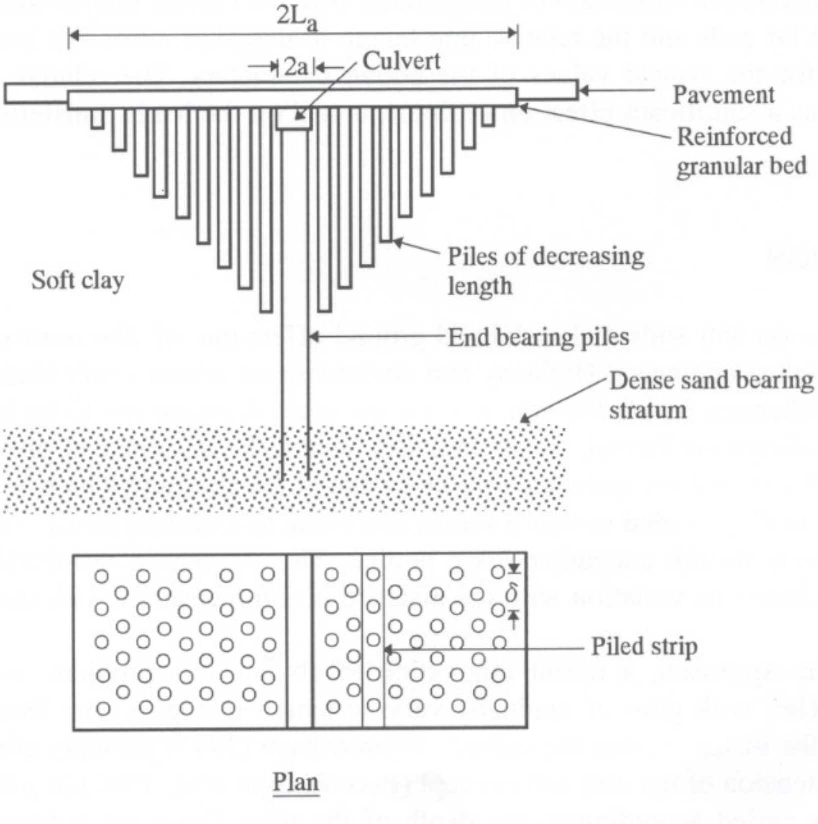

Fig. 1 Culvert on end bearing piles and approaches with piles of varying length

FORMULATION

Figure 1 depicts a culvert founded on end bearing piles with approaches on either side supported by floating piles with their lengths decreasing with the distance from the culvert. The length of the approach on each side is L_a and the floating piles are arranged in a triangular pattern with a spacing of S. For the purpose of analysis, the approaches are divided into piled-strips in Fig. 2 of width, B (= 0.866S), and length, L.

Using the boundary element method, Brown and Weisner (1975) obtain solutions for the piled-strip uniformly loaded with a stress of intensity, q, in Fig. 2, in terms of the relative stiffness of the strip, K_{st} [$=16\ E_{st}I_{st}\ (1-\nu_s^2)/(\pi E_s L^2)$], the relative pile-soil stiffness, K_p ($= E_p/E_s$), the ratios L/B and L_p/d, where $E_{st}I_{st}$ is the flexural stiffness, E_{st} - the modulus of elasticity and I_{st} - the moment of inertia of the strip, E_s and ν_s - the modulus of deformation and Poisson's ratio of the soil, L and B the length and width of the strip, and L_p and d - the length and the diameter of the piles, respectively. The variation of the maximum strip displacement with the relative stiffness, K_{st}, is presented in Fig. 3 for relative pile stiffness ratios of 100, 1,000 and 10,000 and various numbers of piles for $B/d = 5$ and $L/d = 50$. The displacements are accurate within 4% error. The results presented can be corrected for the other B/d values. Thus from Fig. 3 and the various corrections for the lengths of the piles and the widths of the strip, the settlement, S_S, and the equivalent modulus of subgrade reaction, k_{st}, ($= q/S_S$) of a uniformly loaded piled strip are estimated.

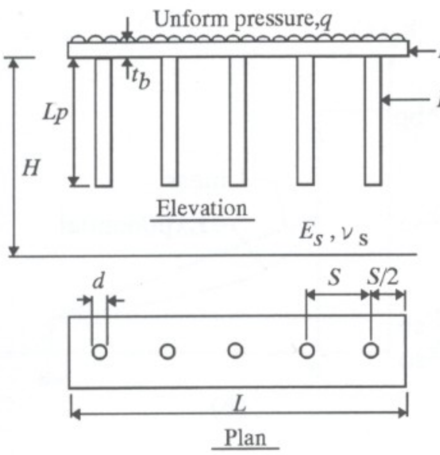

Fig. 2 Pile strip foundation

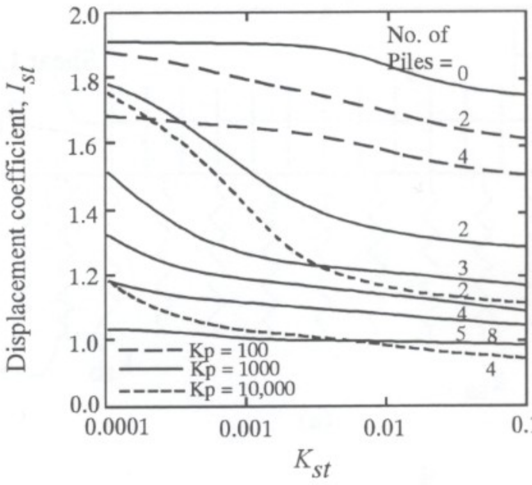

Fig. 3 Displacement influence coefficients for piles strip (after Brown and Weisner 1975)

MODELLING OF THE CULVERT APPROACHES

Having obtained settlement or the equivalent modulus of subgrade reaction of the piled-strip by the method described above, the approaches are modelled as shown in Fig. 4(a). The model consists of a shear layer representing the base course, the sub-base and the wearing coat of the pavement while the Winkler springs with the modulus, k_{st}, decreasing with the distance, x, from the culvert, represent the effect of the piled foundation. The equivalent modulus of subgrade reaction of the culvert with the end bearing piles is k_{bp}, while that of the approaches decreases from a value of k_{sa} at a distance 'a' corresponding to the strip with the longest piles to k_{sb} at a distance L_a from the centre of the culvert. Two forms of variations in Fig. 4(b), linear and exponential, of the modulus with the distance are considered. For the modulus decreasing linearly with the distance,

$$k_s(x) = k_{sa} - (k_{sa} - k_{sb})(x-a)(L_a - a) \tag{3a}$$

or

$$k_s(x) = k_{s0}\{R_k - (R_k - R_{kb})(x-a)/(L_a - a)\} \tag{3b}$$

where $k_s(x)$, k_{sa} and k_{sb}, are the equivalent moduli of the foundation at distances x, 'a' and L_a respectively, $R_k = k_{sa}/k_{s0}$, $R_{kb} = k_{sb}/k_{s0}$ and k_{s0} is the modulus of subgrade reaction of the soft soil.

For the modulus of the approaches decreasing exponentially with the distance, x, the corresponding relations are derived as:

$$k_s(x) = k_{sa} e^{-(\ln k_{sa}/k_{sb})(x-a)(L_a-a)} \tag{4a}$$

or

$$k_s(x) = k_{s0}\left\{R_k e^{-(\ln k_{sa}/k_{sb})(x-a)(L_a-a)}\right\} \tag{4b}$$

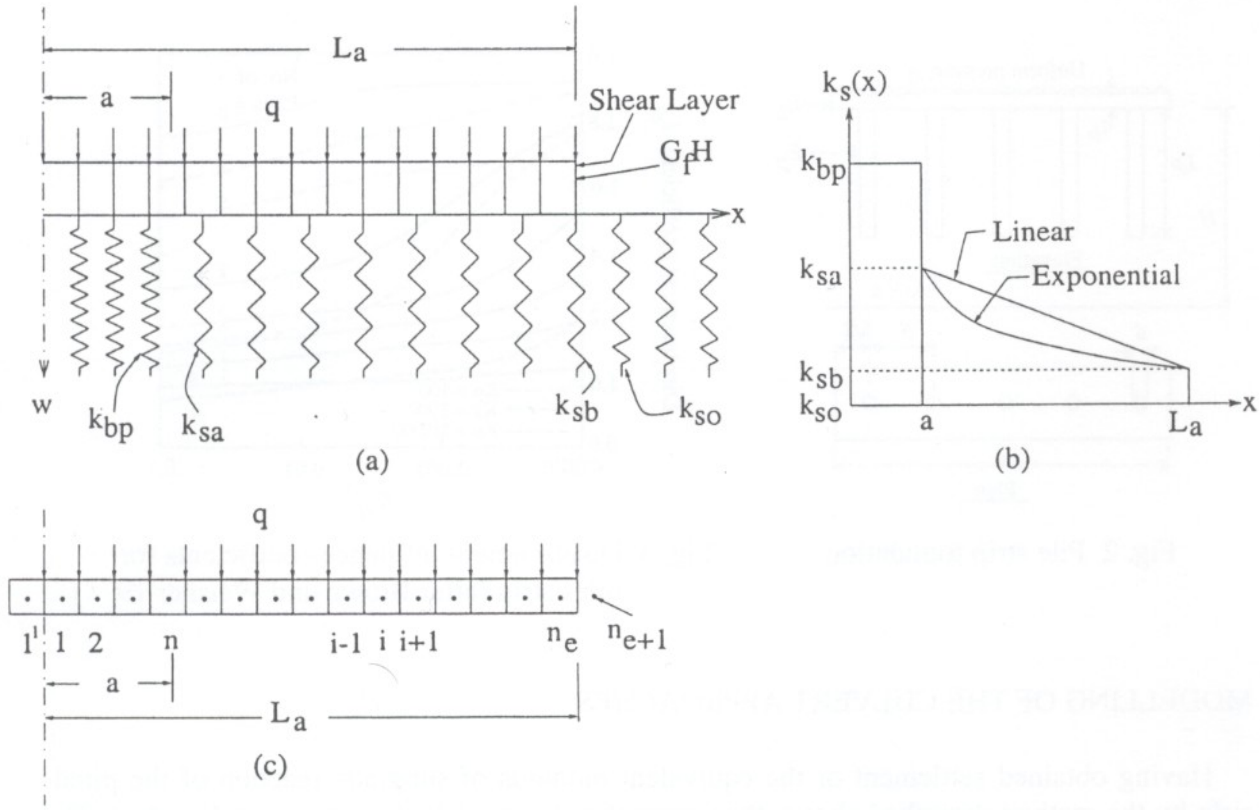

Fig. 4 Culvert with approaches on piles of varying length; (a) model, (b) variation of stiffness with distance and (c) finite difference scheme

The linear and exponential decreases of $k_s(x)$ can be visualised as the upper and the lower bounds for the actual variation. The culvert approaches consisting of a stiff granular bed overlying piles whose lengths taper with the distance, are modelled as a Pasternak foundation (Madhav & Van Impe 1993) with the variable Winkler spring stiffness. The governing equations can be derived (Sujatha 1998) as:

I the Culvert Region:

$$q_0 = k_{bp} w - G_f H \frac{d^2 w}{dx^2} \qquad \text{for } 0 < x < a \tag{5}$$

and II the Culvert Approaches:

$$q_0 = k_s(x)w - G_f H \frac{d^2 w}{dx^2} \qquad \text{for } a < x < L_a \qquad (6)$$

where q_0 is the applied stress, w is the displacement at distance, x, k_{bp} - the stiffness of the end bearing piles, G_t - the shear stiffness of the granular bed, H - its thickness. All the other parameters are defined earlier. The above equations are non-dimensionalised as:

$$\lambda_{s0} = R_{sp}\lambda_{s0}W - \frac{d^2W}{dX^2} \qquad \text{for } 0 < X < 1 \qquad (7)$$

and for linear variation of $k_s(x)$ with x as

$$\lambda_{s0} = \lambda_{s0}R_{kb}\left\{R_k / R_{kb} - (R_k / R_{kb} - 1)(X-1)(L_R - 1)\right\}W - \frac{d^2W}{dX^2} \qquad \text{for } 1 < X < L_R \qquad (8)$$

and for the exponential decrease in $k_s(x)$ with x, as:

$$\lambda_{s0} = \lambda_{s0}R_k \exp\left[-\ln(R_k / R_{kb})\{(X-1)(L_R - 1)\}\right]W - \frac{d^2W}{dX^2} \qquad \text{for } 1 < X < L_R \qquad (9)$$

where $X = x/a$, $W = w/w_w$, $w_w = q_0/k_{s0}$, $\lambda_{s0} = k_{s0}a^2/G_f H$ and $L_R = L_a/a$.

SOLUTION

It is difficult to solve Eqs. (7), (8) and (9) analytically and so a finite difference approach has been adopted. The portion of the reinforced granular bed (the base and the sub base) is discretised in Fig. 4(c) into 'n' elements and the granular bed into 'n_e' elements where $n_e = n\, L_a/a$. The finite difference forms of Eqs. (7) through (9) respectively are:

$$\lambda_{s0} = R_{bp}\lambda_{s0}W_i - \{W_{i-1} - 2W_i + W_{i+1}\}/\Delta X^2 \qquad (10)$$

for the linear decrease case,

$$\lambda_{s0} = R_{kb}\lambda_{s0}f_1(X)W_i - \{W_{i-1} - 2W_i + W_{i+1}\}/\Delta X^2 \qquad (11)$$

and for the exponential decrease case,

$$\lambda_{s0} = R_k\lambda_{s0}f_2(X)W_i - \{W_{i-1} - 2W_i + W_{i+1}\}/\Delta X^2 \qquad (12)$$

where $f_1(X) = \{R_k/R_{kb}-(R_k/R_{kb}-1)(X-1)(L_R-1)\}$ and $f_2(X) = exp\{-(lnR_k/R_{kb})(X-1)/(L_R-1)\}$.

The boundary conditions at $X = 0$ and $X = L_R$ are easily incorporated. At $X = 0$, from symmetry, the slope of the displacement curve is zero, which leads to $W_1' = W_2$, where W_1' is the displacement at the node 1' to the left of node 1. At $X = L_R$, the shear stress is zero which also leads to the slope being zero and $W_{ne+1} = W_{ne}$, where W_{ne+1} is the displacement at node n_e+1 to the right of node n_e. Eqs. (10) through (12) are solved for W_i as:

$$W_i = \left\{\lambda_{s0} + \left(W_{i+1} + W_{i-1}\right)/\Delta X^2\right\}/\left(2/\Delta X^2 + C\right) \tag{13}$$

where $C = \lambda_{s0}R_{bp}$ for $2 \le i \le n$, and $C = \lambda_{s0}R_{bp}f_1(X)$ for $(n+1) \le i \le ne\text{-}1$ for linear decrease of k_s with the distance and $C = \lambda_{s0}R_{bp}f_2(X)$ for $(n+1) \le i \le n_e\text{-}1$ and the exponential decrease of k_s with distance. The above equations get modified for $i = 1$ and $i = n_e$ with the substitution of appropriate boundary conditions.

RESULTS

The above equations are solved by the finite difference method. For numerical decretisation, the number of elements, n, into which the lengths of the culvert approaches on each side are divided, is varied. The value of n equal to 20 is found to provide solutions which converge to a limiting value. Parametric studies have been performed for the settlement variation with distance and for the variation of the percentage load transferred to the piles. The ranges of the parameters considered are R_{bp}: 50 -1000; L_R: 10-50; R_k: 10-50; R_{kb} = 1-4 and $\lambda_c(= K_{bp}\, a^2/G_f H) = 0.05$ -50.

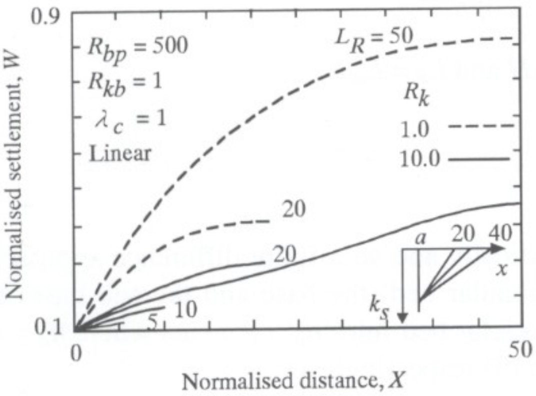

Fig. 5 Normalised settlements for different approach length ratios

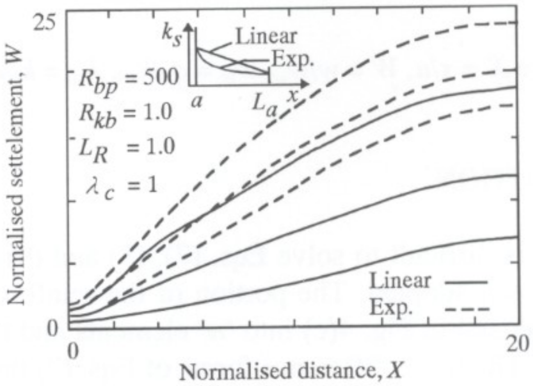

Fig. 6 Comparison of results with linear and exponential variation in stiffness

Figure 5 depicts the variation of normalised settlement, W, with the normalised distance, X, from the culvert, for different length ratios, L_R = (L_a/a) and for the linear variation of equivalent stiffness of the piled soft soil with $\lambda_c = 0.5$, $R_{bp} = 500$, $R_k = 1.0$ (soft soil alone, i.e. no piles) and 10.0 (piled approach) and R_{kb} = 1.0. The results obtained from the present analysis for R_k equal to 1.0 agree closely with those given by Madhav and Van Impe (1993). The response of the culvert approach is sensitive to the length ratio, L_r. The normalised settlements increase with increase in approach length. For length ratios of 20 and 50, the settlements are 0.30 and 0.92 at the farthest end while near the culvert end, they are very small and equal to the settlement of the culvert. For a piled approach, the settlements reduce significantly to 0.18 and 0.36 for the length ratios of 20 and 50, respectively. The trend in the variation of the settlements with distance, exhibits an apparent anomalous results in that the settlements at any distance, X, increase with the length ratios up to 20.0. However, for the longer length ratios, the settlements closer to the culvert are smaller for the higher values of L_R

than those for smaller values of L_R. The equivalent stiffness values at any distance, X, from the culvert, increase with increasing values of L_R, as is evident from the inset in the figure. Higher equivalent stiffness values lead to smaller settlements closer to the culvert, even though the settlements at the farthest end, increase with increasing values of L_R, as is to be expected.

The settlement profiles for linear and exponential variations of the equivalent stiffness with the distance, are compared in Fig. 6 for L_R value of 20, all other parameters being the same as for Fig.5. From the inset, it can be noted that the linear variation of the stiffness with the distance would always lead to higher values of $k_s(x)$ in comparison to the values with exponential variation. A consequence of these variations is that settlements with linear variation in equivalent modulus are considerably smaller than those with exponential decay. For R_k values of 10, 25 and 50, the settlements at the farthest end are 0.24, 0.2 and 0.17 for exponential decay while they are 0.185, 0.12 and 0.08 for linear variation in the equivalent modulus values. The larger the value of R_k greater are the differences in the settlements between the two variations.

A comparison in Fig. 7 of the profiles for R_{kb} values of 1, 2 and 4, indicates the maximum settlement to decrease with increasing values of R_{kb}, the decrease being more in case of exponential decay of the equivalent stiffness of the piled ground. The amount of the load transferred to the culvert by the relative stiffness of the granular pad on top for different near end stiffnesses, R_k, is presented in Fig. 8. Stiffer the granular pad on top, the larger would be the load transferred to the culvert and in turn to the piles supporting the same. The percentage load transferred to the culvert is more in case of exponential decay of the stiffness with the distance than the case with linear variation.

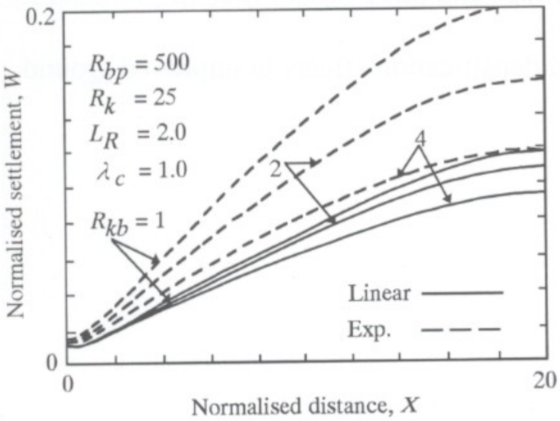

Fig. 7 Effect of the stiffness at the farthest end on normalised settlement

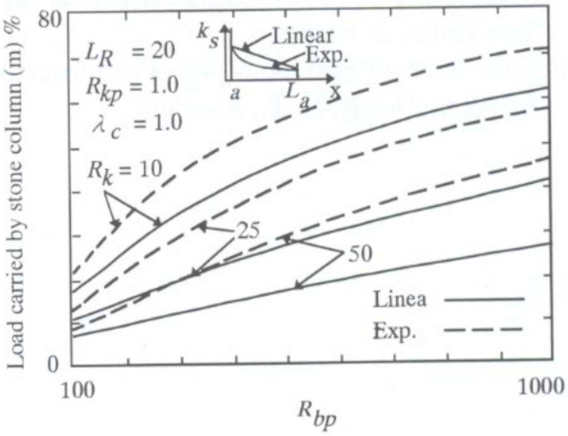

Fig. 8 Percentage load transferred to the culvert foundation

CONCLUSIONS

For the construction of culvert and its approaches on soft soils, the Column Approach of providing piles of varying length, is being practised. The equivalent stiffness of the piled strips as a function of the relative length of the floating piles can be estimated from the analysis of Brown and Weisner (1975). The actual variation of this relative stiffness of the pile length, is bounded by linear and exponential variations with distance. Subsequently, a simple extended Pasternak type model is proposed for the culvert approaches with piles of varying length. The response of the system is governed by the relative stiffnesses of the granular bed, the culvert foundation, the approaches at the near and far ends and the relative pile length and raft width ratios. The exponential decay of equivalent stiffness with distance leads to significantly larger magnitudes of settlements at the farthest end. The relative stiffness of the granular pad has a significant effect on settlements and on the loads transferred to the culvert foundation.

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