Analytical solutions for soil flushing through geotextiles considering filter cake and clogging

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ABSTRACT

Prefabricated vertical drains (PVDs) have been used in finegrained soils remediation system to provide a higher flow rate. However, the migration and accumulation of the fine particles in the soil result in the filter cake adjacent to the geotextile and the pore clogging in the geotextile. An analytical solution for soil flushing through a clogged geotextile with a filter cake is developed. The results obtained by the proposed analytical solution agree well with those obtained from the finite difference method. Results from an illustrative example indicate that the formation of a filter cake at the surface of a geotextile is far more disadvantageous to soil flushing than pore clogging within a geotextile. The 5-hour base concentration at the depth of z = 50mm for the case just considering the pore clogging in the geotextile is just 1.06 times more than that assuming no filter cake and no pore clogging in the geotextile. When the pore clogging combined with a filter cake are considered, the 5-hour base concentration at the depth of z = 50 mm increases by a factor of approximately 4.94.

1. Introduction

Along with the development of society, soils have been significantly polluted by heavy metals and organics in China. Remediation of contaminated low-permeability soils is a major goal of geo-environmental engineering. Prefabricated vertical drains (PVDs) were widely used to accelerate the consolidation process of soft ground (Zhang et al, 2009; Shahiduzzaman et al., 2010; Saowapakpiboon et al., 2011; Wu et al., 2013), and have been introduced in a scheme for removing contaminants from fine-grained soils for decades (Gabr et al., 1996; Quaranta et al., 1997; Kunberger et al., 2003). The cores of the PVDs were wrapped with a geotextile filter to retain a soil. However, the soils experience loss of fine particles due to the action of water flows (Lawson, 1982; Kenney and Lan, 1985), and the fine soil particles move toward the filter, which results in the filter cake and the clogging within the geotextile filter (Giroud, 1994, 2005; Palmeira and Gardoni, 2000; Xu, 2005). Giroud (1994, 2005) investigated the hydraulic characteristics of geotextiles and presented expressions for the hydraulic conductivities of a filter cake and a clogged geotextile, respectively, based on the Kozeny-Carman's equation. Palmeira and Gardoni (2000) presented test results of the hydraulic characteristics of clogged geotextiles, which indicated that the expression

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proposed by Giroud (1994) is useful to predict the hydraulic conductivities of clogged geotextiles. Therefore, it is of great importance to research the effect of the filter cake and the pore clogging on the efficiency of the soil flushing to make the design of the PVD-enhanced system more effectively.

The previous studies for soil remediation using a PVD-enhanced system were traditionally been solved numerically (Welker and Gilbert, 2003; Quaranta et al., 2005; Sharmin et al., 2008; Sharmin and Gabr, 2012; Gabr et al., 2013). However, the lever of numerical sophistication often results in the time cost being unjustifiable, especially at the site selection and preliminary design stages. By contrast, simplified analytical methods can provide an economical and efficient solution, which can be used for a quick preliminary calculation to evaluate the experimental results and to check the complex numerical methods.

Gabr et al. (1996) developed an analytical model for contaminant extraction using PVD-enhanced system, in which the PVDs were installed in a circular configuration. Analytical solutions for contaminant extraction using PVD-enhanced system arranged in a grid formation were also available (Tang et al., 2015). However, no analytical solutions for soil flushing considering the effect of the filter cake and the pore clogging are available due to the sophisticated nature of the problem.

The objective of this paper is to present an analytical model capable of predicting soil flushing through a clogged geotextile with a filter cake. Steady onedimensional flow is assumed in the model. The method of separation of variables is used to obtain the analytical solution for the case with Neumann bottom boundary. The results obtained by the proposed method are compared with those obtained by the finite element method (FEM) provided by GeoStudio 2007. An illustrative example is performed to investigate the effect of filter cake and pore clogging on soil flushing based on the proposed analytical solution.

2. Mathematical model

The governing geometry of the problem is shown in Fig. 1. The flushing solutions are injected through the top of the soil column, and retrieved from the bottom of the soil, aided by a vacuum pump. The initial pore-fluid concentration of contaminant in the soil is assumed to be a constant, C_0 . The accumulation of the fine particles which are lost in the soil forms the filter cake at the surface of the geotextile filter, and the particles entrapped within the filter result in the pore clogging in the geotextile. The model mainly includes the soil column, the filter cake



Fig. 1. Schematic diagram of soil flushing through clogged geotextile with filter cake.

and the clogged geotextile which are all considered to be homogeneous and fully saturated. The positive z-axis is assumed downward. The water head difference between the top of the soil column and the bottom of the geotextile is assumed to be invariable during the soil flushing. Based on Darcy's law, the flushing liquid flows steadily in the z direction.

Layered media are often observed in landfill liner systems (Du and Hayashi, 2005; Du et al., 2009). Similarly to Du and Hayashi (2005), the governing equations for soil flushing through the contaminated soil column at a particular distance z and time t can be expressed as:

$$R_{di} \frac{\partial C_{si}(z,t)}{\partial t} = D_{si} \frac{\partial^2 C_{si}(z,t)}{\partial z^2} - \frac{1}{n_{si}} \frac{q_z \partial C_{si}(z,t)}{\partial z} (i = 1,2)$$
[1]

where R_{d1} , R_{d2} are the retardation factors of the soil column and the filter cake, respectively; C_{s1} is the pore-fluid concentration of contaminant in the soil column; C_{s2} is the pore-fluid concentration of contaminant in the filter cake; D_{s1} , D_{s2} are the coefficients of the hydrodynamic dispersion in the soil column and in the filter cake, respectively; n_{s1} , n_{s2} are the porosity of the soil column and the filter cake, respectively; and q_z is the vertical seepage velocity.

Contaminant concentration in the geotextile is governed by the following equation:

$$\frac{\partial C_{g}(z,t)}{\partial t} = D_{g} \frac{\partial^{2} C_{g}(z,t)}{\partial z^{2}} - \frac{q_{z} \partial C_{g}(z,t)}{\partial z}$$
[2]

where, C_g is the contaminant concentration in the geotextile; and D_g is the coefficient of the hydrodynamic dispersion in the geotextile.

The seepage velocity can be described using Darcy's law:

$$q_{z} = k_{z} \frac{\Delta h}{L_{s} + L_{g}} = \frac{\Delta h}{L_{s1} / k_{s1} + L_{s2} / k_{s2} + L_{g} / k_{g}}$$
[3]

where k_z is the vertical equivalent hydraulic conductivity; $k_{s1},\ k_{s2},\ and\ k_g$ are the hydraulic conductivities of the soil column, the filter cake and the clogged geotextile, respectively; Δh is the water head difference between the top of the soil column and the bottom of the geotextile; $L_{s1},\ L_{s2},\ and\ L_g$ are the lengths of the soil column, the filter cake and the clogged geotextile, respectively.

Using the Henry's law, R_{di} can be represented as follows:

$$R_{di} = 1 + \rho_{di}K_d/n_{si}$$
, (i = 1,2) [4]

where ρ_{d1} , ρ_{d2} are the dry bulk densities of the soil column and the filter cake, respectively; and K_d is the equilibrium distribution coefficient, as the soil type of the soil column and the filter cake is the same, K_d in the both regions are assumed to be the same.

Based on Eq. [3], D_{si}, D_g can be obtained as:

$$D_{si} = D_s^{+} + \alpha_z \frac{q_z}{n_{si}}, \quad (i = 1, 2)$$
 [5]

$$\mathbf{D}_{g} = \mathbf{D}_{g}^{*} + \alpha_{z} \mathbf{q}_{z}$$
 [6]

where, D_{s}^{*} , D_{g}^{*} are the effective diffusion coefficients of the soil and the geotextile, respectively; and α_{z} is the dispersivity.

Initially, the pore-fluid concentration of contaminant in the soil is assumed to be a constant, and the geotextile is assumed to be free of contaminants:

$$C_{si}(z,0) = C_0, \quad (i = 1,2)$$
 [7]

$$C_{g}(z,0) = 0$$
 [8]

For the top boundary, the contaminant concentration of the flushing solution is zero:

$$C_{s1}(0,t) = 0$$
 [9]

The Neumann boundary condition is considered at bottom of the geotextile:

$$\frac{\partial C_g(z,t)}{\partial z}\Big|_{z=L_s+L_g} = 0$$
[10]

At the interface between the soil column and the filter cake, and at the interface between the filter cake and the geotextile, there must be continuities of the contaminant flux and the concentration. Continuity of the contaminant flux can be described as:

$$-n_{s1}D_{s1}\frac{\partial C_{s1}(z,t)}{\partial z} + q_zC_{s1}(z,t) = -n_{s2}D_{s2}\frac{\partial C_{s2}(z,t)}{\partial z} + q_zC_{s2}(z,t)\Big|_{z=z_1}$$
[11]

$$-n_{s2}D_{s2}\frac{\partial C_{s2}(z,t)}{\partial z} + q_zC_{s2}(z,t) = -D_g\frac{\partial C_g(z,t)}{\partial z} + q_zC_g(z,t)\Big|_{z=z_2}$$
[12]

Continuity of the concentration can be described by:

$$C_{s1}(z,t) = C_{s2}(z,t)\Big|_{z=z_1}$$
 [13]

$$C_{s2}(z,t) = C_{g}(z,t)|_{z=z_{2}}$$
 [14]

3. Analytical solutions

The governing equations are second-order partial differential equations which are also the form of the governing equations of the process of the small-strain consolidation. Similarly to the problem of one-dimensional consolidation of layered systems, the method of separation of variables mentioned by Lee et al. (1992) is applied. The analytical solutions for the governing Eqs. [1] and [2] satisfying the boundary conditions given by Eqs. [9] and [10] and the interface conditions given by Eqs. [11] – [14] can be expressed as:

$$C_{si}(z,t) = C_0 e^{\frac{q_z z}{2n_s D_{si} - 4n_s^2 R_s D_{si}}} \sum_{m=1}^{\infty} \xi_m g_{sim}(z) e^{-\beta_{sim} t} \ (i = 1,2)$$
 [15]

$$C_{g}(z,t) = C_{0} e^{\frac{q_{z}z}{2D_{g}} - \frac{q_{z}^{2}t}{4D_{g}}} \sum_{m=1}^{\infty} \xi_{m} g_{gm}(z) e^{-\beta_{gm}t}$$
[16]

where,

$$g_{sim}(z) = A_{sim} sin(\lambda_{sim} z) + B_{sim} cos(\lambda_{sim} z)$$
[17]

$$g_{gm}(z) = A_{gm} \sin(\lambda_{gm} z) + B_{gm} \cos(\lambda_{gm} z)$$
[18]

$$\lambda_{\rm sim}^2 = \beta_{\rm sim} R_{\rm di} / D_{\rm si}$$
[19]

$$\lambda_{gm}^2 = \beta_{gm} / D_g$$
 [20]

The coefficients A_{sim} , B_{sim} , A_{gm} and B_{gm} in Eqs. [17] and [18] can be obtained by the following recurrence equation:

$$\begin{bmatrix} A_{stm} & B_{stm} \end{bmatrix}^{T} = \begin{bmatrix} 1 & 0 \end{bmatrix}^{T}$$

$$\begin{bmatrix} A_{s2m} & B_{s2m} \end{bmatrix}^{T} = S_{tm} \begin{bmatrix} A_{stm} & B_{stm} \end{bmatrix}^{T}$$

$$\begin{bmatrix} A_{gm} & B_{gm} \end{bmatrix}^{T} = S_{2m} \begin{bmatrix} A_{s2m} & B_{s2m} \end{bmatrix}^{T}$$

$$\begin{bmatrix} 21 \end{bmatrix}$$

in which:

$$\begin{split} S_{im} &= \zeta_{i} \begin{bmatrix} E_{im}F_{im} + \eta_{i}G_{im}H_{im} & E_{im}H_{im} - \eta_{i}G_{im}F_{im} \\ G_{im}F_{im} - \eta_{i}H_{im}E_{im} & G_{im}H_{im} + \eta_{i}E_{im}F_{im} \end{bmatrix} \\ & (i = 1,2) \end{split} \\ E_{1m} &= sin(\lambda_{s2m}Z_{1}) & F_{1m} = sin(\lambda_{s1m}Z_{1}) \\ G_{1m} &= cos(\lambda_{s2m}Z_{1}) & H_{1m} = cos(\lambda_{s1m}Z_{1}) \\ E_{2m} &= sin(\lambda_{gm}Z_{2}) & F_{2m} = sin(\lambda_{s2m}Z_{2}) \\ G_{2m} &= cos(\lambda_{gm}Z_{2}) & H_{2m} = cos(\lambda_{s2m}Z_{2}) \\ \eta_{1} &= \frac{n_{s1}}{n_{s2}} \sqrt{\frac{\beta_{s1m}D_{s1}R_{d1}}{\beta_{s2m}D_{s2}R_{d2}}} & \eta_{2} = n_{s2} \sqrt{\frac{\beta_{s2m}D_{s2}R_{d2}}{\beta_{gm}D_{g}}} \\ \zeta_{1m} &= e^{\frac{-q_{s2}Z_{1n}Q_{2n}Z_{2n}Q_{2n}Z_{2n}} & \zeta_{1m} = e^{\frac{-q_{s2}Z_{2m}Q_{2n}Q_{2n}Z_{2n}}{\beta_{gm}D_{g}}} \end{split}$$

The determinant of the following which is equal to zero yields the values of λ_{sim} and λ_{gm} :

$$\begin{vmatrix} S_{3m} \cdot S_{2m} \cdot S_{1m} \cdot \begin{bmatrix} 1 & 0 \end{bmatrix}^T \end{vmatrix} = 0$$
 [22]

in which

$$\begin{split} S_{3m} &= \begin{bmatrix} \lambda_{gm} H_{3m} + \frac{q_z}{2D_g} F_{3m} & -\lambda_{gm} F_{3m} + \frac{q_z}{2D_g} H_{3m} \end{bmatrix} \\ H_{3m} &= \cos \lambda_{gm} (L_s + L_g) \qquad F_{3m} = \sin \lambda_{gm} (L_s + L_g) \\ \frac{D_{s1} \lambda_{stm}^2}{R_{d1}} &= \frac{D_{s2} \lambda_{s2m}^2}{R_{d2}} + (\frac{q_z^2}{4n_{s2}^2 R_{d2} D_{s2}} - \frac{q_z^2}{4n_{s1}^2 R_{d1} D_{s1}}) \\ \frac{D_{s2} \lambda_{s2m}^2}{R_{d2}} &= D_g \lambda_g^2 + (\frac{q_z^2}{4D_g} - \frac{q_z^2}{4n_{s2}^2 R_{d2} D_{s2}}) \end{split}$$

Using the orthogonality property of the Eigen functions, the coefficient ξ m can be expressed as:

$$\xi_{m} = \frac{e^{\frac{q_{z}z_{1}}{n_{s}D_{s1}}\int_{0}^{z_{1}}g_{s1m}(z)e^{-\frac{q_{z}z}{2n_{s}D_{s1}}}dz}}{e^{\frac{q_{z}z_{1}}{n_{s}D_{s1}}\int_{0}^{z_{1}}g_{s1m}^{2}(z)dz + \frac{R_{d2}n_{s2}e^{\frac{q_{z}z_{1}}{n_{s2}D_{s2}}}}{R_{d1}n_{s1}}\int_{z_{1}}^{z_{2}}g_{s2m}^{2}(z)dz}$$

$$= \frac{\frac{R_{d2}n_{s2}e^{\frac{q_{z}z_{1}}{n_{s}D_{s2}}}}{R_{d1}n_{s1}}\int_{z_{1}}^{z_{2}}g_{s2m}(z)e^{\frac{q_{z}z_{1}}{2n_{s}D_{s2}}}dz}$$

$$= \frac{\frac{R_{d2}n_{s2}e^{\frac{q_{z}z_{1}}{n_{s}D_{s2}}}}{R_{d1}n_{s1}}\int_{z_{1}}^{z_{2}}g_{s2m}(z)e^{\frac{q_{z}z_{1}}{2n_{s}D_{s2}}}dz}$$

$$= \frac{R_{d1}n_{s1}}{\frac{q_{z}z_{1}}{q_{z}z_{1}}\frac{q_{z}z_{1}}{q_{z}z_{1}}\frac{q_{z}z_{2}}{q_{z}z_{1}}\frac{q_{z}z_{2}}{q_{z}z_{2}}}$$

$$= \frac{R_{d1}n_{s1}}{R_{d1}n_{s1}}$$

 $+\frac{e^{\frac{D_{g}-n_{s2}D_{s2}-n_{s2}D_{s2}}{D_{s2}}}}{R_{d1}n_{s1}}\int_{z_{2}}^{L_{s}+L_{g}}g_{gm}^{2}(z)dz$

The coefficients A_{sim} , B_{sim} , A_{gm} and B_{gm} and the eigenvalues λ_{sim} and λ_{gm} are obtained directly from Eqs. [21] and [22], respectively. Therefore, the computation is dramatically simplified and can be proved to be efficient (Lee et al. 1992).

4. Comparisons with the finite element method

To demonstrate that the proposed analytical solutions give the correct results, the results calculated by the proposed analytical solutions were compared with that obtained by the finite element method (FEM) provided by GeoStudio 2007 (GEO-SLOPE Ltd., Calgary, Canada). SEEP/W was used to model the flow of solutions, and CTRAN/W was used to model the movement of the contaminants through the multi-layered system. The material and contaminant properties are shown in Table 1. DCM (dichloromethane) was chosen as the contaminant in the soil, and the initial pore-fluid concentration of DCM in the soil was assumed to be 1 mg/L. The material parameters of the soil column, the filter cake and the clogged geotextile were obtained from Giroud (2005) and Xu (2005). The water head difference was maintained at 35 cm. The effect diffusion coefficients of DCM for the geotextile and the soil column and the distribution coefficient of DCM for soil column were taken directly from Rowe et al. (2004). The dispersivity was obtained from Gabr et al. (1996).

The initial pore-fluid concentration of DCM in the soil was assumed to be a constant, and the geotextile was assumed to be free of contaminants (Fig. 2). In the numerical model, the Dirichlet and Neumann boundaries were considered as the top and bottom boundaries, respectively. The finite element (FE) mesh was also illustrated in Fig. 2.

As shown in Fig. 3, the contaminant concentration profiles in the soil column including the filter cake and the clogged geotextile obtained by the presented analytical solution were compared with those obtained by GeoStudio 2007. The results of the presented analytical solutions are agreed well with that obtained by the finite element method. The excellent agreement validates the proposed analytical solution.

5. Results and discussions

An illustrative example is presented to show the effect of the filter cake and the pore clogging on soil flushing. Benzene was selected to represent the contaminant in the soil, and the ethanol solution was selected as the flushing liquid. The initial pore-fluid concentration of



Fig. 2. FE mesh of the numerical model.



Fig. 3. Comparisons of the concentration profiles with GeoStudio 2007

 $\label{eq:table_$

Property		DCM	
n _{s1}		0.47	
n _{s2}		0.4	
$ ho_{d1}$	(kg/m ³)	1.4×10 ³	
$ ho_{d2}$	(kg/m ³)	1.65×10 ³	
Δh	(m)	0.35	
k _{s1}	(m/h)	2.02×10 ⁻³	
k _{s2}	(m/h)	1.72×10 ⁻⁵	
k _g	(m/h)	3.6×10⁻³	
K _d	(m ³ /kg)	1.5×10⁻³	
α _z	(m)	2.0	
D _s *	(m²/h)	2.9×10 ⁻⁶	
D_s^* D_g^*	(m²/h)	1.8×10 ⁻⁹	
Z ₁	(mm)	92	
Ls	(mm)	100	
L_g	(mm)	6	

benzene in the soil was assumed to be 1 mg/L. The material properties of the soil column and the filter cake are the same as those presented in Table 1. The effect diffusion coefficients in the soil column and in the geotextile for Benzene were taken directly from Sangam and Rowe (2005) and Hrapovic (2000). The distribution coefficient of benzene is 7×10^{-4} m³/kg for the soil column. The material and the chemical properties of the media are summarized in Table 2.

When geotextile filters are placed in intimate contact with the soil, the filter cake will not develop, and the geotextiles with a tortuous surface in contact with the soil will restrain the development of a cake (Giroud, 2005). Therefore, three different cases are considered in this study. The case without regard to the filter cake and the pore clogging in the geotextile was chosen as the reference case. The hydraulic conductivity of the geotextile without pore clogging was obtained directly from Giroud (2005). As shown in Table 3, case 2 just considered the effect of the pore clogging combined with a filter cake.

All concentration profiles of the three cases were obtained on the basis that the water head difference was maintained at 5 cm, and correspondingly the seepage velocities of these cases are 1.01×10^{-3} m/h, 9.75×10^{-4} m/h and 9.8×10^{-5} m/h, respectively. The comparisons of the concentration profiles with respect to three different cases are shown in Fig. 4. It is indicated that the effect of the pore clogging within the geotextile on the efficiency of soil flushing is negligible. The 5-hour base concentration at z = 50 mm increases from 0.16 mg/L for the case assuming no pore clogging in the geotextile and no filter cake to 0.17 mg/L for that just considering the pore clogging in the geotextile. The efficiency of soil flushing at the depth of z = 50 mm decreases by just 1.2% due to the pore clogging in the geotextile.

On the other hand, the filter cake has a significant influence on the efficiency of soil flushing as shown in Fig. 4. The 5-hour base concentration at z = 5 cm increases to 0.79 mg/L when the pore clogging combined with a filter cake are considered. The efficiency of soil flushing at the depth of z = 50 mm decreases by approximately 75% due to the filter cake. The results suggest that the formation of a filter cake at the surface of a geotextile is far more disadvantageous to soil flushing than pore clogging within a geotextile. At the stage of PVDs installation, it should be ensured that the geotextile filters are placed in intimate contact with the soil to restrain the development of the filter cake.

Table 2. Parameters of the model for analysis

Property		Benzene	
n _{s1}		0.47	
n _{s2}		0.4	
$ ho_{ m d1}$	(kg/m ³)	1.4×10 ³	
$ ho_{ m d2}$	(kg/m ³)	1.65×10 ³	
Δh	(m)	0.05	
K _d	<i>(</i> m³/kg)	7×10 ⁻⁴	
α _z	(m)	2.0	
D_s^*	(m²/h)	1.8×10 ⁻⁶	
α_z D_s^* D_g^*	(m²/h)	7.2×10 ⁻¹⁰	
k _{s1}	(m/h)	2.02×10 ⁻³	
k _{s2}	(m/h)	1.73×10⁻⁵	

Table 3. Parameters of model for three different cases				
Property		Case 1	Case 2	Case 3
Z 1	(mm)	100	100	92
Ls	(mm)	100	100	100
Lg	(mm)	6	6	6
<i>k</i> g	(m/h)	1.8×10	3.6×10⁻³	3.6×10 ⁻³



Fig. 4. Effect of pore clogging and filter cake on soil remediation

6. Conclusions

1. An analytical solution for one-dimensional soil flushing considering the effect of the filter cake and the pore clogging in the geotextile is presented in this paper. The results obtained by the proposed analytical solutions agree well with those obtained by the finite element method. The excellent agreement validates the proposed analytical solution.

2. Results show that the effect of the pore clogging within the geotextile on the efficiency of soil flushing can be negligible. The 5-hour base concentration at z = 5 cm for the case just considering the pore clogging in the geotextile is just 1.06 times more than that assuming no filter cake and no pore clogging in the geotextile.

3. The filter cake has a significant influence on the efficiency of soil flushing. When the pore clogging combined with a filter cake are considered, the 5-hour base concentration at z = 5 cm increases by a factor of approximately 4.94. The results suggest that the formation of a filter cake at the surface of a geotextile is far more disadvantageous to soil flushing than pore clogging within a geotextile. At the stage of PVDs installation, it should be ensured that the geotextile filters are placed in intimate contact with the soil to restrain the development of the filter cake.

4. The proposed analytical solution is stable, relatively simple, and could be used for checking more complex numerical methods, and making quick preliminary calculation to evaluate experimental results.

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Symbols and abbreviations

R _{d1}	Retardation factor of soil column
R _{d2}	Retardation factor of filter cake
C _{s1}	Pore-fluid concentration of contaminant in soil
C _{s2}	Pore-fluid concentration in filter cake
C _g	Contaminant concentration in geotextile
D _{s1}	Coefficient of hydrodynamic dispersion of soil
	column
D _{s2}	Coefficient of hydrodynamic dispersion of filter
	cake
Dg	Coefficient of hydrodynamic dispersion of
	geotextile
n _{s1}	Porosity of soil column
n _{s2}	Porosity of filter cake
qz	Vertical seepage velocity
k _{s1}	Hydraulic conductivity of soil column
k _{s1}	Hydraulic conductivity of filter cake
k _g	Hydraulic conductivity of geotextile
L _{s1}	Length of soil column
L _{s2}	Length of filter cake
Lg	Length of clogged geotextile
Δh	Water head difference
ρ _{d1}	Dry bulk density of soil column
ρ_{d2}	Dry bulk density of filter cake
K _d	Equilibrium distribution coefficient
D [*] s	Effective diffusion coefficient of soil
D [*] g	Effective diffusion coefficient of geotextile

α _z	Dispersivity	λ_{sim}	Substitute factor
C ₀	Initial pore-fluid concentration of contaminant	β_{sim}	Substitute factor
A _{sim}	Substitute factor	λ_{gm}	Substitute factor
B _{sim}	Substitute factor	β_{gm}	Substitute factor
A _{gm}	Substitute factor	ξm	Integral constant
B _{gm}	Substitute factor		