# PREDICTION OF SOIL WATER CHARACTERISTIC CURVE USING PHYSICALLY BASED SCALING TECHNIQUE

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ABSTRACT: The soil water characteristic curve (SWCC) is an important hydraulic parameter for modeling water flow and contaminant transport in the vadose zone. However, direct measurement of the SWCC is still difficult. The Arya and Paris (AP) model estimates the SWCC from particle-size distribution curve (PSD) based on the shape similarity of the two curves. They introduced an empirical parameter,  $\alpha$ , used to scale pore attributes from hypothetical formations to natural structures. Several approaches have been used to derive  $\alpha$ . However, the calculation procedures of these approaches are either quite complicated or are developed without paying much attention to the physical significance of the soil properties. In the present paper the physically based scaling technique (PBS) was applied to derive  $\alpha$  for the AP model. Fifty soil samples, representing a range of textures that include sand, sandy loam, loam, silt loam, and clay, were selected from UNSODA hydraulic property database for calculating  $\alpha$  using PBS approach. In addition, nineteen soil samples with different textures were used to verify effectiveness of proposed  $\alpha$  values. The results compared with other approaches show that the PBS technique combine with the AP model is a more useful and easier approach to predict SWCC from PSD.

Keywords: Vadose zone, soil water characteristic curve, basic soil properties, Arya and Paris model, empirical parameter.

### INTRODUCTION

The vadose (unsaturated) zone plays a crucial role in the hydrological cycle, and the transport and fate of contaminants (Ireson et al., 2009). The soil water characteristic curve (SWCC) which reflects the storage capacity of soil (Schulze et al. 1985) is an important hydraulic property of vadose zone. The SWCC is often required as input in soil water flow and contaminant migration models supporting hydrologic, environmental engineering (Henry and Smith, 2006; Ireson et al., 2009). However, direct measurement of the SWCC is time consuming, expensive, labor intensive, and subject to numerous errors. As a result, indirect approaches that estimate the SWCC from routinely available taxonomic data (e.g., texture, bulk density, particle density, and organic matter content) using pedotransfer functions have become of interested (Arya et al., 2008).

Arya and Paris (1981) proposed a physico-empirical model (AP model) that is used to estimate the SWCC from particle-size distribution (PSD). The basis for this approach is mainly on the shape similarity of the two curves. The AP procedure introduced an empirical parameter,  $\alpha$ , used to scale pore attributes from hypothetical formations to natural structures. Arya and Paris (1981) initially determined  $\alpha$  as constant, whereas several researchers have suggested that variable  $\alpha$  would improve the predictions of the SWCC (Basile and D'Urso, 1997; Arya et al., 1999; Vaz et al., 2005). Fractal concepts have also been used to derive  $\alpha$  (Tyler and Wheatcraft, 1989). However, the calculation procedures of these approaches are quite complicated or do not pay much attention to the physical significance of the soil properties.

In the past decades, the scaling technique has been used to characterize hydraulic properties of field-scale vadose zones, using measurement scales that are typically much smaller (Miller and Miller, 1956; Peck et al., 1977; Tuli et al., 2001). With growing water quality issues, this scale-transfer question is being asked more frequently than ever (Hopmans et al., 2002). Kosugi and Hopmans (1998) presented an elegant physically based

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scaling technique (PBS) which provides a convenient way to coalesce multiple SWCCs into a single reference SWCC (Tuli et al., 2001; Bhabani et al., 2005).

The objective of this study was to extend the PBS approach to the AP model, yielding  $\alpha$  to estimate SWCC from PSD. A total of 50 experimental soil data representing a range of textures that include sand, sandy loam, loam, silt loam, and clay, were selected from the UNSODA hydraulic property database (Nemes et al., 2001) for this purpose. In addition, 19 soil samples with different textures were used to test this method. The results predicted from the PBS approach were compared with other methods to verify its effectiveness.

## MATERIALS AND METHODS

#### Experimental data

Experimental SWCCs, PSD, bulk density, and particle density data were obtained from the Unsaturated SOil hydraulic DAtabase (UNSODA) (Nemes et al., 2001). The UNSODA contains of SWCC, hydraulic conductivity and water diffusivity data as well as pedological information of some 790 soil samples from around the world (e.g. United States, Netherlands, United Kingdom, Germany, Belgium, Denmark, Russia, Italy, and Australia). Sixty-nine soil samples (group A),

Table 1 Textural	classes a	nd UNSODA	codes for
samples			

Textural class	UNSODA codes	Use	
Sand	1462, 1463, 1464, 1465, 1466, 1467, 3330, 3331, 3332, 3340, 4523, 4660, 4661	Calculating α	
	1460, 2100, 4650, 4651 1161, 1380, 1381, 2532,	Testing $\alpha$	
Sandy loam	3290, 3310, 3320, 3321, 3223	Calculating α	
	3291, 3300, 3301, 3311 1370, 2530, 2531, 4591,	Testing $\alpha$	
Loam	4600, 4610, 4620	Calculating a	
	3293, 3302, 3303, 4592 1280, 1281, 1282, 1341,	Testing $\alpha$	
Silt loam	1342, 1350, 1352, 1490, 2000, 2002, 2010, 2011, 4510, 4671, 4672, 4673	Calculating $\alpha$	
	1340, 1351, 2001, 2012	Testing $\alpha$	
Clay	1162, 1163, 2360, 2362, 4680	Calculating $\alpha$	
Citay	1400, 2361, 4681	Testing $\alpha$	

representing a range of textures that include sand (group S), sandy loam (group SL), loam (group L), silt loam (group SiL), and clay (group C), were selected for this study. Among them, 50 soil samples were used to calculate  $\alpha$  using PBS approach, 19 soil samples used to verify the calculated  $\alpha$  value. All soils are identified in Table 1.

#### Arya and Paris Model

The AP model translates the percentage of particles smaller than the diameter axis of the PSD curve to volumetric water content and the particle diameter axis to suction head (Arya and Paris, 1981; Arya et al., 1999; Arya et al., 2008). First, the PSD is divided into n size fractions that was originally suggested by Arya and Paris (1981) as 20 diameter classes (1, 2, 3, 5, 10, 20, 30, 40, 50, 70, 100, 150, 200, 300, 400, 600, 800, 1000, 1500, and 2000 µm). In each fraction, the solid mass was assembled to form a hypothetical, cubic close-packed structure consisting of uniform-sized spherical particles. Starting with the first fraction (smallest particles), calculated pore volumes are progressively summed and considered filled with water. Each summations of filled pore volumes is divided by the bulk volume of the whole sample to obtain volumetric water content at the upper bound of successive mass fractions. An equivalent pore radius is calculated for each fraction and converted to soil suction head using the capillary equation. Calculated suction heads are sequentially paired with calculated volumetric water contents to obtain a predicted SWCC. The capillary equation that relates soil suction head  $(h_i)$ and pore radius  $(r_i)$  is given as follows:

$$h_i = \frac{2\gamma\cos\Theta}{\rho_w gr_i} \tag{1}$$

where  $\gamma$  is the surface tension at the air-water interfacial (N m<sup>-1</sup>),  $\Theta$  is the contact angle,  $\rho_w$  is the density of water (kg m<sup>-3</sup>), and g is the acceleration due to gravity (m s<sup>-2</sup>).

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The calculation of the volumetric water contents from the PSD as the contribution of each fraction to soil wetting:

$$\theta_i = \phi S_w \sum_{i=1}^{l} W_i \tag{2}$$

where  $\phi$  is the total soil porosity (cm<sup>3</sup> cm<sup>-3</sup>),  $S_w$  is the ratio of measured saturated water content to theoretical porosity and  $W_i$  is the soil mass of the *i*th fraction (*i* =1, ..., *I*). Soil porosity can be calculated from soil bulk density  $\rho_b$  (kg cm<sup>-3</sup>) and particle density  $\rho_s$  (kg cm<sup>-3</sup>):  $\phi = 1 - (\rho_b / \rho_s)$ .

Porous radius of *i*th fraction  $(r_i)$  is determined from soil particle diameter  $(D_i)$  considering packing of

uniform-sized spherical particles and an empirical parameter  $\alpha$  that corrects for natural structure soil

$$r_i = \frac{D_i}{2} \left[ \frac{2en_i^{1-\alpha_i}}{3} \right]^{0.5}$$
(3)

where  $n_i$  is the number of particles of *i*th fraction, and *e* is the void ratio, given as follows:

$$n_i = \frac{6W_i}{\pi D_i^3 \rho_s} \tag{4}$$

$$e = \frac{\rho_s - \rho_b}{\rho_b} \tag{5}$$

The soil suction head is then calculated with the combination of Eq. (1), (3), (4) and (5) as follows:

$$h_{i} = \frac{\gamma}{\rho_{w}gD_{i}\sqrt{\frac{2(\rho_{s}-\rho_{b})}{3\rho_{b}}\left(\frac{W_{i}}{6\pi D_{i}^{3}\rho_{s}}\right)^{1-\alpha_{i}}}}$$
(6)

Once the empirical parameter  $\alpha_i$  is known, the calculated volumetric water contents are paired with the predicted soil suction heads (Eq. (6)) to construct SWCC.

#### Physically based scaling technique

The single objective of scaling is to coalesce a set of functional relationships into a single curve using scaling factors that describes the set as a whole (Tuli et al., 2001). Miller and Miller (1956) introduced a similarmedia concept to conveniently describe soil variability in a unified manner. They assumed that the microscopic structures of two "geometrically similar" soils differ only by a characteristic length  $\lambda$  (Warrick et al., 1977). The scaling factor  $\delta_j$  is defined as the ratio of a characteristic length  $\lambda_j$  of soil sample *j* and the characteristic length  $\lambda_{ref}$  of a reference soil (Peck et al., 1977) as follows:

$$\delta_{j} = \frac{\lambda_{j}}{\lambda_{ref}} \tag{7}$$

Kosugi and Hopmans (1998) presented an elegant physically based scaling (PBS) technique which introduced the median pore radius  $r_m$ , as the characteristic length to scale SWCC for soils that are characterized by a lognormal pore-size distribution, f:

$$f(\ln r) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{\left(\ln r - \ln r_m\right)^2}{2\sigma^2}\right]$$
(8)

where r is the pore radius (cm), and  $\sigma$  is the standard deviation of the frequency distribution. Based on this

assumption, the lognormal SWCC function as a cumulative curve of Eq. (8) was given by Kosugi and Hopmans (1998):

$$S_{e}\left(\ln h\right) = \frac{\theta - \theta_{r}}{\theta_{s} - \theta_{r}} = F_{n}\left[\frac{\left(\ln h_{m} - \ln h\right)}{\sigma}\right]$$
(9)

where  $\theta_s$  and  $\theta_r$  denote the saturated and residual water contents (cm<sup>3</sup>cm<sup>-3</sup>), ln( $h_m$ ) and  $\sigma$  are the mean and the standard deviation of ln(h), respectively. The median suction head  $h_m$  (cm) is related to the median pore radius ( $r_m$ ) by Eq. (1).  $F_n(x)$  is the normal distribution function defined as

$$F_{n} = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-x^{2}/2) dx$$
 (10)

Then, the reference SWCC function  $S_{e,R}$ , is given by the following parametric relation (Kosugi and Hopmans, 1998):

$$S_{e,R}(\ln h) = F_n \left[ \frac{(\ln h_{m,R} - \ln h)}{\sigma_R} \right]$$
(11)

where  $\ln h_{m,R}$  and  $\sigma_R$  represent the mean and standard deviation of  $\ln(h)$  for the reference soil, and are computed from

$$\ln(h_{m,R}) = \frac{1}{J} \sum_{j=1}^{J} \ln h_{m,j}$$
(12)

$$\sigma_R^2 = \frac{1}{J} \sum_{j=1}^J \ln \sigma_j^2$$
(13)

where *J* denotes the number of soil samples in the set and the individual  $\ln(h_{m,j})$  and  $\sigma_j^2$  values are determined from the fitting of Eq. (9) to individual SWCC data. Accordingly, scaling factors for each soil sample, *j*, can be computed directly from (Kosugi and Hopmans, 1998; Tuli et al., 2001):

$$\delta_j = \frac{r_{m,j}}{r_R} = \frac{h_{m,R}}{h_{m,j}} \tag{14}$$

Derive  $\alpha$  using physically based scaling technique

Initially, the PBS technique was used to compute  $\alpha$  value for all 50 calculating soils in Table 1. Later calculating soils were divided into five subpopulations based on the soil texture: sand, sandy loam, loam, silt loam and clay soils. Each textural class subpopulation was then computed to its respective  $\alpha$  value. In the present study, the parameter  $\alpha$  is assumed to have a single value for each soil texture and all soils combined together, respectively. The detailed procedure to derive  $\alpha$  values is as follows:

#### Scaling of measured soil water characteristic curves

The experimental SWCC data points were fitted to lognormal model (Eq. (9)), yielding parameters  $h_{m,measured}$ and  $\sigma_{measured}$  for each soil sample *j*. Subsequently, the measured reference SWCC function  $S_{e,R_{measured}}$  was calculated using Eqs. (11)-(13) for each soil texture and all soils combined together. In this study, we assume that the porosity is equivalent to  $\theta_s$ . For soils that did not provide porosity or  $\theta_s$  value, the first point of the experimental SWCC data that corresponds to the lowest suction head was used as  $\theta_s$  (Chan and Govindaraju, 2004), and  $\theta_r$  was assumed to be zero when the suction being infinity (Fredlund and Xing, 1994).

### Scaling of predicted soil water characteristic curves

The AP model is based mainly on the similarity between the shapes of the cumulative PSD curve and the SWCC. Therefore, PSD data were also fitted to lognormal function to determine the cumulative PSD curve. The function proposed by Buchan (1989) as follows:

$$f(\ln D) = F_{n}\left[\frac{(\ln D - \mu)}{\sigma}\right]$$
(15)

where  $f(\ln D)$  is the cumulated frequency distribution function associated with the natural logarithm of particle-size diameter, D, for particle-size classes i=1, ..., I, and  $\mu$  and  $\sigma$  denote the mean and standard deviation of the ln-transformed particle diameter, respectively. Subsequently, 20 diameter classes were selected as suggested originally by Arya and Paris (1981). A series of potential  $\alpha$  values ( $\alpha_{potential}$ ) were selected for each soil texture and all soils combined together, respectively. Using Eqs. (2)-(6) SWCC can be estimated from PSD for each soil samples. Then, the predicted SWCC data points were fitted to lognormal model (Eq. (9)) to determine the parameters  $h_{m,predicted}$  and  $\sigma_{predicted}$ for each soil core *j*. After that, using Eq. (11) to calculate the predicted reference SWCC function,  $S_{e,R_{predicted}}$ , according potential  $\alpha$  value for each soil group.

#### Calculate optimal $\alpha$ values

An iterative procedure was used that minimized the root mean square error (RMSE) between  $S_{e,R_{measured}}$  and  $S_{e,R_{predicted}}$  to determine optimal  $\alpha$  values for each soil group. The RMSE given by

$$RMSE(\alpha_{potential}) = \left\{ \frac{\sum_{l=1}^{L} \left[ S_{e,R_{measured}}(\ln h_l) - S_{e,R_{protected}}(\ln h_l, \alpha_{potential}) \right]^2}{L-1} \right\}^{0.5} (16)$$

where *L*, denotes the total number of suction head (*h*) values, that were fixed ranging between 0.1 cm and  $10^{10}$  cm in present study. Microsoft Excel 2010 (Microsoft Corporation) was used for all of the nonlinear optimization runs.

## Verification

After obtaining the optimal  $\alpha$  values for each texture and all the soils, testing soils in Table 1 were used to

Table 2 Represented methods to predict SWCCs according AP model

Method	$\alpha$ value and equation			
Constant <i>a</i>	$\alpha$ =1.38 (Arya and Paris, 1981), and 0.938 (Arya and Dierolf, 1992) for all the soil classes. And $\alpha$ = 1.285, 1.459, 1.375, 1.150, and 1.160 for the sand, sandy loam, loam,			
	silt loam, and clay soils (Arya et al., 1999).			
Logistic equation				
(Arya et al., 1999)	$(Y + \Delta Y) = \frac{Y_f Y_i}{Y_i + (Y_f - Y_i) \exp[-\mu(x + \Delta x)]}$			
	where Y is the dependent variable log $N_i$ , $Y_f$ is the final value of log $N_i$ , $Y_i$ is the initial value of log $N_i$ , $\mu$ is the rate coefficient, x is the independent variable log $n_i$ , $\Delta Y = \Delta \log n_i$ , $\Delta x = \Delta \log n_i$ , and $\alpha_i = \log N_i / \log n_i$ . $n_i$ and $N_i$ is the number of spherical particles in the ideal and natural structure soil, respectively. These parameters values were shown in Table 2 of Arya et al. (1999).			
Linear equation (Arya et al., 1999)	$\alpha_i = \left[\frac{a + b \log\left(8W_i/D_i^3\right)}{\log n_i}\right]$			
	Parameters for equation were represented in Table 3 of Arya et al. (1999).			
$\alpha = f(\theta)$	$\alpha_i = 0.947 + 0.427 \exp(-\theta_i/0.129)$			
(Vaz et al., 2005)	where $\theta_i$ is the water content of each fraction (cm <sup>3</sup> cm <sup>-3</sup> ).			

verify the effectiveness of PBS approach. To compare the results with previous similar studies, the SWCCs were also predicted with the methods in Table 2. Statistical comparison of the results was carried out in terms of the coefficient of determination ( $\mathbb{R}^2$ ), and root mean square error (RMSE) to determine the accuracy of these methods and the correlation between the measured and predicted SWCC.

The Table 2 lists the represented methods to predict SWCCs according AP model, including constant  $\alpha$  and variable  $\alpha$  methods. Except methods in the Table 2, there are some approaches to estimate SWCC based on AP model. For example, in Basile and D'Urso (1997),  $\alpha$  was assumed as a function of soil suction head (*h*). However,

the use of the  $\alpha = f(h)$  relationship is quite complicated due to the interdependence of  $\alpha$  and h in the application of the AP model (Vaz et al., 2005). Fractal concepts have also been used to determine  $\alpha$  value (Tyler and Wheatcraft, 1989). However, fractal approaches account only for the effects of the tortuosity of pore lengths but not for other factors that influence the SWCC, such as bulk density. These methods were ignored in present study due to their defects.

## **RESULES AND DISCUSSION**

Scaling of measured soil water characteristic curves



Fig. 1 Unscaled measured SWCCs for each texture and all soils combined together

The measured SWCCs of 50 soils for calculating the optimal  $\alpha$  are shown in Figure 1. All the SWCCs were successfully described by the lognormal model (Eq. (9)) with more than 91.04 % of SWCCs having R<sup>2</sup>> 0.95 and all SWCCs with R<sup>2</sup> > 0.91. Such high values of R<sup>2</sup> indicate the effectiveness of the lognormal model in describing measured SWCC data.

highest when all the 50 SWCCs are scaled together and it is reduced when soils are scaled after grouping them by soil textures. Thus, as expected, soils that are separated by textural group are more similar than when all soils are combined together. Consequently, soil texture may serve as a guide for distinguishing similar media.



Fig. 2 Scaled measured SWCCs for each texture and all soils combined together. The black solid lines represent the reference SWCC

We applied the PBS technique to soils with similar texture. Figure 2 shows scaled SWCCs (open circles) and reference SWCC (solid lines) for each textural class. Scaled SWCCs and reference SWCC for all data of 50 calculating soils are also shown in this figure. Resulting reference SWCC parameters are shown in Table 3. The effectiveness of scaling within respective textural class is evaluated by estimating the RMSE between scaled (open circles) and the reference (solid lines) SWCCs are shown in Table 3. Table 3 indicates the RMSE value is

Table 3 Scaling results for each texture and all soils combined together

Soil texture	$h_{m,ref}$ (cm)	$\sigma_{ref}$	RMSE
Sand	42.08	1.25	0.119
Sandy loam	756.09	3.78	0.105
Loam	3333.71	4.97	0.063
Silt loam	1452.21	4.33	0.122
Clay	43047.72	3.18	0.042
All soils	835.11	3.75	0.226

Optimal  $\alpha$  values ( $\alpha_{opt}$ )

The optimal  $\alpha$  with RMSE between  $S_{e,R_{measured}}$  and  $S_{e,R_{predicted}}$  for each texture and all soils combined together are shown in the Table 4. The better performance of sandy loam and loam soil textures are expressed by the lower RMSE values, which is likely caused by the smaller variations in soils compared with other textural classes.

Table 4 Optimal  $\alpha$  value ( $\alpha_{opt}$ ) for each soil each texture and all soils combined together

Soil	Sand	Sandy	Loam	Silt	Clay	All
texture		loam		loam	5	data
$\alpha_{opt}$	1.43	1.76	1.58	1.39	1.30	1.48
RMSE	0.034	0.012	0.017	0.085	0.033	0.038

Figure 3 shows the measured reference soil water characteristic curve (reference SWCC<sub>m</sub>) and the optimal predicted reference soil water characteristic curve (optimal reference SWCC<sub>p</sub>) correspond to  $\alpha_{opt}$  for each texture and all soils. Figure 3 also includes the predicted reference soil water characteristic curve for the possible  $\alpha$  values (possible reference SWCC<sub>p</sub>), allowing a qualitative evaluation of the sensitivity of the parameter  $\alpha$  on the predicted SWCC. From Figure 3, the prediction approach seems to fit the measured SWCC data better for sandy loam and loam soil textures, as reflected by the better agreement between the experimental with calculated reference curves (Fig. 3b and c).

## Verification results

The SWCCs of all testing soils were predicted using optimal  $\alpha$  values in Table 4 for each texture and all the soils. The results were compared with the predictions of



Fig. 3 Reference SWCC<sub>m</sub> (black dash lines), possible reference SWCC<sub>p</sub> (gray solid lines) and optimal reference SWCC<sub>p</sub> (black solid lines) correspond to  $\alpha_{opt}$  for each texture and all the soils

the methods in Table 2. Figure 4 shows the ability of eight  $\alpha$  equations to predict the data for five textural soils. Typical examples of predicted and measured SWCCs for sand (code: 4650), sandy loam (code: 3311), loam (code: 3302), silt loam (code: 2012), and clay (code: 2361) soils are presented in Figure 4. Figure 4 shows that, the use of proposed  $\alpha_{opt}$  values for each texture ( $\alpha_{opt, T}$ , black solid lines in Figure 4) and all the soils ( $\alpha_{opt, A}$ , black dash lines in Figure 4) improve the AP estimation compare with other approaches. It is worth noting that when applied the  $\alpha_{opt, T}$  values for corresponding textural class obtain the best agreement between the experimental with calculated SWCCs in all cases. For other approaches, using linear equation causes a great overestimation of water content,  $\theta$ , in all cases. On the contrary, using  $\alpha = 0.938$ , logistic equation and  $\alpha = f(\theta)$ cause underestimation of water content,  $\theta$ , in the higher suction range.

The overall predictive ability of the eight  $\alpha$  equations are presented in Figure 5, which is evaluated by comparing measured and predicted water content,  $\theta$ , at the applied suction heads for all testing soils in Table 1. And Table 5 shows the root mean square error (RMSE) between measured and estimated  $\theta$ . Table 5 indicates that the best AP model estimation is obtained with the optimal  $\alpha$  values for corresponding textural class (i.e.  $\alpha_{opt}$ .  $_{T}$ ) that provides RMSE of 0.032, followed by the optimal  $\alpha$  values for all the soils (i.e.  $\alpha_{opt, A}$ ) that provide RMSE of 0.038. The results indicate that more accurate predicted results will be obtained when calculating  $\alpha$  for each textural class respectively. This is probably due to the fact that the soils that are separated by textural class are more similar than when all soils are considered together. The worst estimation is obtained with logistic equation that provides RMSE of 0.235, although it works relatively well for clay soils (see Figure 4e).



Fig. 4 Comparison of SWCC prediction methods with experimental data for a) sand, b) sandy loam, c) loam, d) silt loam, and e) clay (page 14 line 10)



Fig. 5 Comparison of measured and predicted volumetric water content,  $\theta$ , using different  $\alpha$  equations for all testing soils. The 1:1 line is a gray solid line and the best-fit line is a black solid line

Table 5 Comparison of RMSE values for different SWCCs prediction re
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Method	a=1.38	<i>α</i> =0.938	Linear	Logistic	$\alpha = f(\theta)$	$\alpha = \alpha_{c, T}^{a}$	$\alpha = \alpha_{opt, A}^{b}$	$\alpha = \alpha_{opt, T}^{c}$
RMSE	0.054	0.128	0.095	0.235	0.093	0.049	0.038	0.032

<sup>a</sup> $\alpha_{opt, T}$  is constant  $\alpha$  value for each soil texture (i.e.  $\alpha$ =1.285, 1.459, 1.375, 1.150, and 1.160 for the S, SL, L, SiL, and C soils) proposed by Arya et al., (1999); <sup>b</sup> $\alpha_{opt, A}$  is optimal  $\alpha$  value for all soils combined together (i.e.  $\alpha$ =1.48); <sup>c</sup> $\alpha_{c, T}$  is optimal  $\alpha$  value for each soil texture (i.e.  $\alpha$ =1.43, 1.76, 1.58, 1.39, and 1.30 for the S, SL, L, SiL, and C soils).

Figure 5 shows that the proposed  $\alpha_{opt, T}$  values for corresponding textural class (Figure 5h) gives an overall good agreement between measured and predicted water content,  $\theta$ . The linear regression has the highest coefficients of determination  $(R^2)$  of 0.964 and the regression line differed only slightly from the 1:1 line. Although the regression appears to be good with an  $R^2$  of 0.962, the  $\alpha_{opt, A}$  values for all the soils (Figure 5g) led to underestimation in the dry range and overestimation in the wet range (Figure 5h also has this phenomenon but appear slightly). This can partially be explained by the fact that at low suction heads, AP model assumes complete desorption of all pores. However, it is impossible in the practical case. And at high suction heads, a significant percentage of water may be held as film and in poorly connected pores. As a result, the model will tend to underestimate the water content in the high suction regions and overestimate the water content in the low suction regions.

Regarding other methods, some approaches appear to produce great bias. Figure 5 shows that use of  $\alpha$ =0.938 (Figure 5b), logistic equation (Figure 5c), and  $\alpha$ = $f(\theta)$  (Figure 5e) underestimate soil water content both in the dry range and wet range. In contrast, linear equation (Figure 5d) overestimates soil water content in all ranges.

In summary, compared with other predicted approaches the PBS approach exhibited better agreement between the measured and estimated SWCC, especially when applied to soils with similar texture. We conclude that the PBS technique combined with the AP model is a more effective and feasible approach to predict SWCC from PSD.

## CONCLUSIONS

In present study, the physically based scaling technique (Kosugi and Hopmans, 1998) was extended to the Arya and Paris model, calculating  $\alpha$  to predict soil water characteristic curve from particle-size distribution. A total of 50 experimental soil data representing a range of textures that include sand, sandy loam, loam, silt loam, and clay, were selected from the Unsaturated Soil hydraulic Database (Nemes et al., 2001) for this purpose. In addition, 19 soil samples with different textures were used to test this method. Results showed that the physically based scaling technique improved the Arva and Paris estimation and outperformed other approaches especially when applied to soils with similar texture. It should be noted that this study has examined only Unsaturated Soil hydraulic Database. Notwithstanding its limitation, this study can clearly demonstrate the potential capability to apply the physically based scaling

technique for estimating the soil water characteristic curve with a robust method in soil hydrologic studies.

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