

Minimum Weight Design of Composite Panel under Aeroelastic Constraint

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Abstract

The present study minimizes the number of layers of wing shaped plates fabricated by laminated composite (CFRP) under aeroelastic constraint which keeps stable flutter speed. The finite element analysis (FEA) for the aeroelastic characteristics requires considerable calculation effort. Therefore, the response surface method (RSM) is utilized to reduce the computational time for optimization where an approximate expression uses bending rigidity of composite plates as input and flutter speed as output. Then, distributed genetic algorithm (DGA) is employed as an optimizer and minimizes numbers of layers of composite plates by designing fiber orientation angles. The optimum wing model shows large amount of weight reduction and enough flutter speed in less computational time than without RSM.

Keywords: Aeroelastic flutter; genetic algorithm; laminated composite; optimum design

1. Introduction

In aerospace industry, weight saving of airframes is the most important issue due to increasing fuel costs, and use of composite materials, especially carbon fiber reinforced plastics (CFRP), are mainstream of aircraft development. CFRP are utilized for structures by stacking orthotropic layers because they have high specific strength and stiffness [1]. Design of its lay-up configurations enables aeroelastic tailoring, which controls stiffness of wings to meet aeroelastic requirements. Aeroelasticity is a field of study focusing on interactions among aerodynamic force, elastic force and inertia force for airframes [2]. Weisshaar [3] showed that composite materials are superior in aeroelastic tailoring. In aeroelastic problems, flutter [4] is critical in particular. Kameyama and Fukunaga [5] observed flutter properties of composite plates by using lamination parameters. Guo [6] optimized composite wings to satisfy weight and flutter requirements. These reports conducted aeroelastic tailoring for composite wings, however, there are few publications that aim at weight minimization of

composite wings and search practical lay-up configurations in consideration of lay-up constraint.

The present paper proposes the practical optimization method for lightweight aircrafts. Distributed Genetic Algorithm (DGA) [7] is employed as an optimizer. In the optimization, fiber orientation angles of laminated composites are assigned to design variables, and the critical value of flutter speed is imposed as the constraint.

The optimization problem requires a lot of calculation effort for aerodynamic analyses of the wing shaped models by the finite element analysis (FEA). Therefore, Response Surface Method (RSM) is involved in the optimization. The RSM is available to various optimization problems in order to reduce computational cost. Todoroki and Ishikawa [8] studied the optimization of laminate stacking sequence for maximizing buckling load with Genetic Algorithm (GA) and the RSM. The optimum wing model obtained by the optimization in less computational time showed the effective weight reduction and enough resistance to the flutter. The present study employs MD. Nastran for aerodynamic analysis and MATLAB for other optimization process.

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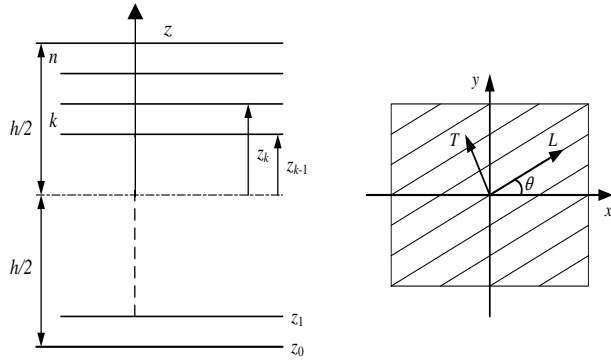


Figure 1. The coordinate systems of laminated plates.

and minor Poisson ratios, z_k is the distance from the plate middle surface to the upper surface of the k th layer, and θ is the fiber orientation angle in the k th layer.

2.2. Finite element analysis

The finite element model with four-node elements as shown in Fig. 2 is used for structural and aerodynamic finite element analyses (FEA) by MD. Nastran. In structural analysis, the lowest four vibration mode shapes and natural frequencies are calculated. In aerodynamic analysis, critical air speed leading to flutter V_F is evaluated with the $\Delta V_F = 1$ m/s. Dimensions of this model and material constants for CFRP are defined as

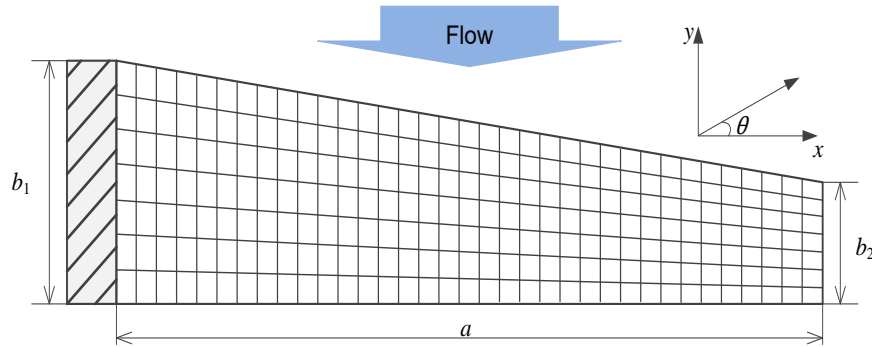


Figure 2. The finite element mesh model.

2. Analysis method

2.1. Bending stiffness of laminated plates

A symmetrically N -laminated plate is considered as shown in Fig. 1. The bending stiffness D_{ij} of this plate is given by

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3), \quad (i, j = 1, 2, 6)$$

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}l^4 + 2(Q_{12} + 2Q_{66})l^2m^2 + Q_{22}m^4 \\ \bar{Q}_{12} &= Q_{12}(l^4 + m^4) + (Q_{11} + Q_{22} - 4Q_{66})l^2m^2 \\ \bar{Q}_{22} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})l^2m^2 + Q_{22}l^4 \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})l^3m - (Q_{22} - Q_{12} - 2Q_{66})lm^3 \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})lm^3 - (Q_{22} - Q_{12} - 2Q_{66})l^3m \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})l^2m^2 + Q_{66}(l^4 + m^4) \end{aligned} \quad (1)$$

$$Q_{11} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}}, \quad Q_{22} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}},$$

$$Q_{12} = \frac{\nu_{TL}E_L}{1 - \nu_{LT}\nu_{TL}} = \frac{\nu_{LT}E_T}{1 - \nu_{LT}\nu_{TL}},$$

$$Q_{66} = G_{LT}, \quad l = \cos \theta, \quad m = \sin \theta$$

where E_L and E_T are the moduli of elasticity in the L and T directions, G_{LT} is the shear modulus, ν_{LT} and ν_{TL} are the major

$$\begin{aligned} a &= 1000 \text{ mm}, & b_1 &= 300 \text{ mm}, \\ b_2 &= 150 \text{ mm}, & t &= 0.19 \text{ mm} \\ E_L &= 152 \text{ GPa}, & E_T &= 8.00 \text{ GPa}, \\ G_{LT} &= 4.00 \text{ GPa} \\ \nu_{LT} &= 0.340, & \rho &= 1600 \text{ kg/m}^3 \end{aligned} \quad (2)$$

where the thickness of one ply is denoted by t .

In order to compare with optimized models, the benchmark model with $[45^\circ/0^\circ/-45^\circ/90^\circ]_{4s}$ lay-up is introduced here. As a result of aerodynamic analysis, V_F of the benchmark model is obtained as

$$V_{F \text{ required}} = 165 \text{ m/s} \quad (3)$$

3. Response surface method

RSM gives an approximate equation between responses y and predictor variables x . The present study employs the response surface equation written as

$$\begin{aligned} y &= \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 \\ &+ \beta_5x_5 + \beta_6x_6 + \beta_7x_7 + \beta_8x_8 + \beta_9x_9 \\ &+ \beta_{10}x_{10} + \beta_{11}x_{11} + \beta_{12}x_{12} + \beta_{13}x_{13} \\ &+ \beta_{14}x_{14} + \beta_{15}x_{15} + \beta_{16}x_{16} + \beta_{17}x_{17} \\ &+ \beta_{18}x_{18} + \beta_{19}x_{19} + \beta_{20}x_{20} \end{aligned}$$

with

$$\begin{aligned}
 x_1 &= D_{11}, x_2 = D_{16}, x_3 = D_{22}, x_4 = D_{26}, \\
 x_5 &= D_{66}, x_6 = D_{11}^2, x_7 = D_{16}^2, x_8 = D_{22}^2, \\
 x_9 &= D_{26}^2, x_{10} = D_{66}^2, \\
 x_{11} &= D_{11} \times D_{16}, x_{12} = D_{11} \times D_{22}, \\
 x_{13} &= D_{11} \times D_{26}, x_{14} = D_{11} \times D_{66}, \\
 x_{15} &= D_{16} \times D_{22}, x_{16} = D_{16} \times D_{26}, \\
 x_{17} &= D_{16} \times D_{66}, x_{18} = D_{22} \times D_{26}, \\
 x_{19} &= D_{22} \times D_{66}, x_{20} = D_{26} \times D_{66}
 \end{aligned} \tag{4}$$

Table 1. Values and F-statistics of the coefficients.

	Values	F-statistics
b_0	154.0	1996270
b_1	20.01	7330
b_2	-40.41	7680
b_3	4.933	567.2
b_4	22.92	2869
b_5	49.51	21543
b_6	5.388	184.0
b_7	-168.8	26486
b_8	3.867	142.9
b_9	-72.15	6029
b_{10}	-8.355	155.7
b_{11}	-9.854	85.91
b_{12}	4.720	96.53
b_{13}	13.07	221.0
b_{14}	4.470	37.65
b_{15}	-13.58	225.2
b_{16}	112.6	7619
b_{17}	9.373	45.33
b_{18}	-2.947	9.587
b_{19}	-	0.02932
b_{20}	-5.311	16.86

where y , x and β represent flutter speed, bending stiffness and unknown coefficients respectively. Here, D_{12} is omitted due to linear relationship between D_{12} and D_{66} . The linear multiple regression model is expressed as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{5}$$

$$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix}$$

$$\boldsymbol{\beta} = \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{Bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix}$$

where n is the number of sampling point, k is the number of variables, and ε is an error vector. The unbiased estimator \mathbf{b} of $\boldsymbol{\beta}$ is obtained by the least square method as follows [8].

$$\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} \tag{6}$$

$$\mathbf{b} = \begin{Bmatrix} b_0 \\ b_1 \\ \vdots \\ b_k \end{Bmatrix}$$

The RSM equation is created along the following three steps.

1. The database of V_F derived random bending stiffness of the wing model shown in Fig. 2 is created by aerodynamic analysis. In order to create this database, 4500 times of FEA trials are conducted. Then, the data quantity is reduced to 3923 due to erasing of overlapped data, and the values of bending stiffness are normalized based on the maximum and minimum value.
2. The unbiased estimator \mathbf{b} is given by Eq. (6) for the database, where $k = 20$ and $n = 3923$.
3. F-test is performed to verify the validity of \mathbf{b} with significance level 0.01 as shown in Table 1. Finally, the response surface equation is written as follows.

$$\begin{aligned}
 y &= 154.0 + 20.01x_1 - 40.41x_2 + 4.933x_3 \\
 &+ 22.92x_4 + 49.51x_5 + 5.388x_6 - 168.8x_7 \\
 &+ 3.867x_8 - 72.15x_9 - 8.355x_{10} - 9.854x_{11} \\
 &+ 4.720x_{12} + 13.07x_{13} + 4.470x_{14} - 13.58x_{15} \\
 &+ 112.6x_{16} + 9.373x_{17} - 2.947x_{18} - 5.311x_{20}
 \end{aligned} \tag{7}$$

The adjusted determination coefficient is 0.8915 and indicates excellent accuracy of this equation.

4. Optimization method

Distributed Genetic Algorithm (DGA) [7] is adopted to solve the stacking sequence optimization problem as follows.

- [Objective function] Minimize n ($1 \leq n \leq 32$)
- [Subject to] $V_F \geq V_{F \text{ required}}$
- Symmetric lamination
- Less than 3 consecutive lamination of the same θ (8)
- [Design variable] θ_i ($i = 1, 2, \dots, n/2$)
- $-85 \leq \theta \leq 90$ ($\Delta\theta = 5^\circ$)

In this DGA, a large population is distributed to some islands labeled by each number of lamination. Thus, weight reduction is performed by selection of labeled islands as the number of lamination. At the same time, lay-up configurations in each island are optimized. Fitness H is evaluated independently in each islands as follows.

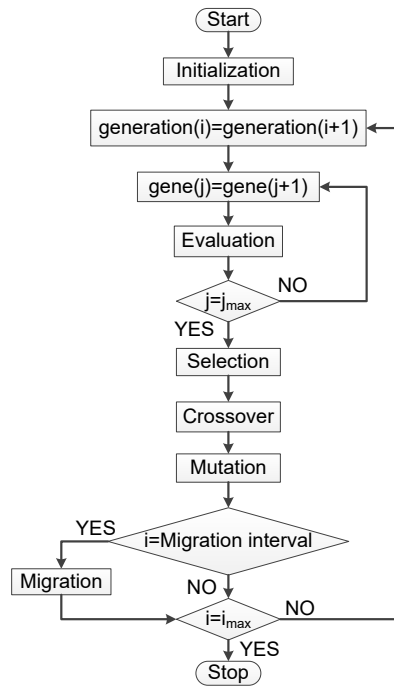


Figure 3. Flowchart of DGA

$$H = V_F \tag{9}$$

Moreover, DGA has the unique operation named as migration. This operation exchanges some individuals among some islands, and it contributes to keep diversity of populations. As DGA parameters, the number of generation is 100, total population is 150, crossover rate is 0.9, mutation rate is 0.1, the number of islands in one generation is two and migration interval is 10. This optimization adopts elitist strategy, roulette wheel selection and 2-point crossover. Fig. 3 illustrates flowchart of DGA. Table 2 represents decoding in DGA.

The optimization described above is performed in two cases Opt. A and Opt. B. In Opt. A, V_F is calculated by FEA in each time. On the other hand, estimated value y of Eq. (7) is substituted for V_F to reduce computational load in Opt. B.

5. Results and discussions

Table 3 shows the comparison of the benchmark model and two optimum models. The optimum solutions in Opt. A and B present 12.5 % reduced weight from the benchmark model and keep enough V_F . In Opt. B, $y = 179$ m/s is derived as the optimum solution with estimation equation. The calculated value by FEA for the obtained solution is 176 m/s, and the error is only 1.70 % in spite of 70 % reduction of FEA trials. Since the lay-up configurations of the optimum models contain the angles strong to torsion such as 45° , -45° , 40° and -40° , it is revealed that the optimum solutions have high D_{66} values, resulting in better resistance to flutter. Moreover, the values of D_{16} decrease compared to the benchmark model. These results indicate the importance of

Table 2. Decoding

Senary	Decimal	Phenotype (Fiber orientation angle)
00	0	0
01	1	5
11	7	10
10	6	15
12	8	20
02	2	25
22	14	30
21	13	35
20	12	40
23	15	45
13	9	50
03	3	55
33	21	60
32	20	65
31	19	70
30	18	75
34	22	80
24	16	85
14	10	90
04	4	-85
44	28	-80
43	27	-75
42	26	-70
41	25	-65
40	24	-60
45	29	-55
35	23	-50
25	17	-45
15	11	-40
05	5	-35
55	35	-30
54	34	-25
53	33	-20
52	32	-15
51	31	-10
50	30	-5

D_{66} and unimportance of D_{16} as observed from the F-statistics of b_5 and b_7 in Table 1.

Figures 4-6 show natural frequencies and vibration mode shapes for Bench mark model and both solutions of Opt. A and B. It is known from Figs. 4-6 that the natural frequencies for the bending modes decrease and torsional modes increase compared with the benchmark model since the lay-up configurations are strengthened against torsion.

6. Conclusions

The present study proposed the optimization method to reduce weight of wing shaped composite plates by using DGA and RSM. This method yielded the excellent optimum solution precisely in less computational time compared to the case without RSM. The obtained results revealed that assigning bending stiffness to predictor variables is effective to derive V_F as responses for RSM. In addition, the tested coefficients of the response surface equation showed the significant factors to keep enough V_F .

Table 3. Comparison between three models

	The benchmark model	Opt. A	Opt. B
Lay-up	[45/0/-45/90] _{4s}	[-40/45/45/-45/-30/40/-40/35/-65/-50/-5/-40/0/-65] _s	[-40/40/45/-40/-40/45/-40/30/-40/45/10/45/65/25] _s
V_F [m/s]	165	175	176
D_{11} [N·m]	1197	708.6	731.5
D_{12} [N·m]	385.5	441.0	452.1
D_{16} [N·m]	19.87	-56.13	-32.49
D_{22} [N·m]	1149	498.0	452.9
D_{26} [N·m]	19.87	-19.22	-1.610
D_{66} [N·m]	409.2	456.9	468.0
The number of layers	32	28	28
Weight [kg]	2.189	1.915	1.915
Reduced weight [%]	Reference value	12.5	12.5
aerodynamic FEA trials	-	15000	4500
Reduced FEA trials [%]	-	Reference value	70

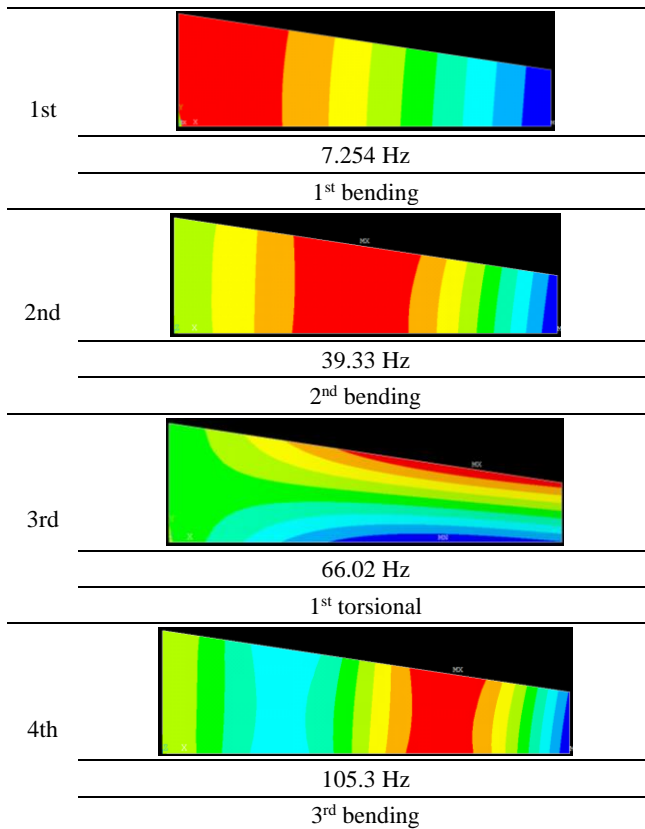


Figure 4. Vibration mode shapes and natural frequencies for benchmark model

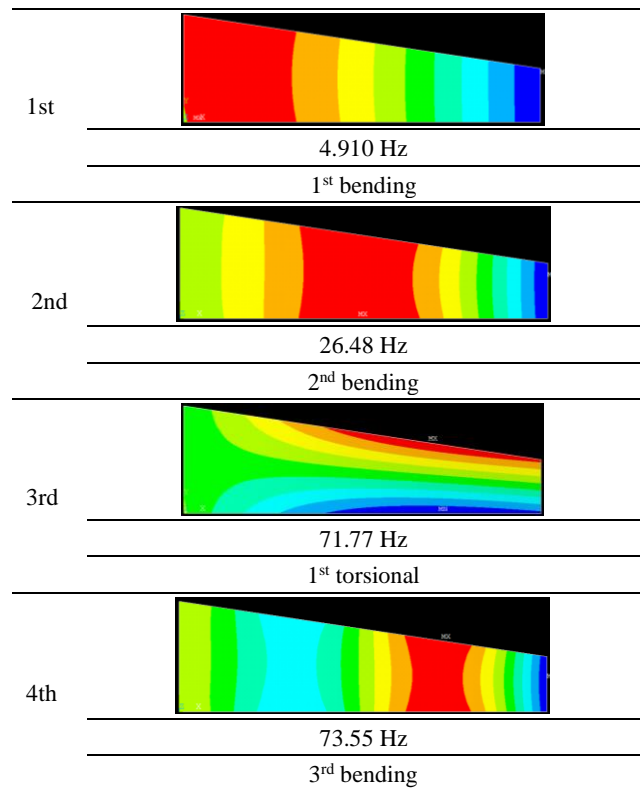


Figure 5. Vibration mode shapes and natural frequencies for Opt A

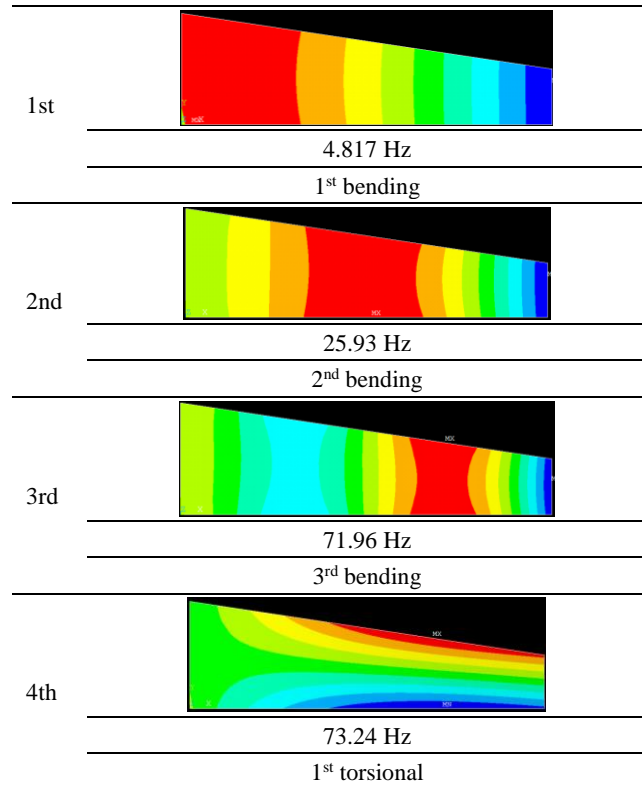


Figure 6. Vibration mode shapes and natural frequencies for Opt B

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