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Vibration Analysis of Cracked Structures as a Roving Body Passes a Crack Using the Rayleigh-Ritz Method

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Abstract

The natural frequencies of a cracked plate with a roving mass were computed using the Rayleigh-Ritz Method for various sets of boundary condition. The obtained frequencies exhibit a sudden shift as a roving body crosses a crack. If the crack is only partial and continuity of translation is maintained, then the frequency shift occurs only when the body possesses a rotary inertia. If the crack is a complete one (through thickness) which permits differential translation to occur on either side of the crack, a particle having mass only (translatory inertia) is sufficient to cause a sudden shift. There is no need for a rotary inertia. This is potentially useful in detecting cracks in structures, as it is possible to track the changes in the natural frequencies of a structure as a test body such as a vehicle on a bridge moves and identify points where sudden frequency changes occur.

Keywords: Cracked plate; natural frequencies; rayleigh-ritz method; roving mass; vibration

1. Introduction

Identifying cracks through frequency measurements has been a subject of research for decades [1-14], but it still remains a challenge due to two main reasons. The frequency changes due to cracks are usually very small and the inverse problem of identifying cracks is further complicated by the fact that the frequencies depend on both the number, severity and locations of cracks. However, recent work [15] shows that in a beam, a roving body that has a rotary inertia causes a sudden shift in frequencies as it passes a crack.

In this paper, this phenomena is investigated for a plate with a roving body. Vibration analysis of plates with cracks also attract many researchers for decades [16-22]. We show that the frequencies of a plate with a crack will change abruptly as a mass attached to the plate is moved from one side of the crack to the other. This is potentially useful in detecting cracks in structures, as it is possible to track the changes in the natural frequencies of a structure as a test body such as a vehicle on a bridge, moves and identify points where sudden frequency changes occur. These would then correspond to potential crack locations irrespective of the number and severity of the cracks. To identify a crack and its location all that is needed is an observation of a sudden change in the natural frequencies. The location of the roving body then corresponds to a crack location. This sudden shift in frequency occurs in all modes with the exception of certain cases where the crack is at the nodal line and the use of a cumulative frequency shift parameter also helps to address the difficulty due to frequency changes being too small.

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2. Method

The Rayleigh-Ritz Method is used to find the natural frequencies of rectangular plates with cracks and a roving body (Fig. 1). The type of crack considered here is that there is a discontinuity in flexural rotation but the translation is continuous such as those considered in beams [15]. The differential rotation is related to the bending moment at the crack and a rotational spring stiffness representing the effective stiffness of the joint. In plates, the crack can also go through the full thickness and in this case, both translation and rotation are discontinuous.

The plate is also subject to a roving body, that is, a body whose location is changed to track any change in the frequencies but the body has no velocity relative to the plate. The Rayleigh-Ritz Method is then applied to calculate the natural frequencies of the plate and the results are then plotted against the location of the roving body. The crack is

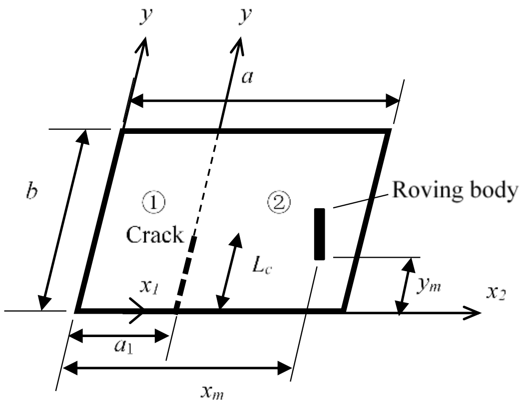

Figure 1. A cracked plate with a roving body

introduced as follows: The plate is formed by assembling two rectangular plates and the coupling between the two plates is enforced through distributed penalty stiffness that control the relative translations and rotations between the two components. A length along which a crack is present is subject to zero or low penalty stiffness but elsewhere along the joint sufficiently high penalty stiffness is applied. To represent a complete (through thickness) crack both translational and rotational penalty stiffness are set to zero while for flexural cracks similar to those in beams, the translational stiffness is set to a high value but rotational stiffness is set to a smaller value. Suitable magnitude of penalty stiffness is determined by using positive and negative stiffness values [23] which help to ensure that any error due to violation of the continuity is kept within the required accuracy.

The plate was subdivided into two rectangular segments (Segment 1 and 2) that have the separate coordinates (x_1, y) and (x_2, y), and for each segment the admissible functions in x, y directions consisted of a constant, a linear function, a quadratic function and a cosine series [24]. The out-plane displacement plate of a segment of a completely free plate, W_k (k = 1 or 2) can be defined by the following equations.

$$w_k\left(x_k, y, t\right) = W_k\left(x_k, y\right) \ \sin\omega t \tag{1}$$

with

$$W_k\left(x_k, y\right) = \sum_{i=1}^{N}\sum_{j=1}^{N} G_{ij}\phi_i\left(x_k\right)\phi_j\left(y\right) \tag{2}$$

and

$$\phi_i\left(x_k\right) = \left(\frac{x_k}{a_k}\right)^{i-1} \qquad \text{for } i = 1, 2 \text{ and } 3$$

$$\phi_i\left(x_k\right) = \cos\left(\frac{\left(i-3\right)\pi x_k}{a_k}\right) \qquad \text{for } i \geq 4$$

where ω is the circular frequency and t is time. $G_{i,j}$ are undetermined weighting coefficients. The above equations are substituted into the strain energy expression, V_k and kinetic energy expression, T_k given by Eqs. (3) and (4) respectively.

$$Vk = \frac{1}{2}D\int_0^{a_k}\int_0^{b}\left[\left(\frac{\partial^2 W_k}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W_k}{\partial y^2}\right)^2 + 2v\frac{\partial^2 W_k}{\partial x^2}\frac{\partial^2 W_k}{\partial y^2} + \ 2\left(1-v\right)\left(\frac{\partial^2 W_k}{\partial x \partial y}\right)^2\right] + dx_k d_y \tag{3}$$

and

$$T_k = \frac{\omega^2 \rho h}{2}\int_0^{a_k}\int_0^{b} {W_k}^2 dx_k dy \tag{4}$$

where,

$$D = \frac{Eh^3}{12\left(1-v^2\right)}$$

Here, D is the plate rigidity, E is Young's modulus, v is Poisson's ratio, ρ is the density of the material and h is the thickness of the plate.

The two segments were joined together by translational and rotational springs. For the length of the crack the rotational stiffness was of low magnitude for partial flexural cracks but for full cracks the stiffness values were set to zero. Elsewhere very high stiffness values of the order 10^6 times that of the typical plate stiffness was used to enforce continuity. The strain energy due to the springs between the plate segments are given by Eqs. (5) and (6)

$$V_t = \frac{1}{2}\int_0^{b} k_t\left(W_2\left(0, y\right) - W_1\left(a_1, y\right)\right)^2 dy \tag{5}$$

$$V_r = \frac{1}{2}\int_0^{b} k_r\left(W_2^{'}\left(0, y\right) - W_1^{'}\left(a_1, y\right)\right)^2 dy \tag{6}$$

where k_t and k_r are translational and rotational spring constants of artificial springs attached between the two segments respectively.

The translational and rotational kinetic energy of the roving mass are given by Eqs. (7) and (8) respectively.

$$T_t = \frac{1}{2}M_m\left(W_k\left(x_m, y_m\right)\right)^2 \qquad (k = 1 \text{ or } 2) \tag{7}$$

$$T_r = \frac{1}{2}I_m\left(W_k^{'}\left(x_m, y_m\right)\right)^2 \qquad (k = 1 \text{ or } 2) \tag{8}$$

By attaching translational and rotational springs along the plate edges, any typical set of boundary conditions can be incorporated with the penalty method [25]. The strain energy due to attached springs along the plate edges, V_e is given by Eq. (9).

$$\begin{aligned} V_e = &\frac{1}{2}\int_0^{b} K_t\left\{W_1\left(0, y\right)^2 + W_2\left(a_2, y\right)^2\right\}dy \\ &+ \sum_{k=1,2}\frac{1}{2}\int_0^{a_k} K_t\left\{W_k\left(x_k, 0\right)^2 + W_k\left(x_k, b\right)^2\right\}dx_k \\ &+ \frac{1}{2}\int_0^{b} K_r\left\{W_1^{'}\left(0, y\right)^2 + W_2^{'}\left(a_2, y\right)^2\right\}dy \\ &+ \sum_{k=1,2}\frac{1}{2}\int_0^{a_k} Kr\left\{W_k^{'}\left(x_k, 0\right)^2 + W_k^{'}\left(x_k, b\right)^2\right\}dx_k \end{aligned} \tag{9}$$

where, $a_2 = a - a_1$, and K_t is translational spring constant and K_r is rotational spring constant of the attached spring on the edges.

Equations (10) and (11) give the total strain and kinetic energy.

$$V_{total} = V_1 + V_2 + V_t + V_r + V_e \qquad (10)$$
$$T_{total} = T_1 + T_2 + T_t + T_r$$

The stiffness matrix and mass matrix used in the Rayleigh – Ritz analysis are derived from the total strain and kinetic energy equations.

3. Results and Discussion

The natural frequencies of a rectangular plate with a crack running parallel to one edge were computed using the Rayleigh-Ritz Method. The results were generated for various sets of boundary conditions with a partial flexural crack at a_1=0.4a running the full width of the plate (L_c=b) for the non-zero natural frequency. When the mass is exactly at the location of the crack, (i.e. x_m/a = 0.4) the natural frequencies were calculated for two cases, one is where the mass is on the edge of Segment 1 and the other is where it is on Segment 2. Roving body had a mass of 5% of the plate mass. For the rotary inertia a radius of gyration of 0.1a was used.

A convergence study was carried out for a completely free square plate with a crack that located at $a_1 = 0.4a$. Table 1 shows frequency parameters against the order of polynomial in Eq. (2). It shows that the results for first three bending modes are converged in four significant figures by the order of polynomial of 10 × 10.

Figure 2 shows the variation of a non-dimensional first frequency parameter $\Omega = \omega a^2(\rho h/D)^{0.5}$ for a completely free square plate against the location (x_m/a) of a roving body with (continuous line) and without (dotted line) rotary inertia, for $y_m = 0.3b$. With the roving mass having rotary inertia, it can be seen that there is a sudden change in the frequency parameter when the roving mass passes the crack.

Table 1. Convergence of frequency parameters for a completely free square plate with a crack (a_1 = 0.4a)

Mode	Order of polynomial (i × j)					
	5x5	**6x6**	**7x7**	**8x8**	**9x9**	**10x10**
1	12.23	12.23	12.23	12.23	12.23	12.23
2	13.50	13.44	13.44	13.43	13.43	13.43
3	21.71	21.71	21.70	21.70	21.70	21.70

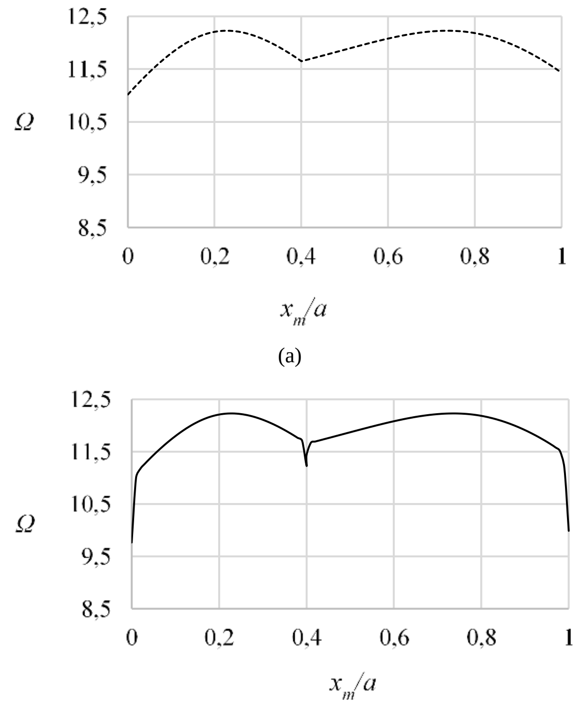

Figure 2. The frequency parameter against the location of the roving mass for a completely free square plate (a) without rotary inertia, (b) with rotary inertia.

Figures 3 and 4 show the first frequency parameter computed including the rotary inertia for simply supported and clamped square plates respectively. Those results were obtained using the penalty method with Eq. (9). It can also be seen that there is a sudden change in the frequency parameter when the roving mass passes the crack.

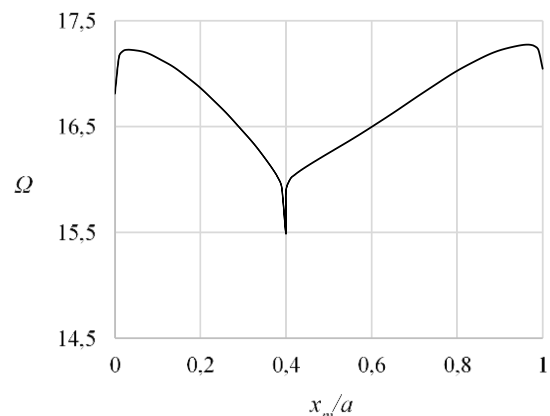

Figure 3. The frequency parameter against the location of the roving mass for a simply supported square plate.

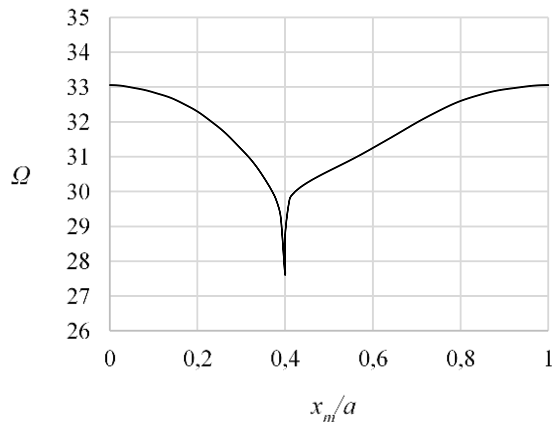

Figure 4. The frequency parameter against the location of the roving mass for a clamped square plate.

For the case of the clamped plate (Fig. 4), there is no effect of the roving mass when it is on the plate edges, (i.e. x_m/a = 0 and 1) since there is no translation and rotation on the clamped edge. However, this is not the case for completely free and simply supported plates. The effect of the roving mass when it is on the edges of these plates are observed in Figs. 2 and 3.

Figure 5 shows the first frequency parameter for a cantilever plate where the crack is perpendicular to the clamped edge. An abrupt change in the frequency is observed however, the change is smaller than those of the simply supported and clamped plates.

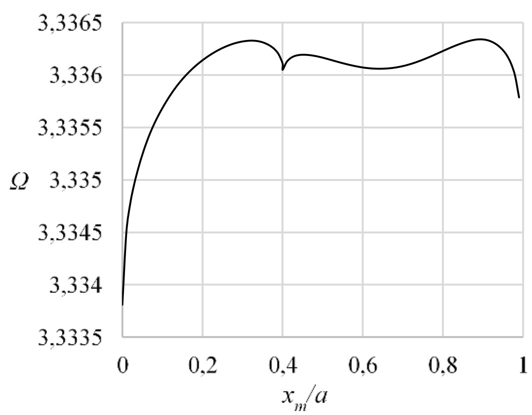

Figure 5. The frequency parameter against the location of the roving mass for a cantilever square plate.

4. Conclusions

The natural frequencies of a thin square plate under various boundary conditions with a crack parallel to an edge were computed using the Rayleigh-Ritz Method. The computed frequencies exhibit a sudden shift as a roving body crosses a crack. If the crack is only partial and continuity of translation is maintained, then the frequency shift occurs only when the body possesses a rotary inertia, as has been observed in beams. If the crack is a complete one (through thickness) which permits differential translation to occur on either side of the crack, a particle having mass only (translatory inertia) is sufficient to cause a sudden shift. There is no need for a rotary inertia. Future work would be to study the effect of cracks that are not parallel to an edge, and non-straight cracks. The body used in this study was assumed to possess mass and rotary inertia at a point. The effect of a body of small but finite dimensions also would be investigated.

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