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OPTIMAL DESIGN OF GEOTECHNICAL STRUCTURES FOR LOWLAND AREAS

S. Valliappan[1], A. Hakam[2] and V. Tandjiria[3]

ABSTRACT: The applications of optimization technique for raft foundations built on soft soils and retaining walls are presented in this paper. The objective function of the optimization problem is the cost of the structures (which is a function of thickness, dimensions and material of stabilized soil). The constraints are the displacement, differential displacement and stabilities. The finite difference sensitivity analysis and the combination of extended bi-point constraint and Lagrangian constraint approximation are carried out during the structural optimization process. The finite element method has been used to analyse the response of the structures. The results of the numerical examples show that the structures can be designed both economically and effectively using the proposed method.

INTRODUCTION

Optimization methods have been developed rapidly over the last thirty years. However, their application to geotechnical structures is still rare. In this study the coupled optimization and finite element technique has been used to the case of raft foundations built on a soft ground to achieve an economical design by satisfying the design criteria. Further, the optimization technique has also been applied to the retaining wall structures.

Apparently with the increasing population trend around the world, the land resources especially in urban areas are limited. In spite of the fact, the costs of such land resources are very high, the demand also grows up significantly in modern society. Unfortunately, the existing land on which structures are built naturally have their special problems. The soil strata in some areas are good and stable but there are soft and unstable soils in other areas. Therefore, it is necessary to provide safe designs (structures built on soft ground) with economical costs.

In order to design economically, it is necessary to propose appropriate design variables, so that the structural responses such as displacements and stresses in the system are within the allowable values. To obtain an optimum solution without performing a number of analyses, the combination of the structural optimization and the finite element method has been successfully applied to linear analysis of raft-pile foundations (Tandjiria et al. 1996). Further, the method was extended to nonlinear analysis of raft-pile foundations (Valliappan et al. 1997).

BASIC MODULES FOR STRUCTURAL OPTIMIZATION

The three main modules required in structural optimization problems are analysis module,

1 Professor, School of Civil and Environmental Engineering, University of New South Wales, P.O. Box 1, Kensington, NSW 2033, AUSTRALIA.
2 Ph.D. Student, ditto.
3 Lecturer, Petra University, Surabaya, INDONESIA.
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optimization module and interface module. In the analysis module, either analytical or numerical techniques, such as the finite element method, are used to determine the response of the structure due to the applied loads. The optimization module involves the selection of the optimization method. The generalized reduced gradient (GRG) method is used as the optimization solver (Lasdon et al. 1978). The interface module couples the analysis and the optimization modules. This is an important stage in the optimization process, which involves processes such as sensitivity analysis and constraint approximation. Depending on the constraint formulation chosen, more initial values of each design variable may be required.

The automatic mesh generation has also been developed in this study. In addition, the design element concept has also been applied for representing the shape variables (Imam 1982).

SENSITIVITY ANALYSIS

The sensitivity analysis is carried out to calculate the gradients of the constraints with respect to each design variable, which are necessary for constructing constraint approximation functions. The finite difference procedure which is simple and straightforward is adopted in this study.

The sensitivity value of a constraint function $g(x)$ with respect to the a design variable, x, is defined as:

$$\frac{\partial g(x)}{\partial x} = \frac{g(x+\Delta x) - g(x)}{\partial x} \tag{1}$$

where Δx is a perturbation of the design variable, x.

CONSTRAINT APPROXIMATION

In order to provide correlations between the design variables and the responses of the structure, one or more constraint approximations should be generated. In this study, the combination of the extended bi-point constraint approximation and the Lagrangian polynomial constraint approximation is presented (Tandjiria et al. 1998).

Bi-point Constraint Approximation

Bi-point constraint approximation involves two initial points for each design variable. Let x_1 and x_2 be the two initial points of a design variable with values of $g(x_1)$ and $g(x_2)$ and their first derivatives $g'(x_1)$ and $g'(x_2)$. The constraint approximation at any point within or outside the ranges x_1 and x_2 is calculated using the following formula.

- for the range between x_1 and x_2 the approximation function may be given as:

$$g_{BP}(x) = \alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x + \alpha_4 \tag{2}$$

where α_1, α_2, α_3, and α_4 are cubic coefficients which are the functions of $g(x_1)$, $g(x_2)$, $g'(x_1)$ and $g'(x_2)$ and x is the design variable at which the constraint value is being estimated .

- for points beyond x_1 and x_2 two conditions may be identified:

a) Condition 1 : $g'(x_1)\,g'(x_2) > 0$

$$g_{BP}(x) = g(x_1) + (x - x_1)\,g'(x_1)\theta_1\,, \quad \text{for } x_L \le x \le x_1 \qquad (3)$$
$$g_{BP}(x) = g(x_2) + (x - x_2)\,g'(x_2)\theta_2\,, \quad \text{for } x_2 \le x \le x_U \qquad (4)$$

where $g_{BP}(x)$ is bi-point approximation of the constraint at x, x_L is the lower bound of the design variable and x_U is the upper bound of the design variable. The parameters θ_1 and θ_2 are functions of $g(x_1)$, $g(x_2)$, $g'(x_1)$, $g'(x_2)$ and g' which is the gradient between the two points.

b) Condition 2 : $g'(x_1)\,g'(x_2) \le 0$

$$g_{BP}(x) = \beta_1 x^2 + \beta_2 x + \beta_3 \qquad (5)$$

where parameters β_1, β_2 and β_3 are functions of $g(x_1)$, $g(x_2)$, $g'(x_1)$ and $g'(x_2)$.

Lagrangian Polynomial Constraint Approximation

Lagrangian polynomial constraint approximation for N number of design variables can be expressed as:

$$g_{LP}(\mathbf{X}) = g(\mathbf{X_o}) + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(L_{ij}\left(g_{ij} - g_{io}\right)\right) \qquad (6)$$

where $(\mathbf{X})$ is a vector containing all design variables, i.e. $[x_1, x_2, x_3, \dots, x_N)$, $(\mathbf{X_o})$ is the vector of initial design variables of $(\mathbf{X})$ about which the approximation is created, n_i is the number of design points chosen in the i^{th} design variable, L_{ij} is the Lagrangian shape function at point j of the i^{th} design variable, g_{ij} is the constraint value at point j of the i^{th} design variable and g_{io} is the initial value of the i^{th} design variable.

Extended Bi-point Constraint Approximation

It is evident that when more design points are chosen, the approximated values will be improved. However, using more than three design points is not efficient due to expensive analyses to be performed. Thus, the extended bi-point constraint approximation, which uses only three basic design variables, i.e. x_1, x_2 and x_3, is an alternative.

Bi-point constraint approximation which is applied separately for two ranges of the design variable, i.e. between x_L and x_2 and between x_2 and x_U forms the extended bi-point constraint approximation. By combining the results of the two ranges, global results of the approximation are obtained.

Combination of Extended Bi-point and Lagrangian Polynomial Constraint Approximation

Although the extended bi-point approximation can be carried out by selecting three reference values of each design variable, the formulation is only for a single design variable and the functions are discontinuous. On the other hand, Lagrangian approximation requires more data points for each design variable and hence is not economical when dealing with large problems especially in nonlinear analysis. Therefore, a special procedure is developed here to combine the two approximations in order to predict the constraint values accurately and effectively. Firstly, the extended bi-point constraint approximation is carried out for each design variable. Then, several basic points for each design variable which are required for formulating the Lagrangian polynomial constraint approximation are obtained from the results produced by the extended bi-point approximation.

NUMERICAL RESULTS

Raft Foundation on Soft Soils

It is proposed to design optimally a raft foundation subjected to uniformly distributed load. For this particular example, the dimension of the foundation has been fixed. The foundation is subjected to a vertical load of 30 kPa. The width of the foundation is 10 m. Figure 1 shows the configuration of the foundation. Due to symmetry, only a half of the system is considered in the finite element analysis.

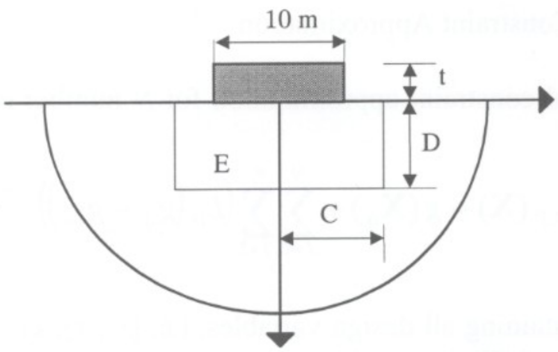

Fig. 1 Configuration of raft foundation

The existing soil layer is a very soft clay having a modulus of elasticity of 2 MPa and Poisson's ratio of 0.499. The ratio between the elastic modulus and the cohesion of soil is 500. The concrete has a compressive strength of 25 MPa, elastic modulus of 24000 MPa and Poisson's ratio of 0.2.

The objective of the design is to minimize the cost of the raft foundation. The cost of the foundation including costs for labour, material and plant requirements is based on the commercial industrial building cost guide, NSW, Australia (1995). The design variables chosen in this problem are the raft thickness, the width and the depth of the stabilized soil and the material used for the stabilized soil which is represented using the values of elastic modulus. Besides the constraints mentioned previously, there are size limits of the design variables which are between 0.5 and 1.5 m for the raft thickness, between 5 and 8 m for C, between 0.5 and 1.5 m for D and between 2 and 100 MPa for elastic modulus of stabilized soil. It is assumed that the material chosen for the stabilized soil can be achieved by selecting better materials and a compaction process. Table 1 shows the different types of materials and their elastic moduli used for soil improvement. Since the nonlinear behaviour of both the soil and the concrete has been already considered in the analysis with the use of Drucker-Prager criterion, only the maximum settlement and differential settlement are chosen as the constraints.

Table 1 Elastic modulus and type of soil

Types of soil	Elastic modulus (kPa)
Existing soil	2000
Compacted soil	15000
Sand	22000
River gravel	100000

A raft thickness of t = 1 m, C = 8 m, D = 3 m and E = 100 MPa are selected as the initial design variables for this problem. Besides the above initial design variables, two other supplementary design points for each design variable have been selected for the purpose of constraint approximation.

For this case, only one design cycle was required to obtain the optimal result and satisfy the convergence criteria. The optimal cost of the foundation and its improvement is about A$ 2380.00 whereas the cost of the original design was A$ 3920.00 and hence, A$ 1540.00 can be saved. The corresponding design parameters are raft thickness of 0.67 m, width of soil improvement of 10 m, depth of soil improvement of 3 m and elastic modulus of the stabilized soil of 45 MPa. It is noted that the error between the results of the approximated constraints and their corresponding results from the finite element analysis is only 5%. Table 2 shows the constraint ratio values for the initial and optimal design of the raft foundation. The constraint ratio is defined as the ratio of the value of a particular constraint to its corresponding allowable value. It can be seen that both the settlement and the differential settlement constraints are nearly at their bounds. It can be understood that the constraints at the optimal design are higher than those at the initial design because the dimension of the foundation and the stabilized soil are smaller and the quality of material chosen for the stabilized soil under the foundation is lower at the optimal design than those at the initial design.

Table 2 Constraint ratio values for the initial design and final design

Constraint ratio	Initial design	Final design
Settlement	0.40	0.97
Differential settlement	0.61	0.98

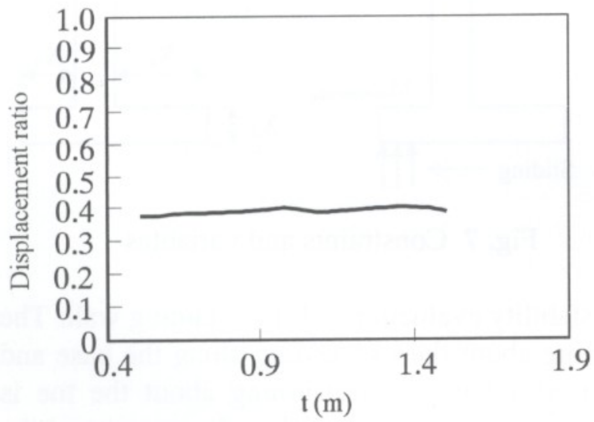

Fig. 2 Displacement ratio versus raft thickness

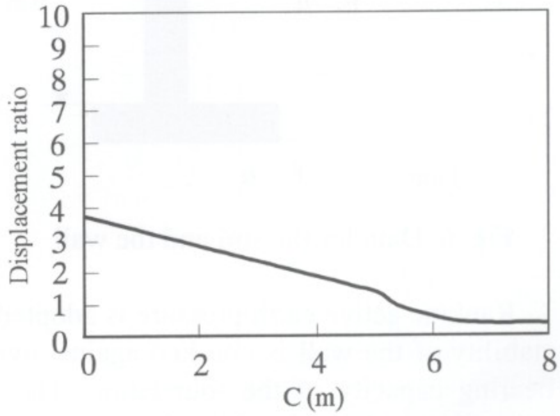

Fig. 3 Displacement ratio versus half width of stabilized soil

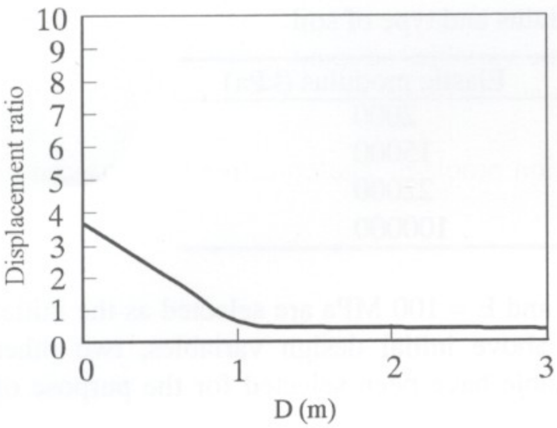

Fig. 4 Displacement ratio versus depth of stabilized soil

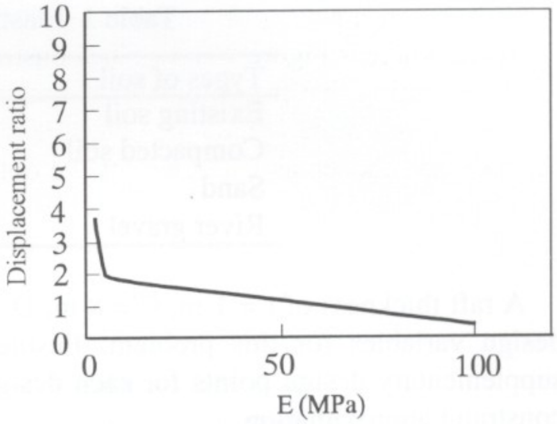

Fig. 5 Displacement ratio versus elastic modulus of stabilized soil

Figures 2 to 5 show the displacement ratio versus raft thickness and width, depth and elastic modulus of the stabilized soil, respectively. All figures were obtained from the extended bi-point constraint approximation.

Cantilever Retaining Walls

The retaining wall is of 9 m height and the base of the wall is embedded into firm soil. Four cases are considered in this analysis, with a combination of sand and clay back fill soil and inclination of its surface. The clay back fill has the unit weight of 17.3 kN/m^3, cohesive strength of 48 kPa, modulus of elasticity of 14400 kPa and Poisson's ratio of 0.3. The unit weight of the sand is 19 kN/m^3 with internal friction angle of 30^o, modulus of elasticity of 23500 kPa, and Poisson's ratio of 0.25. The cohesion of the firm clay under the foundation is 167.6 kPa with modulus of elasticity and Poisson's ratio of 34480 kPa and 0.3 respectively. The wall is made of reinforced concrete with unit weight of the concrete as 24 kN/m^3.

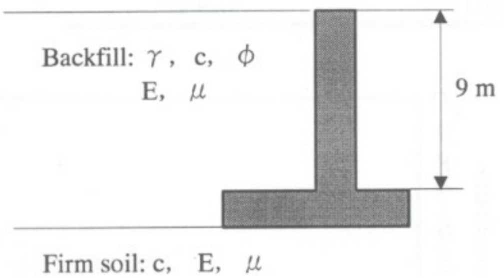

Fig. 6 Data for the soil and the wall

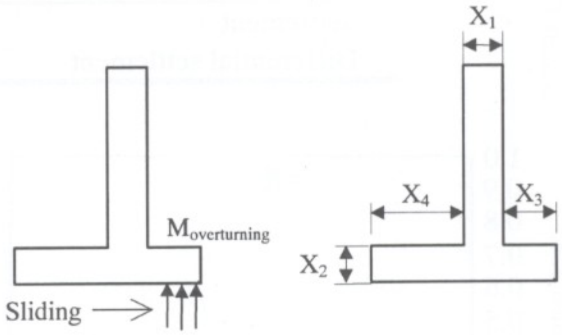

Fig. 7 Constraints and variables

Rankine active earth pressure is adopted for stability evaluations of the retaining wall. The stability of the wall is checked against overturning about the toe, sliding along the base and bearing capacity of the foundation. The factor of safety for overturning about the toe is expressed as the ratio of the sum of the resisting moment about the toe to the moment of the active force about the same point. The factor of safety against sliding is the maximum resisting force along the base divided by the horizontal active force. The factor of safety for bearing capacity is determined by dividing the ultimate bearing capacity of the base soil by the used as constraints in optimization analysis. Figure 6 shows the geometry of the system analysed whereas Figure 7 shows the constraints and variables adopted.

By assuming that the volume of the retaining wall is proportional to the cost, optimization is aimed at minimizing the volume of the material. The thickness of the wall and the length of the base are chosen as variables for the optimization problem. The objective function can be written as :

$$f(x) = h \cdot x_1 + x_2 \, (x_1 + x_3 + x_4) \qquad (7)$$

where h is height of the wall (9 m), x_1 is thickness of the wall, x_2 is thickness of the base, x_3 and x_4 length of the toe and the heel respectively.

The size limits of the design variables are between 0.3 and 1 m for the thickness of the wall, between 0.3 and 1 m for the length of the toe and between 1 and 5 m for the length of the heel. The minimum values of the constraints are 2.0 for both the factor of safety against overturning and sliding and 3.5 for the factor of safety for bearing capacity.

Sensitivities of the stability to changes of the design variable are evaluated and then applied for gradient of constraint functions. Each variable is perturbed in stability analysis to develop the constraint functions. For this purpose, the finite difference technique is adopted. The constraint function can be written according to the formula :

$$g_i(x) = g_{i,0}+dg/dx(x_1-x_{1,0})+dg/dx(x_2-x_{2,0})+dg/dx(x_3-x_{3,0})+dg/dx(x_4-x_{4,0}) \qquad (8)$$

Tables 3 to 6 present the results of the coupled analyses of stability and optimization evaluations. Three design cycles were required to obtain the optimal result for the cases in Tables 3, 4 and 5 whilst only two design cycles for the case in Table 6. It can be seen that for the case in Table 3 even though cycle 1 provides the same value as in cycle 2, the factor of safety for sliding is violated. In cycle 2 and 3 this has been achieved. However for practical purposes the value of 3.0 m for the length of the heel in cycle 2 will be adopted instead of 2.83 m in cycle 3. For the case in Table 4, even though the volume in cycle 1 is less, the factor of safety for bearing is violated. Hence, cycle 2 and 3 have been proceeded with. Again, for practical purposes, the heel length of 3.5 m (cycle 2) will be adopted instead of 3.45 m (cycle 3). For the case in Table 5, only cycle 3 provides the required factor of safety. For the case in Table 6, the required safety factor are obtained in cycle 2. But for practical purposes, using the value of 3.75 m for heel length in cycle 2 and achieving a factor of safety of 3.49 for bearing is good enough.

Table 3 For clay back fill

	Initial	Cycle 1	Cycle 2	Cycle 3
Wall thickness (m)	0.5	0.3	0.3	0.3
Base thickness (m)	0.5	0.3	0.3	0.3
Toe length (m)	1.0	1.0	0.5	0.5
Heel length (m)	3.0	2.5	3.0	2.83
FS overturning	3.87	2.83	3.09	2.81
FS sliding	2.31	1.99	2.10	2.00
FS bearing	4.88	3.50	3.66	3.50
Volume (m^3)	8.75	3.84	3.84	3.79

Table 4 For sand back fill

	Initial	Cycle 1	Cycle 2	Cycle 3
Wall thickness (m)	0.3	0.3	0.3	0.3
Base thickness (m)	0.3	0.3	0.3	0.3
Toe length (m)	1.0	1.0	1.0	1.0
Heel length (m)	4.8	3.2	3.5	3.45
FS overturning	4.15	2.15	2.53	2.49
FS sliding	4.03	2.98	3.17	3.15
FS bearing	4.48	3.27	3.54	3.50
Volume (m^3)	4.53	4.05	4.15	4.13

Table 5 For clay back fill with a slope of 15 degrees

	Initial	Cycle 1	Cycle 2	Cycle 3
Wall thickness (m)	0.3	0.3	0.3	0.3
Base thickness (m)	0.3	0.3	0.3	0.3
Toe length (m)	1.0	0.5	0.5	0.5
Heel length (m)	5.0	4.1	4.3	4.4
FS overturning	5.83	4.08	4.30	4.41
FS sliding	2.30	1.94	1.99	2.01
FS bearing	6.18	4.05	4.16	4.21
Volume (m^3)	4.59	4.17	4.25	4.26

Table 6 For sand back fill with a slope of 15 degrees

	Initial	Cycle 1	Cycle 2
Wall thickness (m)	0.3	0.3	0.3
Base thickness (m)	0.3	0.3	0.3
Toe length (m)	1.0	1.0	1.0
Heel length (m)	3.5	3.75	3.76
FS overturning	2.56	2.80	2.81
FS sliding	2.62	2.72	2.73
FS bearing	3.30	3.49	3.50
Volume (m^3)	4.14	4.20	4.22

Table 7 Maximum displacements

Back fill	Slope of surface	Maximum displacement
Clay	0^o	42 cm
Sand	0^o	15.6 cm
Clay	15^o	42 cm
Sand	15^o	20.6 cm

The finite element simulation by displacing the wall away from the backfill to get the maximum displacement is conducted based on optimal design of the retaining wall structure. The analysis is plane strain condition and the wall is assumed to behave as elastic. The soil is assumed to behave as elastic perfectly plastic. The finite element mesh and the yield surfaces of the soil behind the wall are shown in Fig. 8. The finite element analyses indicate that the slope of the yield zones on the back-fill behind the wall with respect to the horizontal is about 45° for clay (ϕ=0) and about 60° for sand (ϕ=30°). These failure lines are about 45 + ϕ/2 with respect to the horizontal and hence agree with Rankine's slip lines for active condition. Table 7 shows the maximum displacement of the retaining structures.

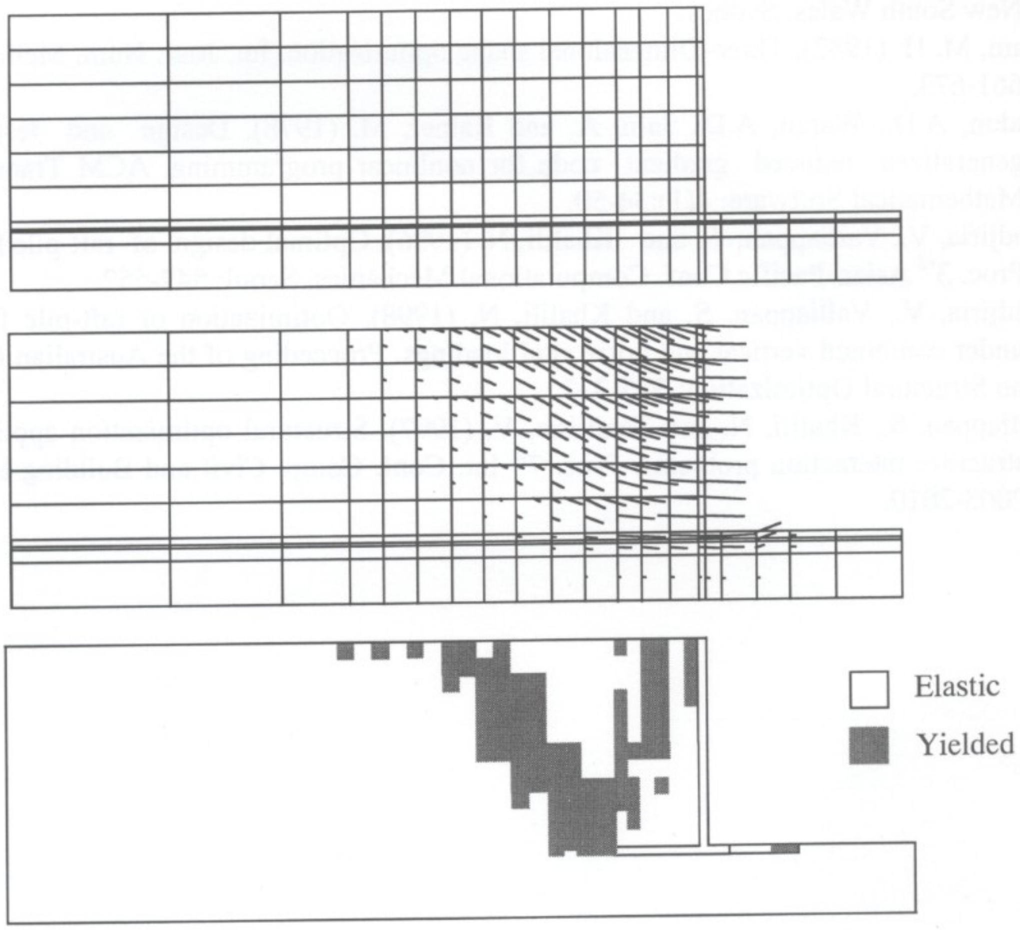

Fig. 8 Finite element mesh, displacement and yielded zones

CONCLUSIONS

The optimization technique applied to geotechnical structures has been presented in this paper. It is found that using the optimization technique, the cost of the geotechnical structures can be minimised by taking into consideration various design variables such as the dimensions and the material parameters.

The finite difference sensitivity method and the combination of extended bi-point constraint approximation and Lagrangian polynomial constraint approximation are found to be very useful for the combined finite element and optimization approach.

For a raft foundation under the uniformly distributed load, it has been found that both displacement and differential displacement are close to their bounds. The yield zones behind the retaining walls with the optimal design variables, agree with Rankine's active slip lines. It is concluded that the proposed combination of optimization technique and finite element method is efficient in designing geotechnical structures in lowland areas.

REFERENCES

Cordell Building Information Services (1995). Commercial Industrial Building Cost Guide New South Wales, Sydney.

Imam, M. H. (1982). Three-Dimensional shape optimization. Int. Jour. Num. Meth. Eng., 18: 661-673.

Lasdon, A.D., Waren, A.D., Jain, A. and Ratner, M. (1978). Design and testing of a generalized reduced gradient code for nonlinear programming. ACM Transactions on Mathematical Software: 4(1):34-50.

Tandjiria, V., Valliappan, S. and Khalili, N. (1996). Optimal design of raft-pile foundation. Proc. 3rd Asian-Pacific Conf. Computational Mechanics, Seoul: 547-552.

Tandjiria, V., Valliappan, S. and Khalili, N. (1998). Optimization or raft-pile foundations under combined vertical and horizontal loadings. Proceeding of the Australian Conference on Structural Optimization: 325-332.

Valliappan, S., Khalili, N. and Tandjiria, V. (1997). Structural optimization applied to soil-structure interaction problems. Proc. 7th Int. Conf. Comp. Civil and Building Eng, Seoul: 2003-2010.